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CURVES, VERTICES, KNOTS AND SUCH.

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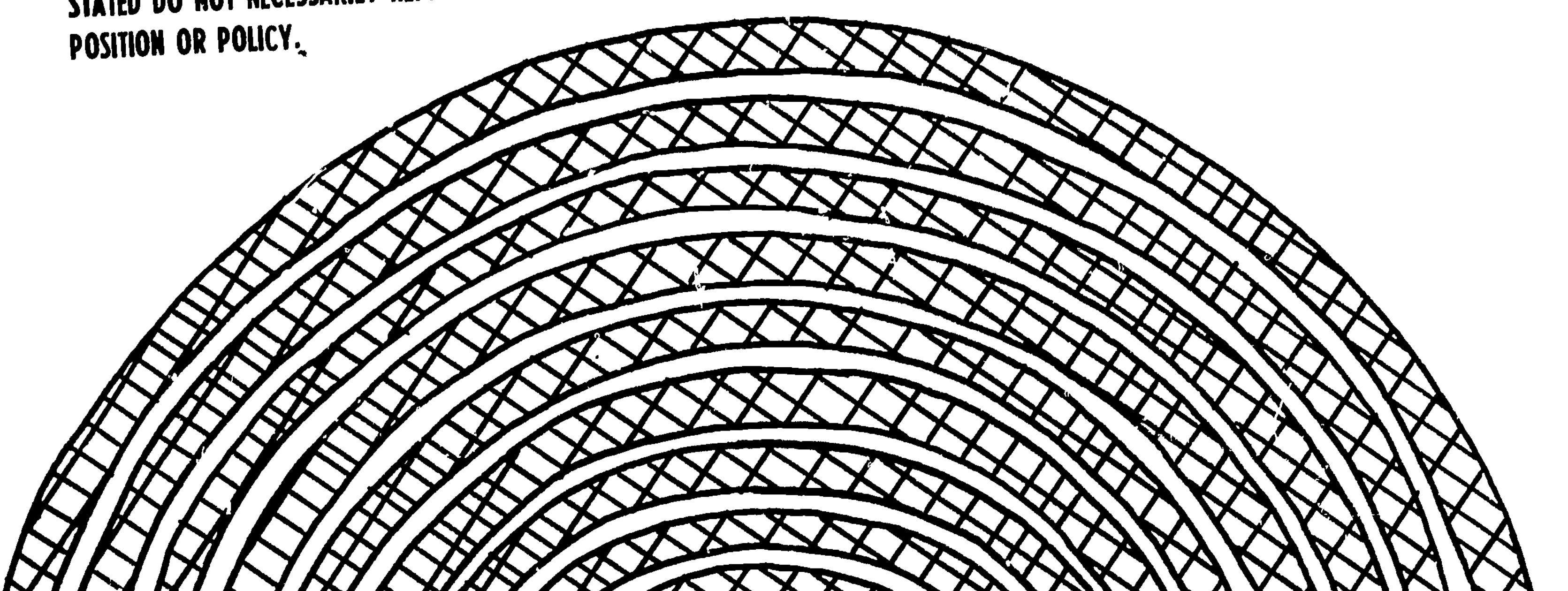
THIS BOOKLET, ONE OF A SERIES, HAS BEEN DEVELOPED FOR THE PROJECT, A PROGRAM FOR MATHEMATICALLY UNDERDEVELOPED PUPILS. A PROJECT TEAM, INCLUDING INSERVICE TEACHERS, IS BEING USED TO WRITE AND DEVELOP THE MATERIALS FOR THIS PROGRAM. THE MATERIALS DEVELOPED IN THIS BOOKLET INCLUDE SUCH CONCEPTS AS (1) SIMPLE CLOSED CURVES, (2) NETWORKS, (3) MAP COLORING, (4) TOPOLOGICAL TRANSFORMATIONS, (5) THREE DIMENSIONAL TOPOLOGY, AND (6) KNOTS. ACCOMPANYING THESE BOOKLETS WILL BE A "TEACHING STRATEGY BOOKLET" WHICH WILL INCLUDE A DESCRIPTION OF TEACHER TECHNIQUES, METHODS, SUGGESTED SEQUENCES, ACADEMIC GAMES, AND SUGGESTED VISUAL MATERIALS. (RP)

CURVES, VERTICES, KNOTS



AND SUCH

ESEA Title III

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CURVES, VERTICES, KNOTS AND SUCH

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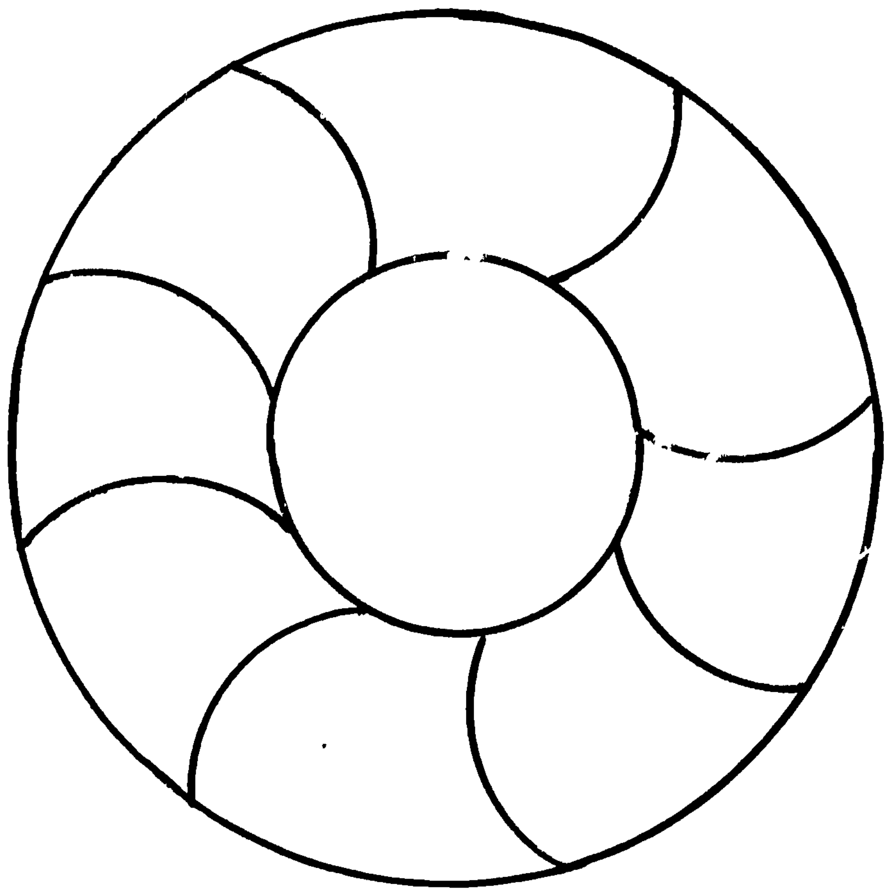
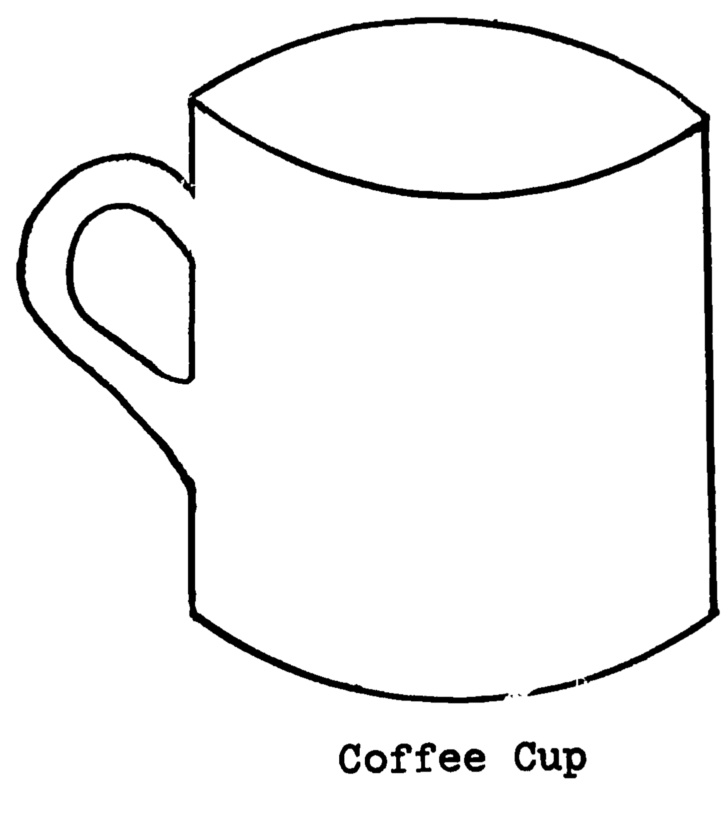
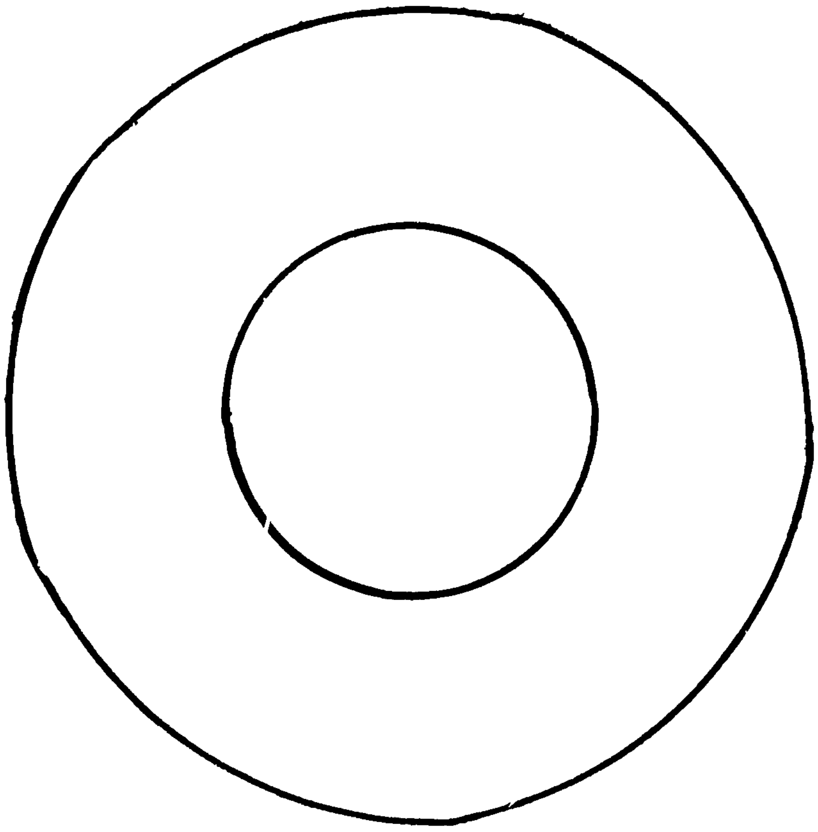


Doughnut

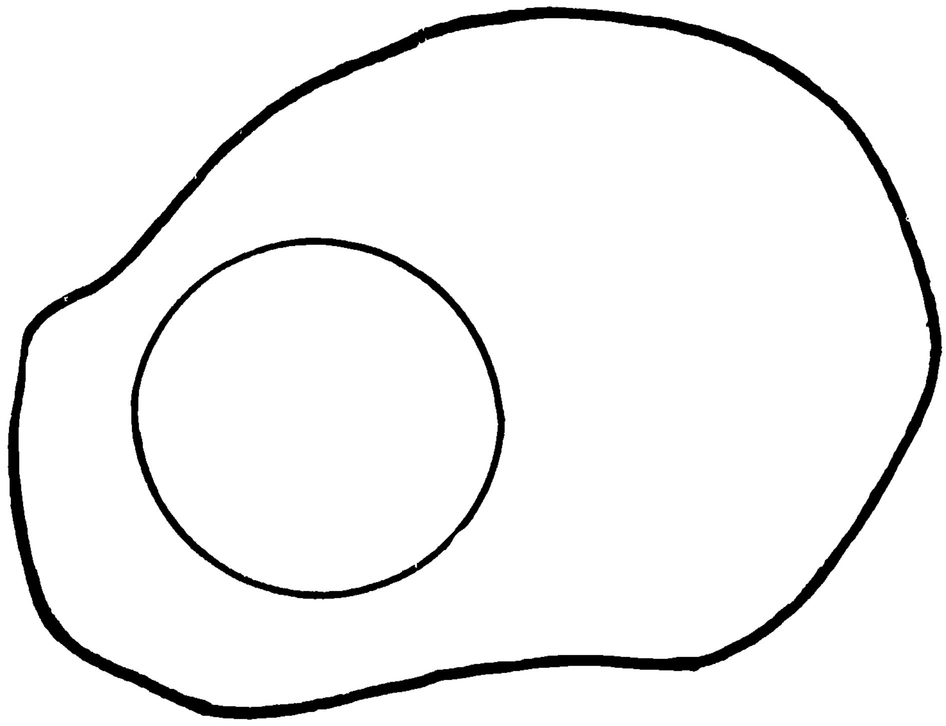
Can you see anything similar between these two?



Coffee Cup



From this doughnut



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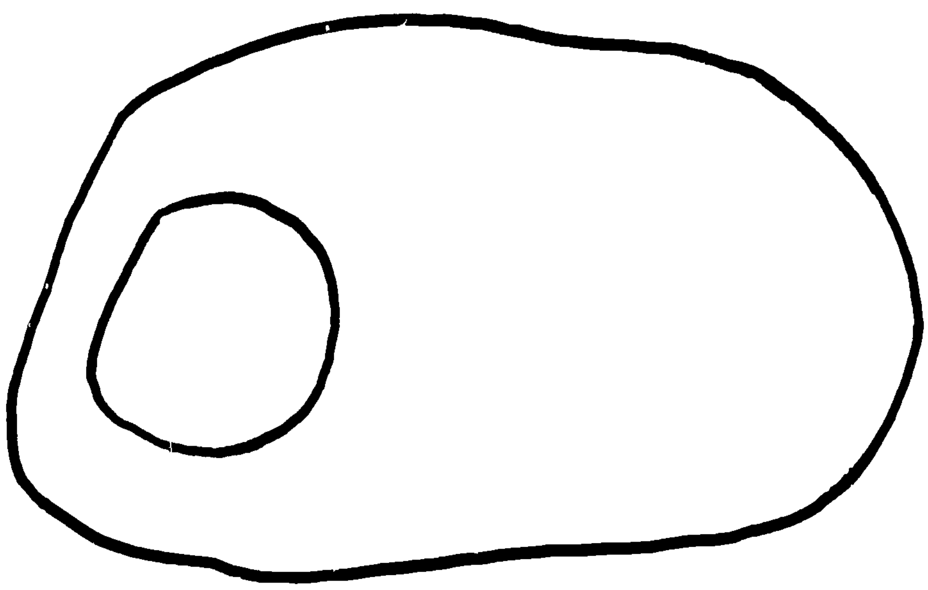

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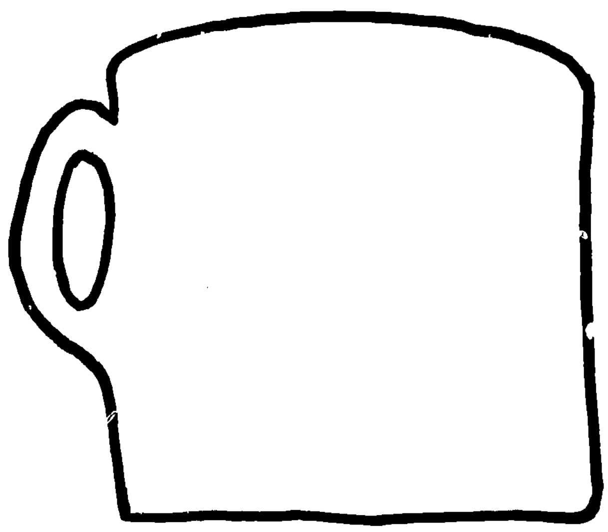
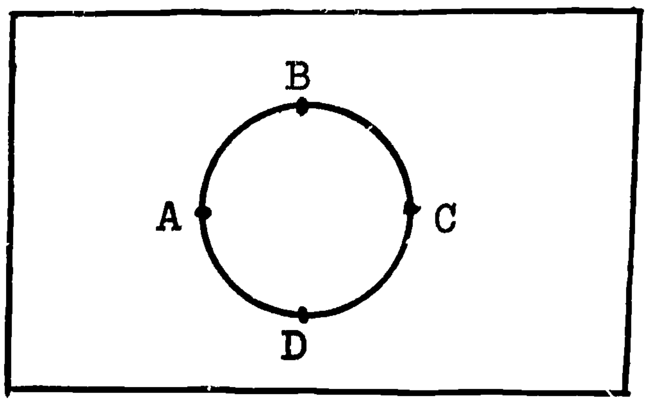


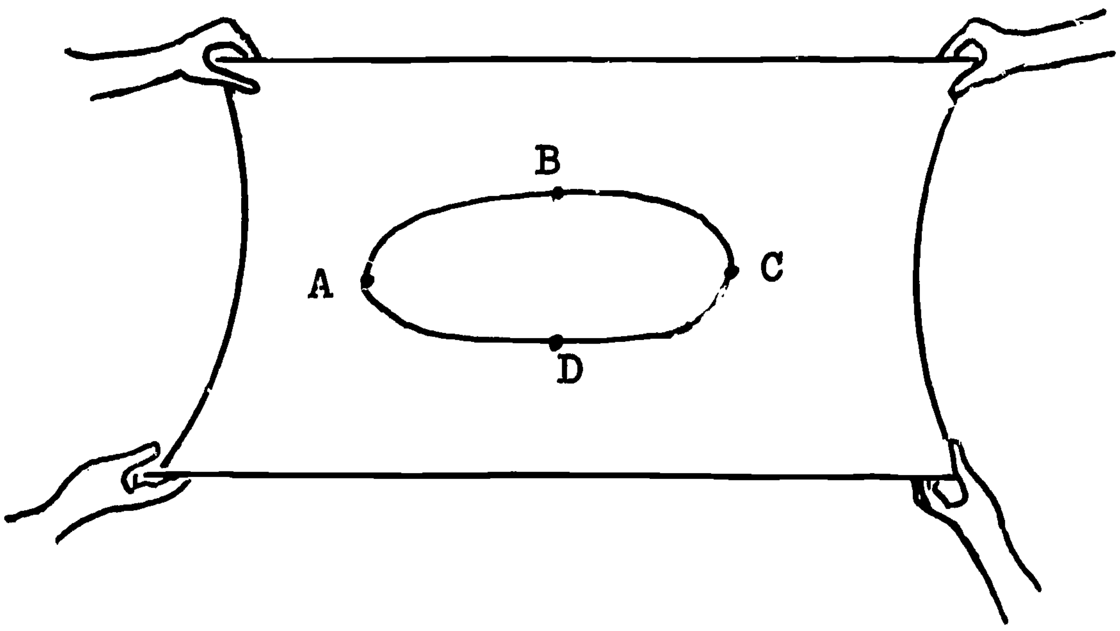
To this coffee cup

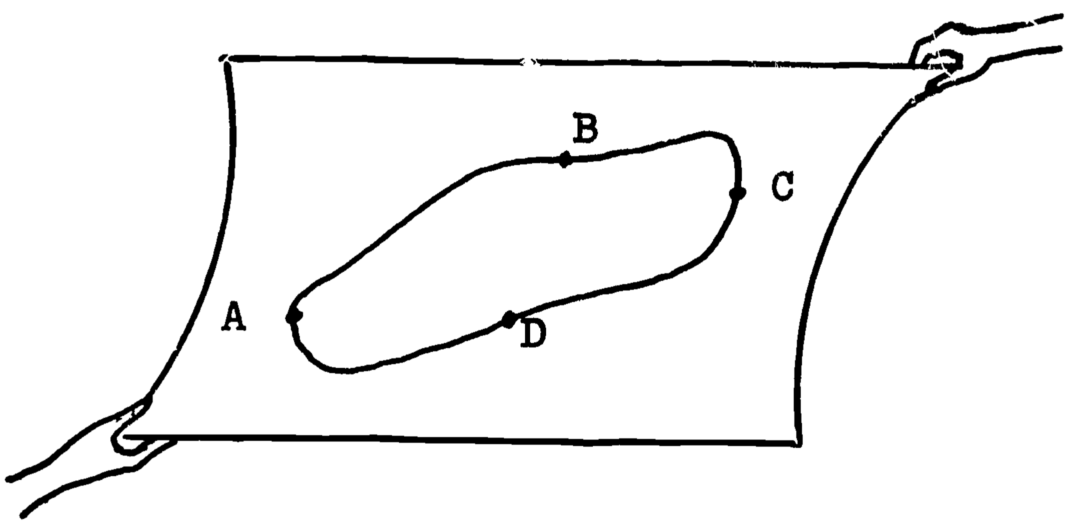
From a doughnut to a coffee cup is a regional transformation.

DOUGHNUT OR COFFEE CUP--A REGIONAL TRANSFORMATION

If a circle is drawn on a rubber sheet, is it still a circle if the sheet is pulled out of shape? What about the circumference of the circle?







The points of the rubber sheet circle have undergone a surface transformation.

Simple Closed Curves

Topological Surfaces

A simple closed curve divides a plane into three sets of points.

First set of points--inside curve

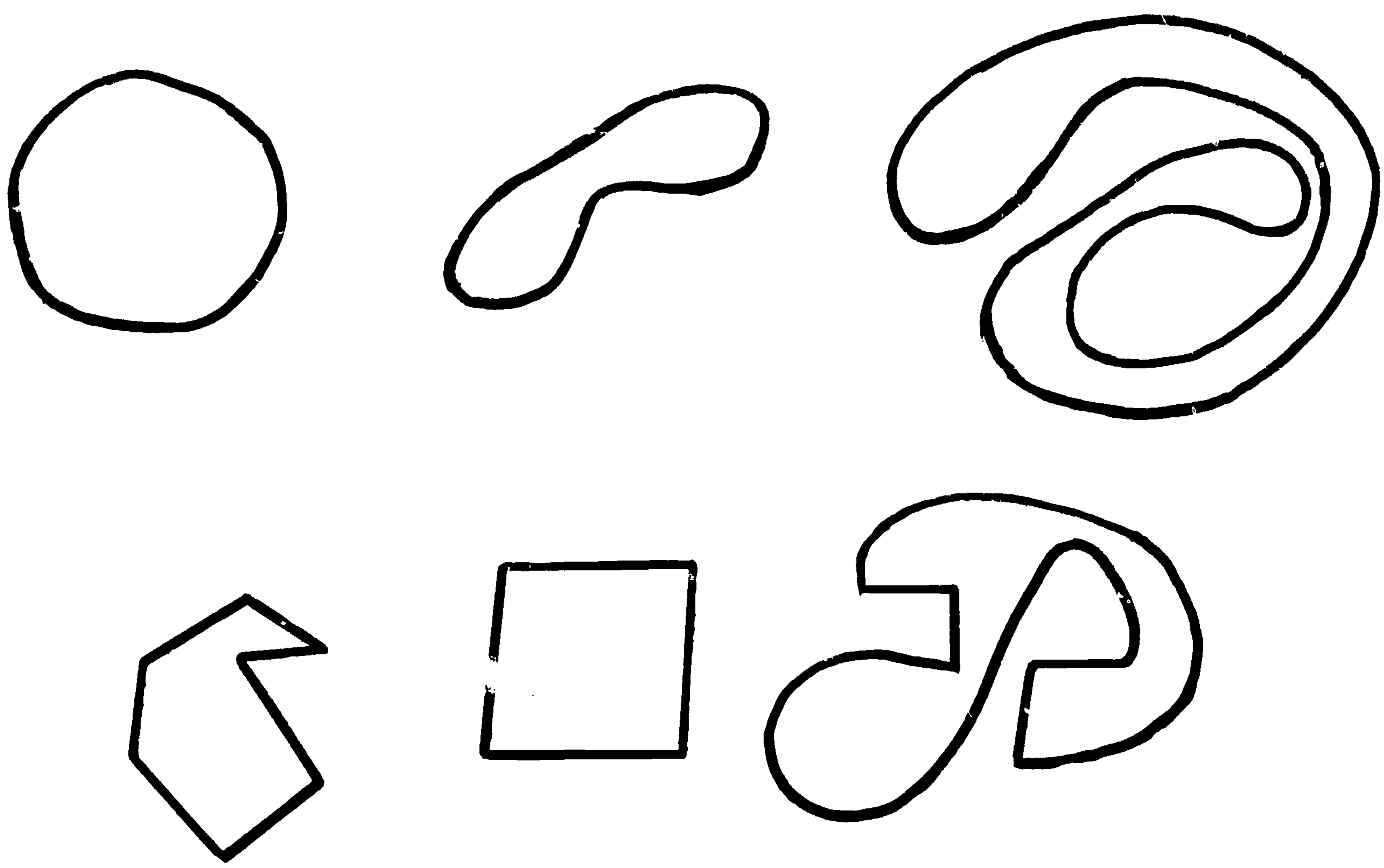
Second set of points--curve itself

Third set of points--all points outside curve

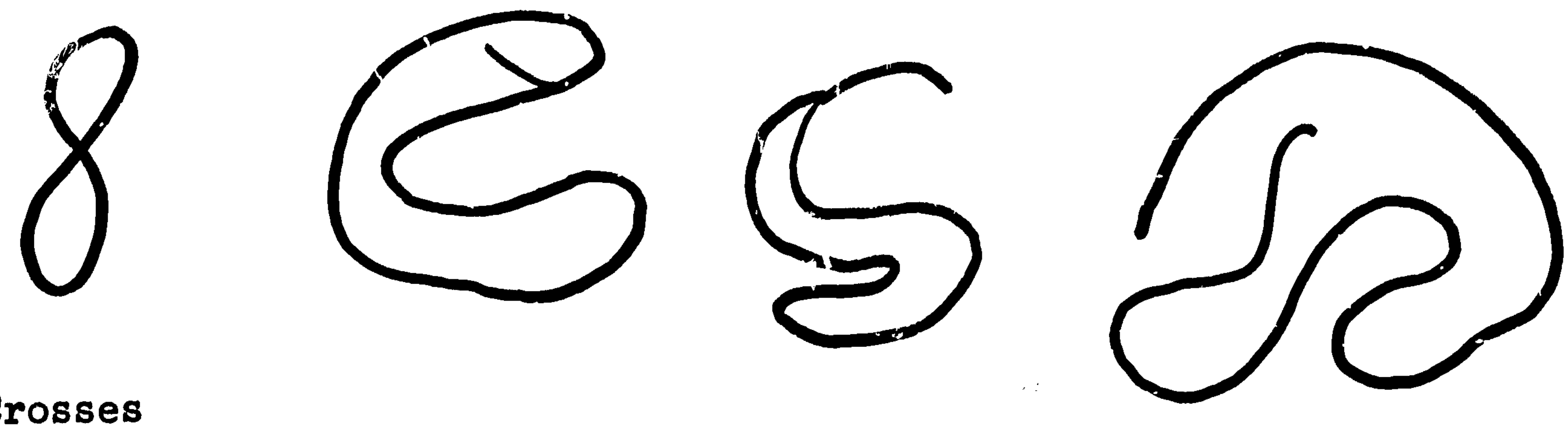
Simple closed curves are easy to draw. To make one you must:

Never lift your pencil from the paper.
Never cross the curve.
Return to your starting point.

These are examples of simple closed curves:



These curves are not simple closed curves. Under each write the reason it is not.



Crosses itself

A simple closed curve must have only one inside and one outside.

See if you can draw some simple closed curves.

Does a simple closed curve have just one inside area?

In these simple closed curves can you go from any point inside the curve to another point inside the curve without ever crossing the curve itself? Try. Draw a line from point A to point B using any path. It will not necessarily be a straight line, but it must be <u>continuous</u> and not ever cross the curve itself.

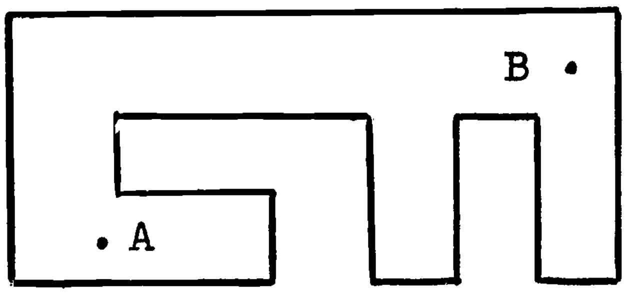


Figure 1

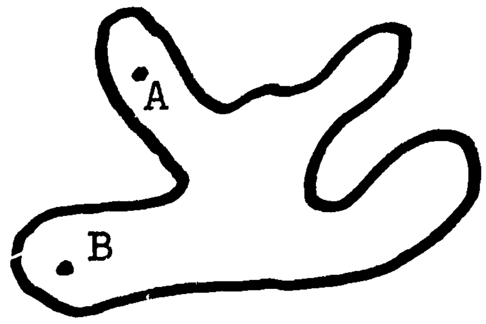


Figure 2

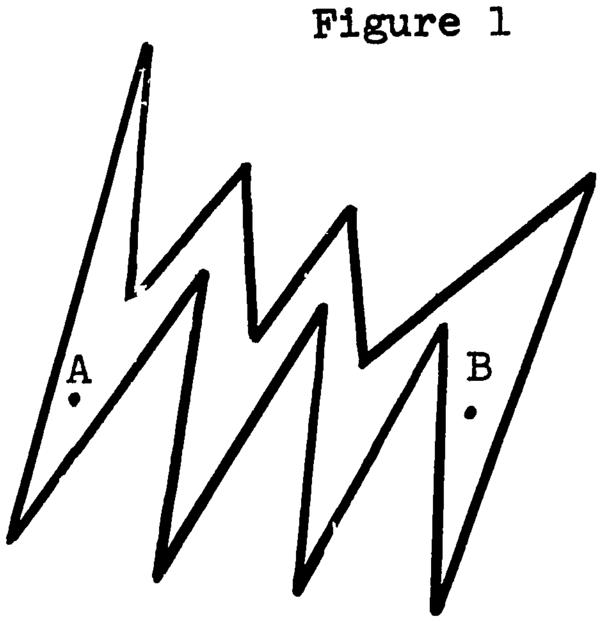


Figure 3

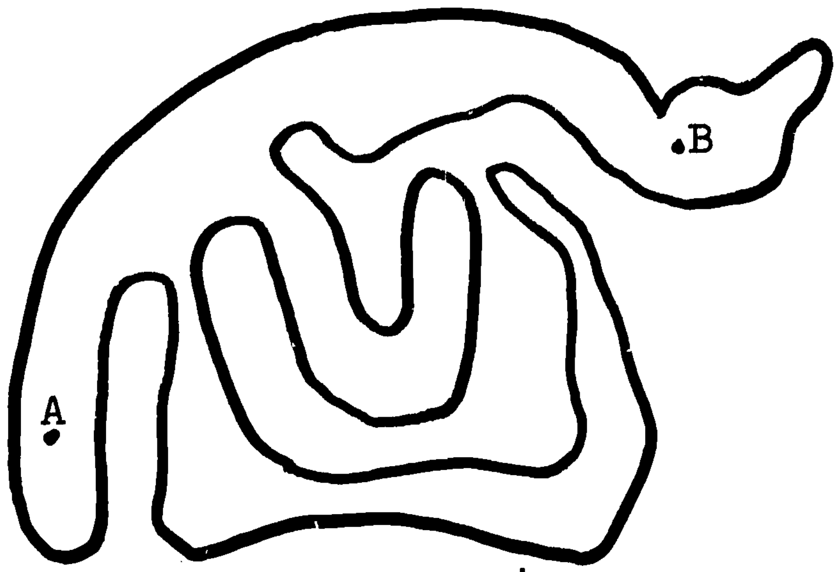


Figure 4

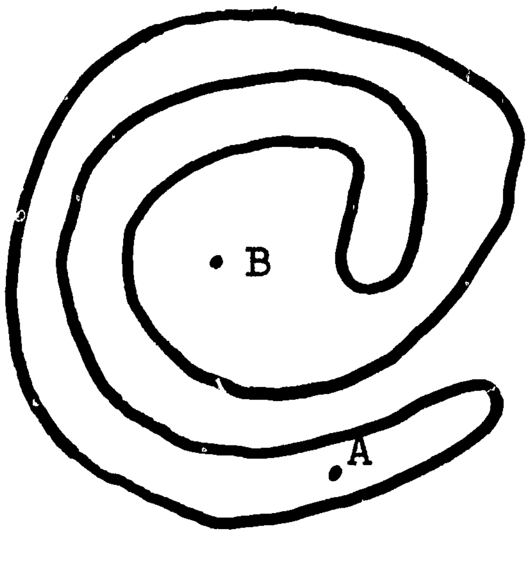


Figure 5

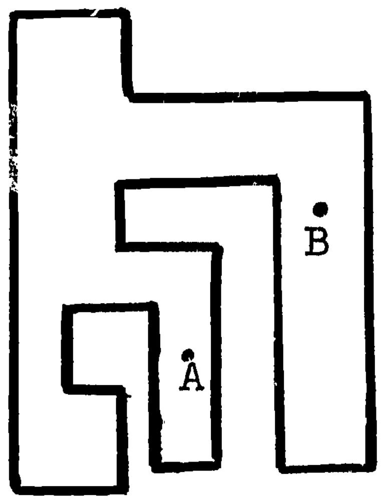


Figure 6

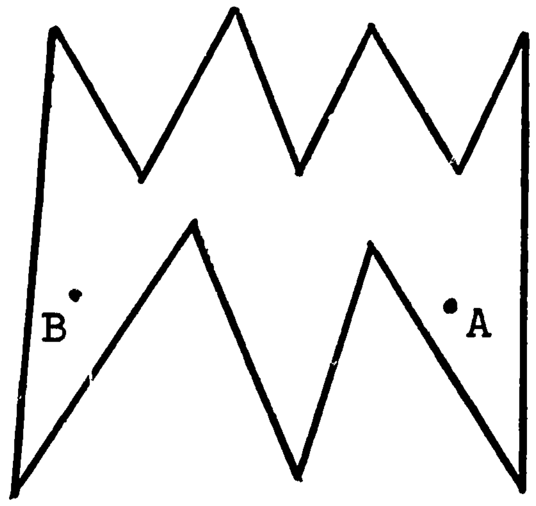


Figure 7

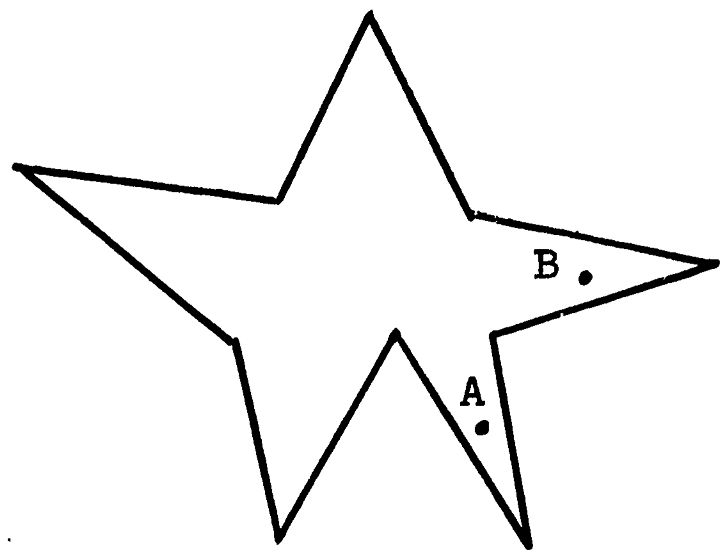


Figure 8

Now go back and draw a <u>straight</u> line from point A to point B. Count the number of times you crossed the curve itself.

Do you feel that Figures 1, 3, 6, 7, and 8 should not be called curves? In mathematics a straight line is considered to be a special form of a curve. Any path between two points is a curve. The shortest path between two points is a curve we call a <u>straight line</u>.

Fill in the following table with the number of lines crossed when a straight line is drawn from point A to point B.

Figure 1	
Figure 2	
Figure 3	
Figure 4	
Figure 5	
Figure 6	
Figure 7	
Figure 8	

In your counting did you always get an even number? Why?

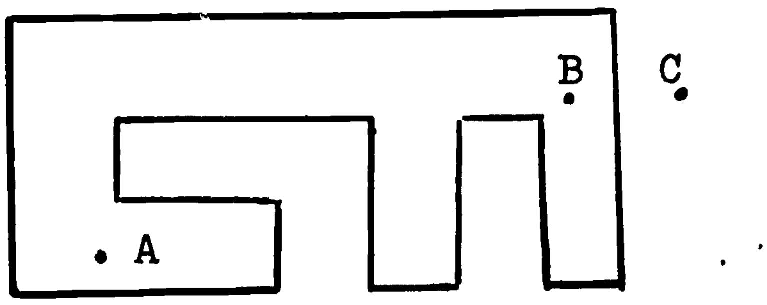


Figure 1

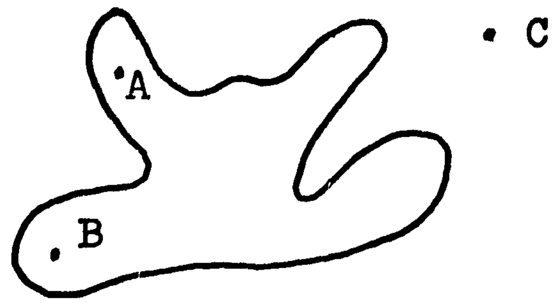


Figure 2

Draw a straight line from point A to C and point B to point C on each curve and count the number of times you crossed the curve. Complete the following table:

Number of Times Curve is Crossed.

Figure 1	A to C	
	B to C	
Figure 2	A to C	
	B to C	
Figure 3	A to C	
	B to C	
Figure 4	A to C	
	B to C	
Figure 5	A to C	
	B to C	
Figure 6	A to C	
	B to C	
Figure 7	A to C	
	B to C	
Figure 8	A to C	
	B to C	

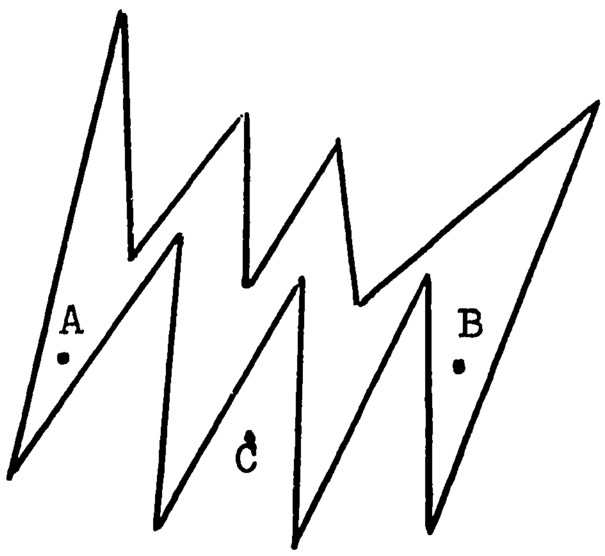


Figure 3

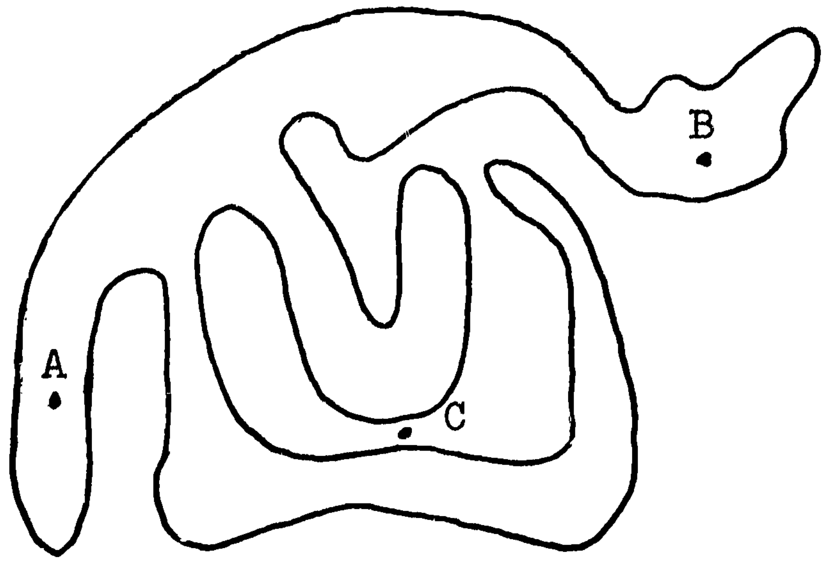


Figure 4

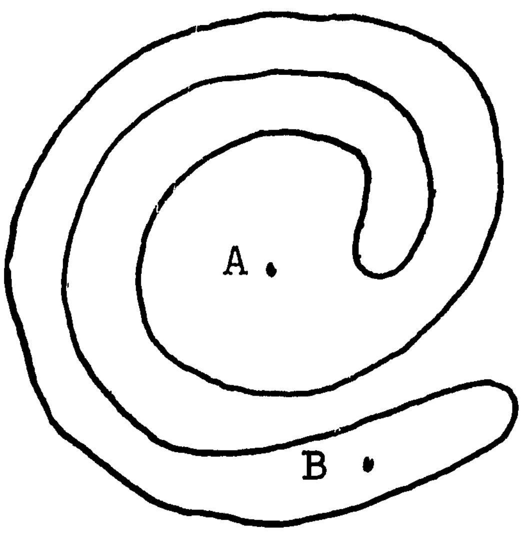


Figure 5

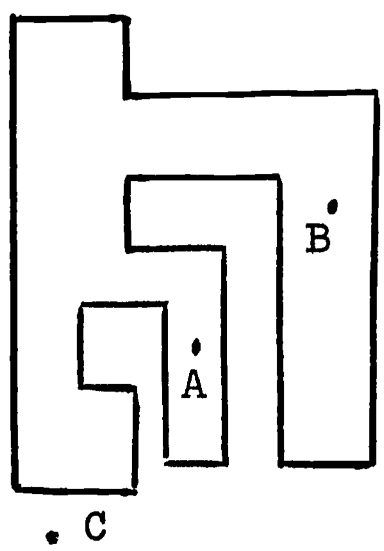


Figure 6

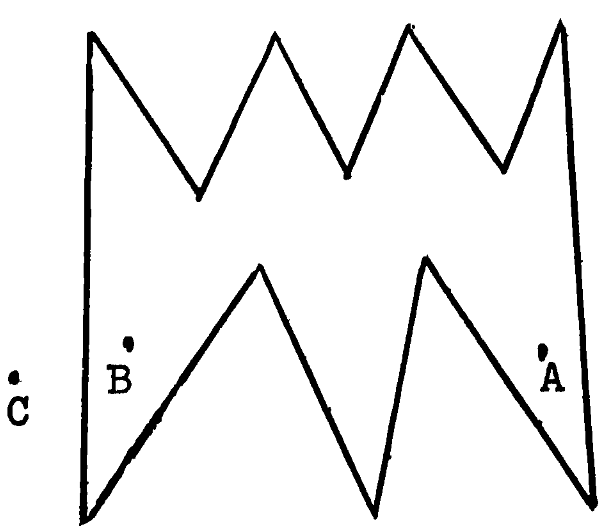


Figure 7

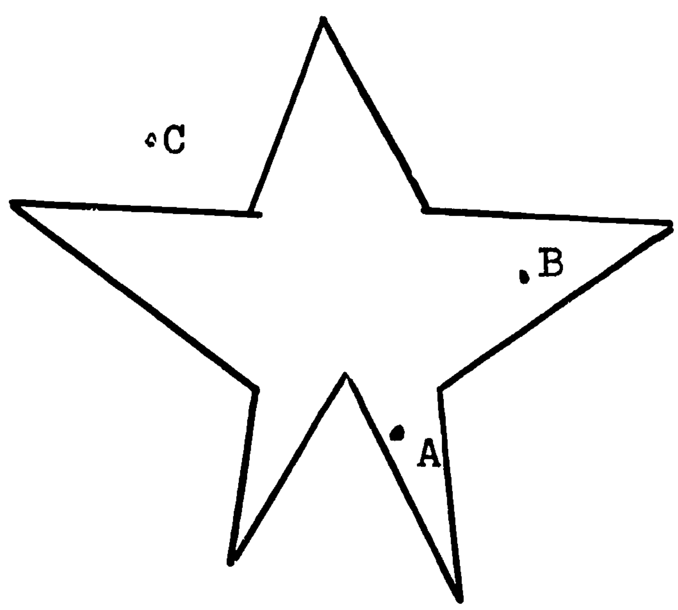


Figure 8


What shape is this?
Man, it's a circle.
So, what do you call this?
Every point of the circle is still there.
You're so smart, what's this?
It still has the same points the circle has.
How about your points now?
They're still there.
I suppose every point is still there?
Not any more. Now two points are together.
You can't call all these circles. What are they?
All except the figure 8 one are called simple closed curves.

If you go from this room in the school to another room in the school, how many doorways do you go through? ______

What if you continued to go from this room to many rooms but always ended up inside a room? Would you have passed through an even or an odd number of doorways? ______

Could we guess a way to tell always whether a point is inside a curve or outside just by drawing a straight line from the point inside to a point definitely outside?

If it is inside, a straight line from the inside point to a point directly outside will cross an __________ number of lines. (even) (odd)

From all this we could say a SIMPLE CLOSED CURVE is:

> A set of points which can be put into one-to-one correspondence with the points of a circle in such a way that the correspondence is continuous in both directions.
>
> (definition from a mathematics dictionary)

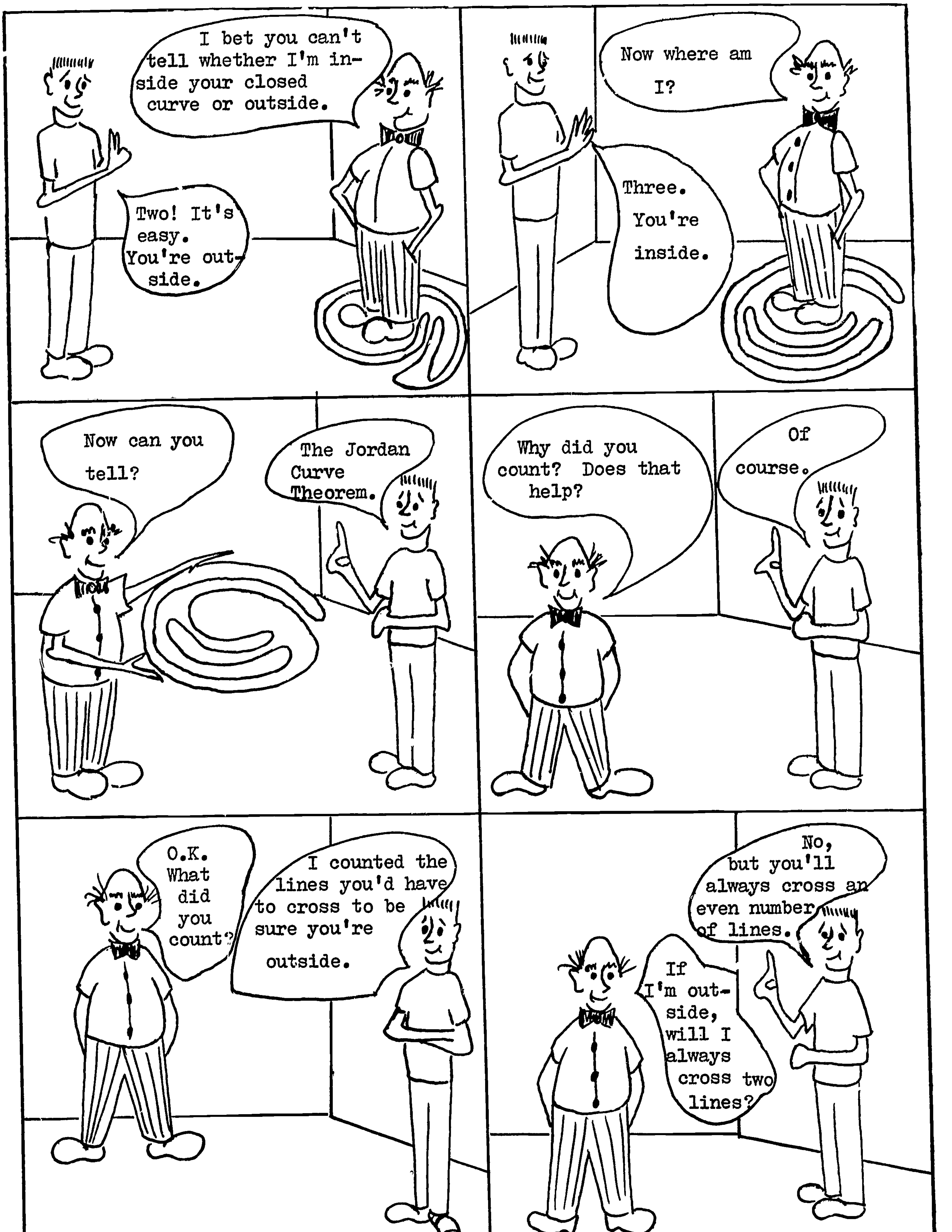
There is a theorem, called the Jordan Curve Theorem, that says:

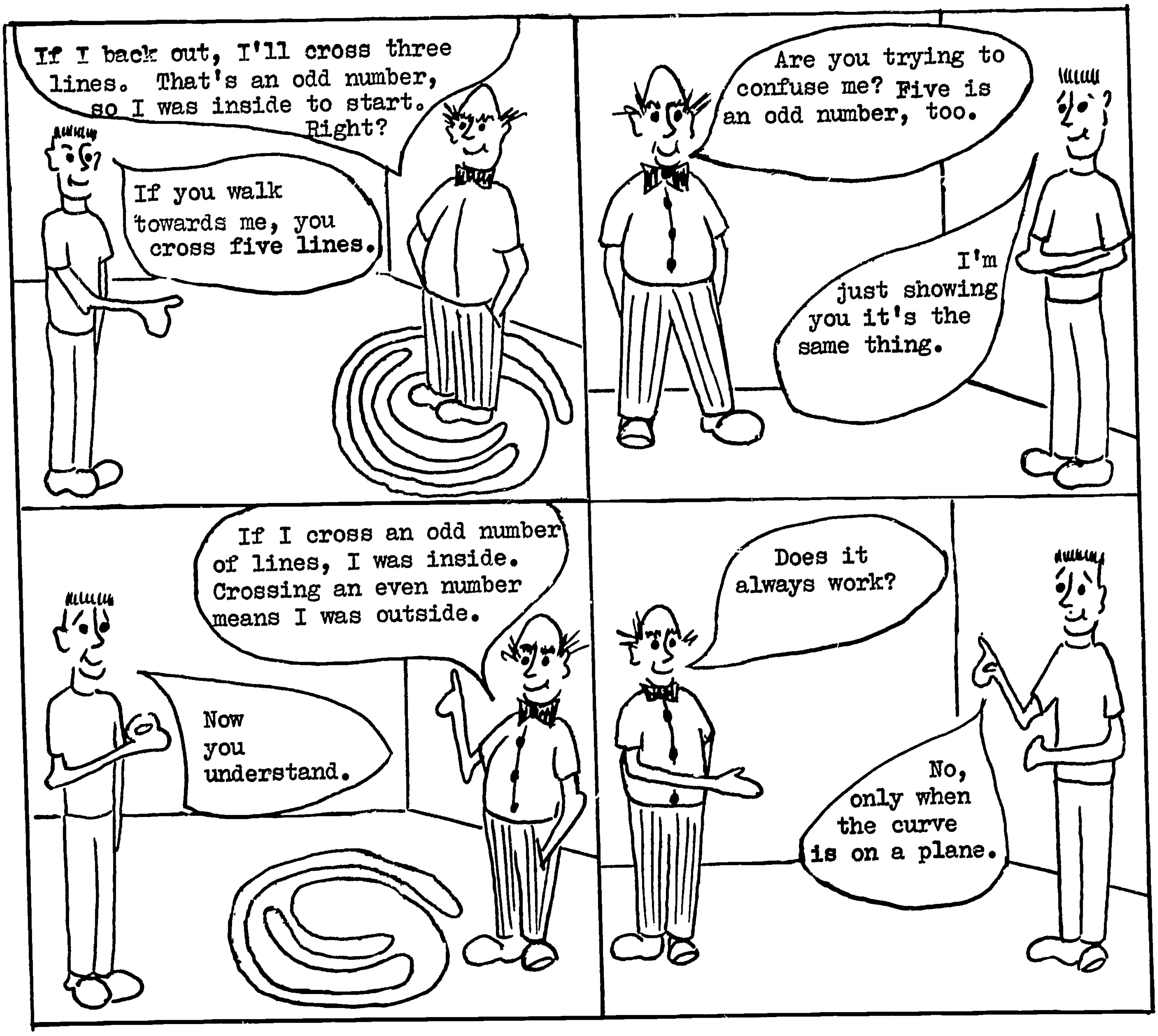
> A simple closed curve in a plane determines two regions, of which it is the common frontier.
>
> (Jordan made a mistake when he tried to prove this theorem, and the first correct proof was given by Veblen in 1905.)

On the next page is a curve called the Jordan Curve. See if you can use the information you have about inside and outside to determine whether this is a simple closed curve and also whether the points labeled A and B are inside the curve or outside.

Below is a curve called the Jordan Curve. See if you can use the information you have about inside and outside to determine whether this is a simple closed curve or not.

Are the points labeled A and B inside the curve or outside the curve?

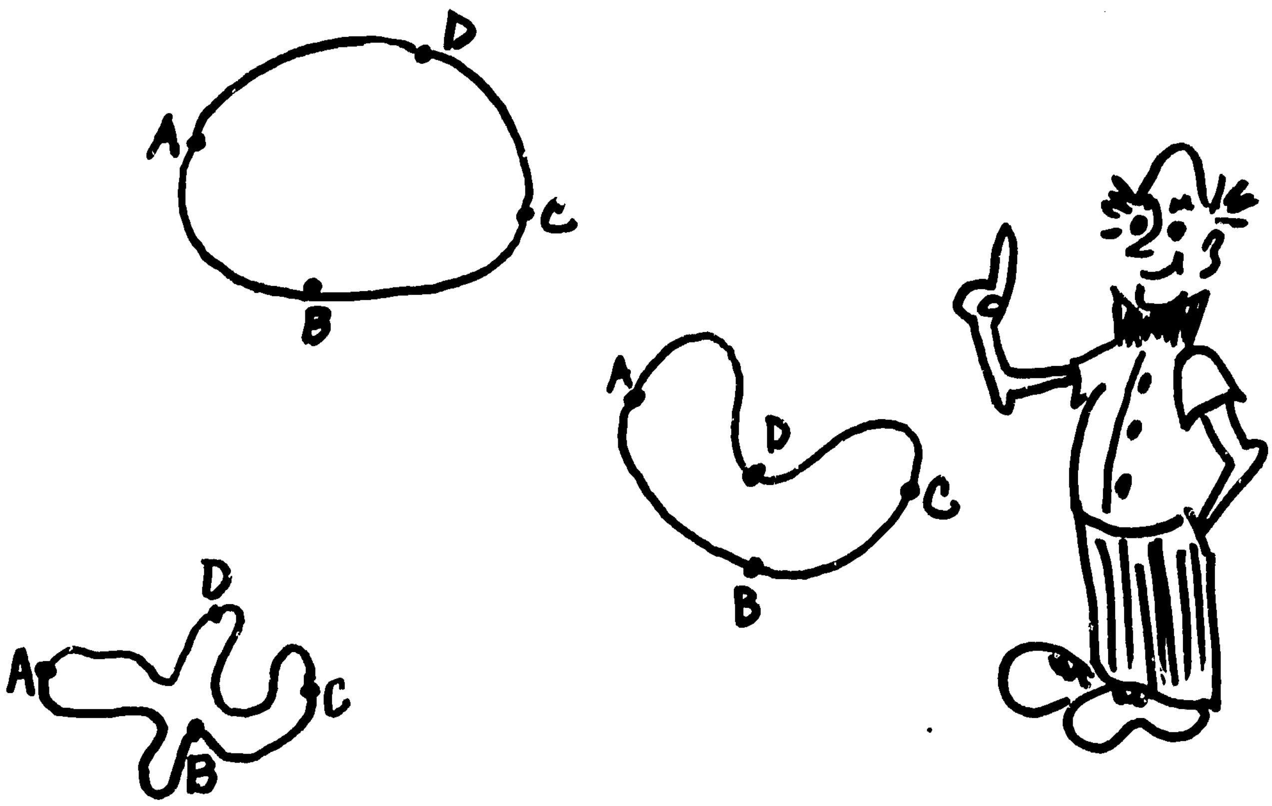

I bet you can't tell whether I'm inside your closed curve or outside.
Two! It's easy. You're outside.
Now where am I?
Three. You're inside.
Now can you tell?
The Jordan Curve Theorem.
Why did you count? Does that help?
Of course.
O.K. What did you count?
I counted the lines you'd have to cross to be sure you're outside.
If I'm outside, will I always cross two lines?
No, but you'll always cross an even number of lines.


If I back out, I'll cross three lines. That's an odd number, so I was inside to start. Right?
If you walk towards me, you cross five lines.
Are you trying to confuse me? Five is an odd number, too.
I'm just showing you it's the same thing.
If I cross an odd number of lines, I was inside. Crossing an even number means I was outside.
Now you understand.
Does it always work?
No, only when the curve is on a plane.

Simple closed curve by definition
(Check it for yourself.)

Let's go back to the very formal definition we copied from a dictionary and see what it means. Remember that topology is called rubber sheet geometry.

Suppose we had a rubber band. It could look like this:

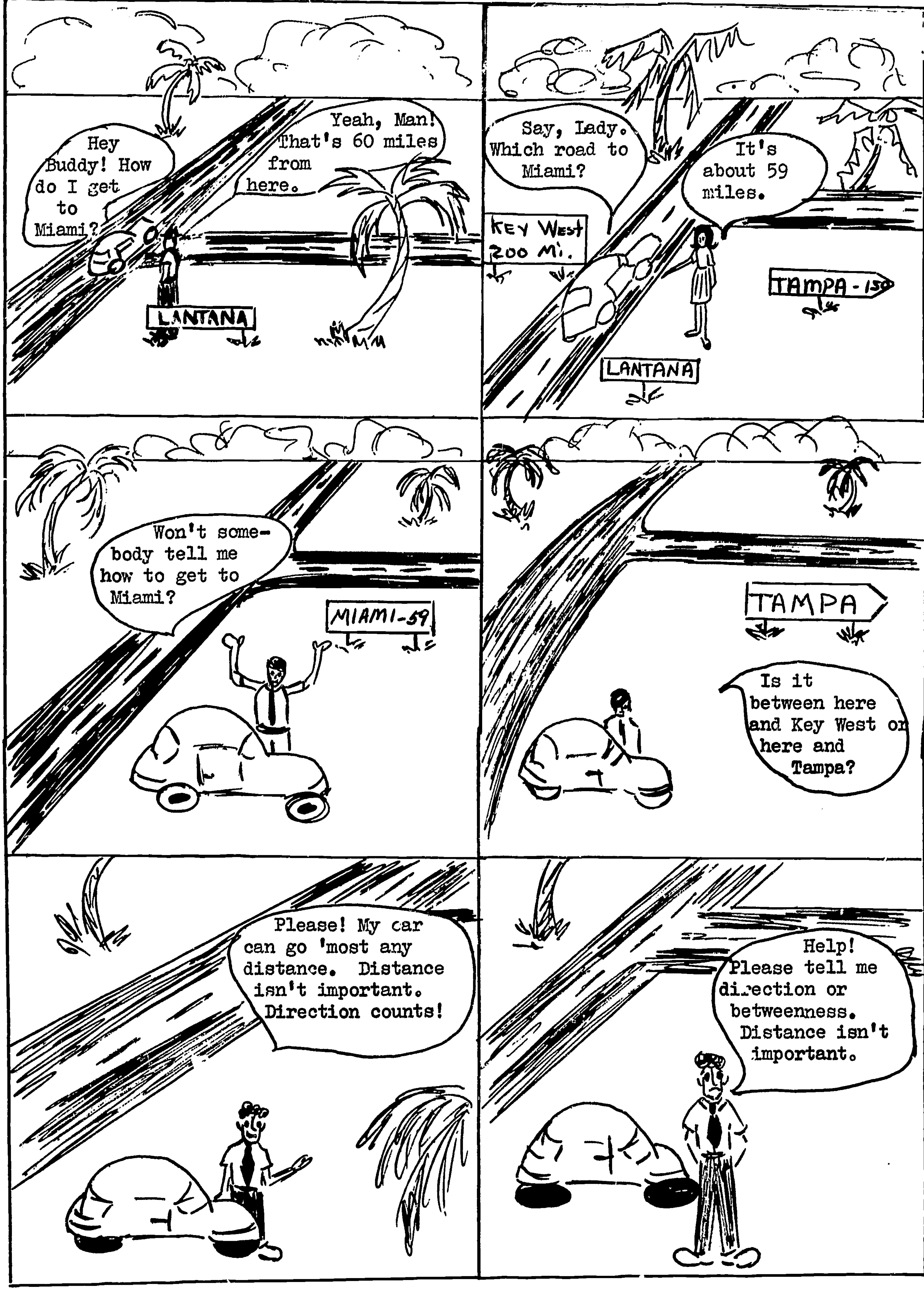


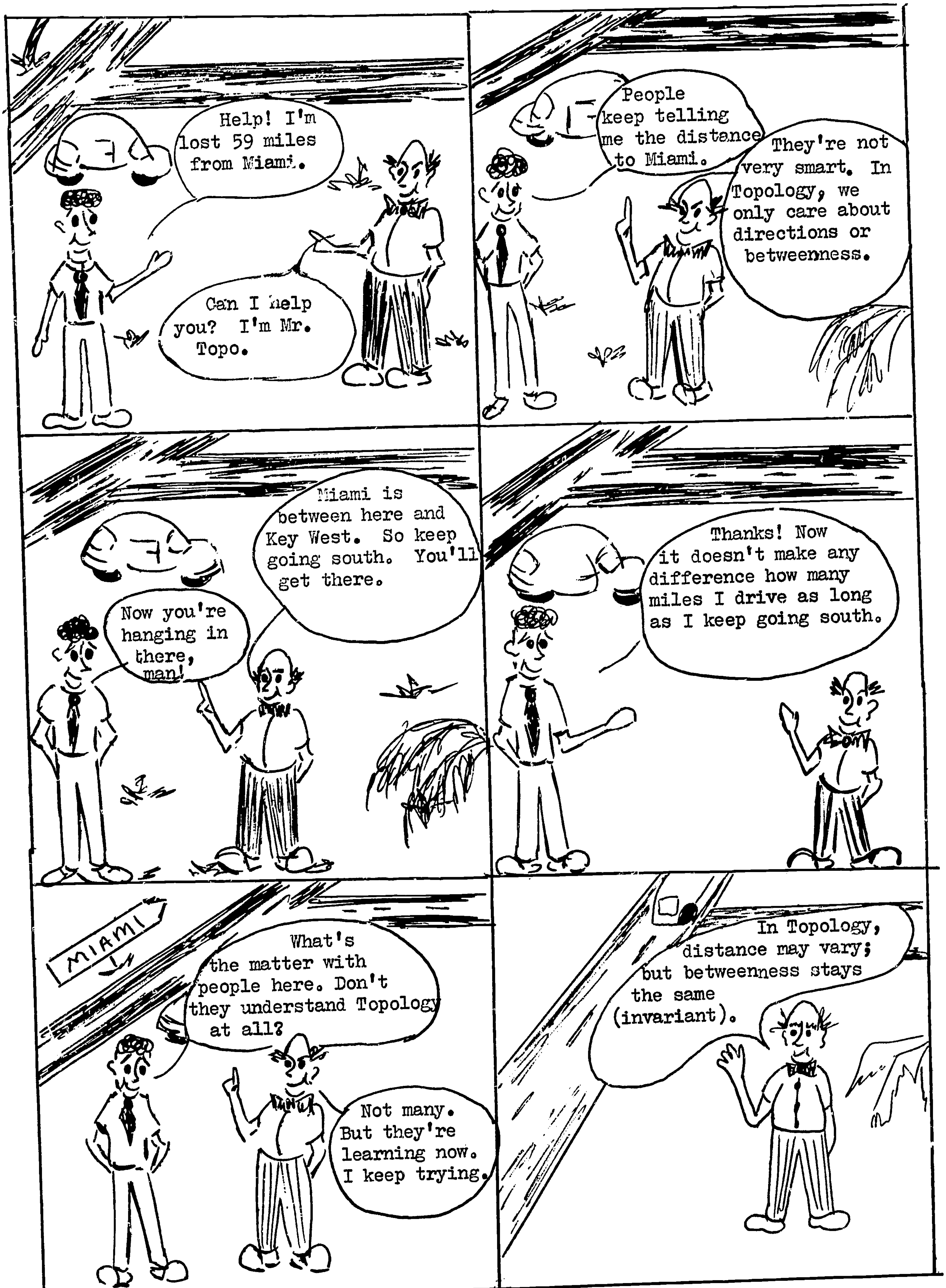
With a lot of work, it could look like any simple closed curve we can make. Point B will always be between point A and point C. The distance may change and the curve will appear completely changed; but the <u>order</u> of the points around the curve can never change or disappear. In topology we are not interested in the distance between points, but rather the fact that the points stay in the same order. This order is the same even though we start with a simple closed curve, perhaps a circle, and end with a strangely shaped closed curve.

Topology is the study of points that do not change their positions in a figure even though we change the shape or size of the figure. Suppose you see a puddle of oil. Suddenly the wind blows the oil so it seems to move to the side away from the wind. Do you suppose there is some particle of oil that never changes location? In topology, when we refer to properties that do not change, we call these "invariants."

There are many times when distance is not important to us but direction and location are.

DIRECTION vs. DISTANCE

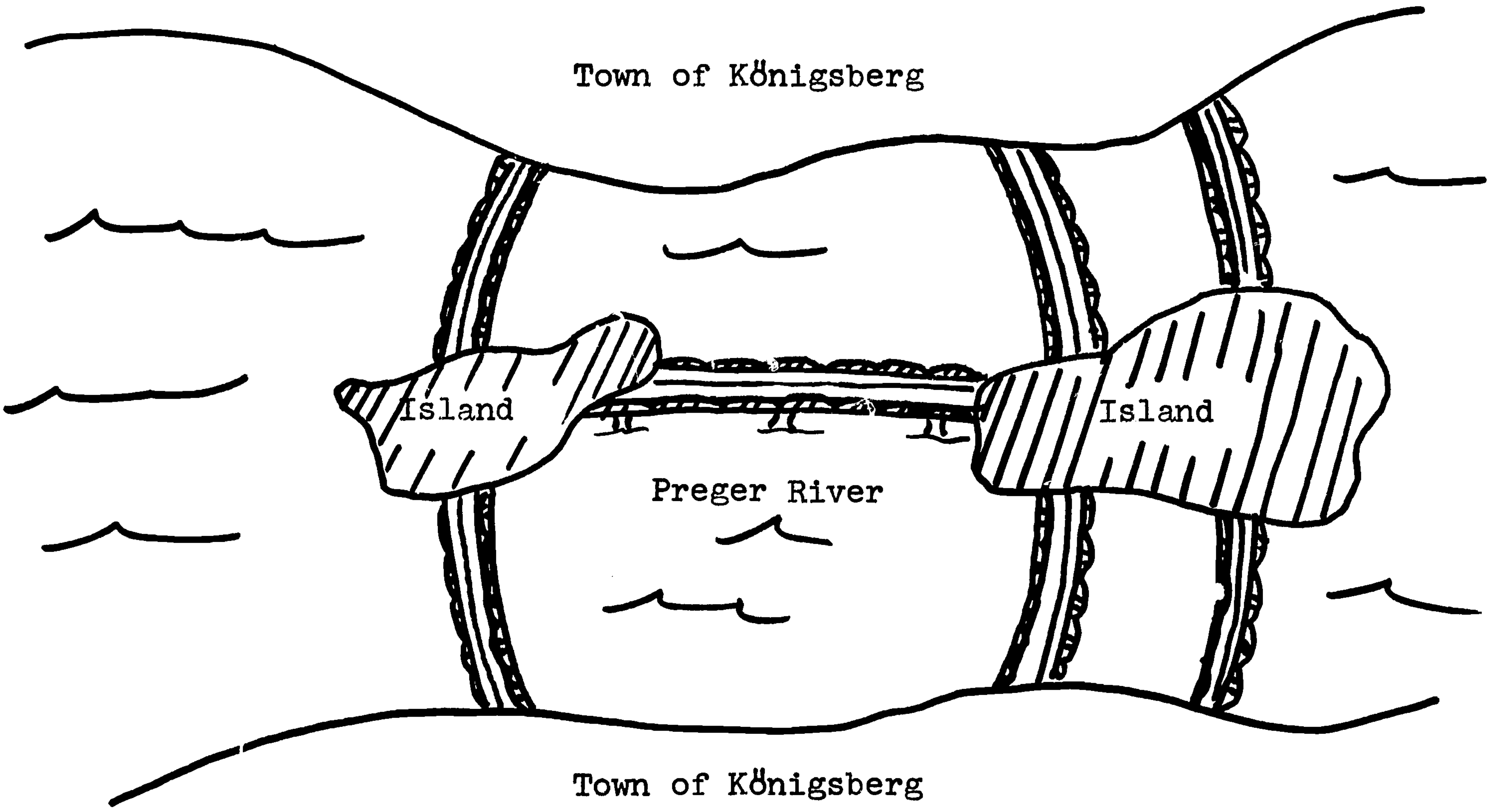




Help! I'm lost 59 miles from Miami.
Can I help you? I'm Mr. Topo.
People keep telling me the distance to Miami.
They're not very smart. In Topology, we only care about directions or betweenness.
Now you're hanging in there, man!
Miami is between here and Key West. So keep going south. You'll get there.
Thanks! Now it doesn't make any difference how many miles I drive as long as I keep going south.
MIAMI
What's the matter with people here. Don't they understand Topology at all?
Not many. But they're learning now. I keep trying.
In Topology, distance may vary; but betweenness stays the same (invariant).

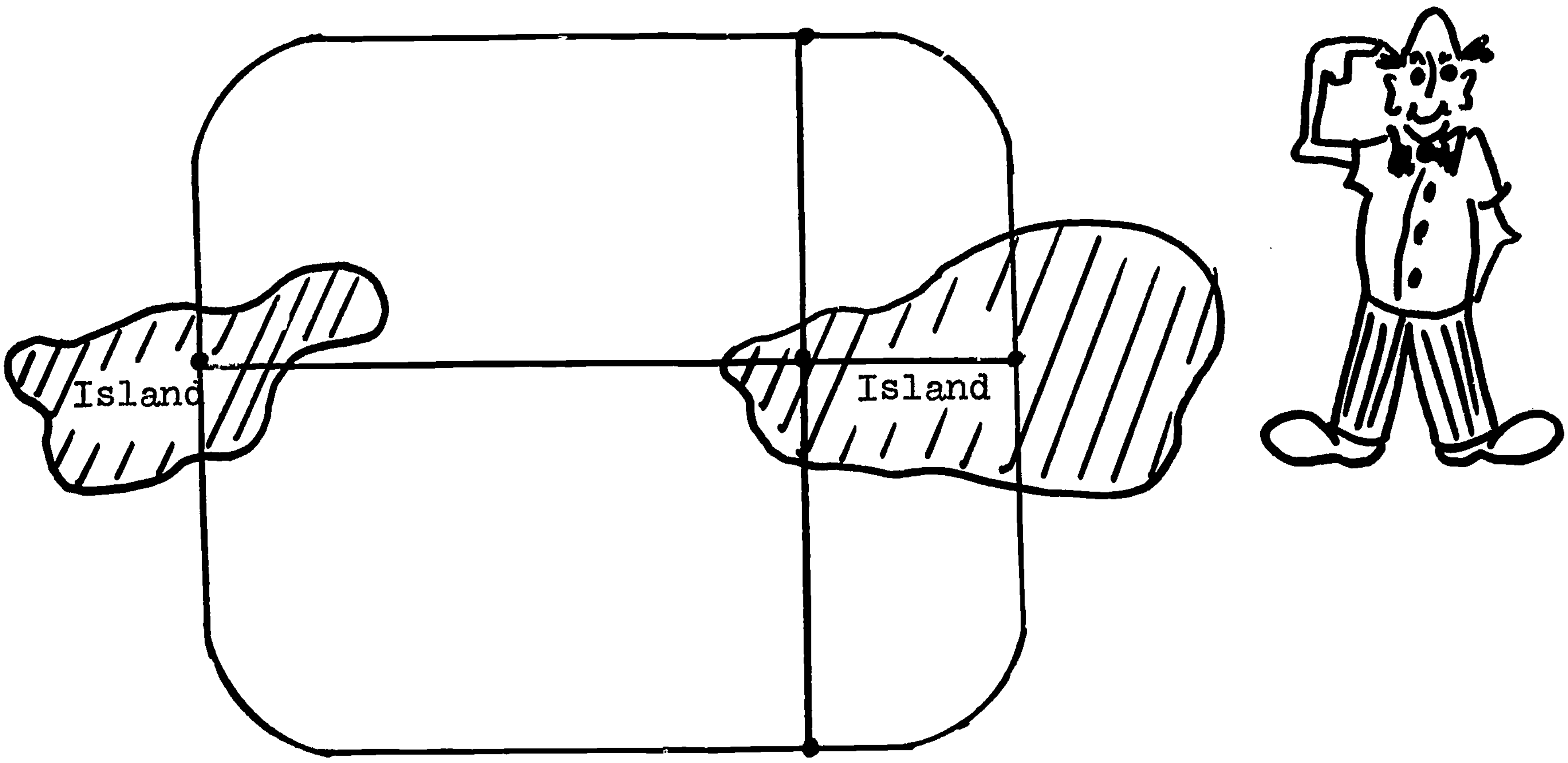
Networks

One area of topology may have been started because of the bridges in Königsberg, Germany. The Preger River flows through this town. There are two islands in the river which were joined to each other and the banks by bridges.



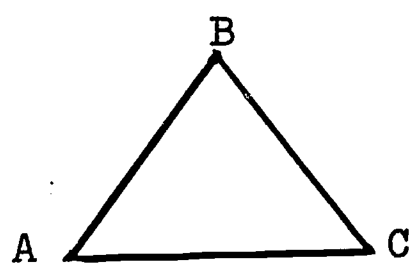
Everyone in Königsberg was once interested in how a person could take a walk and cross each of the bridges only once. Can you figure out how? If you don't try to end up where you start, it is not too difficult to cross six bridges without retracing your steps. However, crossing all seven of the bridges without retracing seems very difficult.

Back in 1735 when the people of Königsberg were trying to solve this problem, there was one German who was very good at problems. His name was Euler. However, he was in Russia acting as a teacher to Catherine the Great. Someone sent the problem to him. He wrote back and said that it was impossible. He studied the problem further and developed some rules that would help to work all problems of this nature. Instead of thinking about islands and rivers, he thought about lines and points.



Now, there are five points where you have a choice of two or more paths. At four places there are three paths coming into the intersection, and at one place four paths meet at the intersection. The points where you have more than one way to move on are called vertices. (A vertex is the point of intersection of two or more paths.)

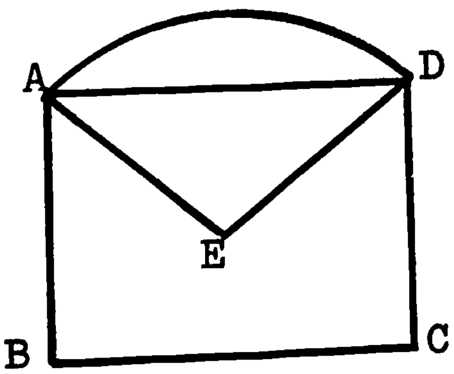
In this path:



there are three vertices, but at each vertex there are only two possible directions to follow. (Two is an even number, and these are even vertices.) In this figure, then, we can start at A and go to B and C and return to A without retracing our steps.

Try these different paths. See if you can start at any one point, visit every other point without retracing your path, and still return to your starting point. Remember you must not lift your pencil.

1.



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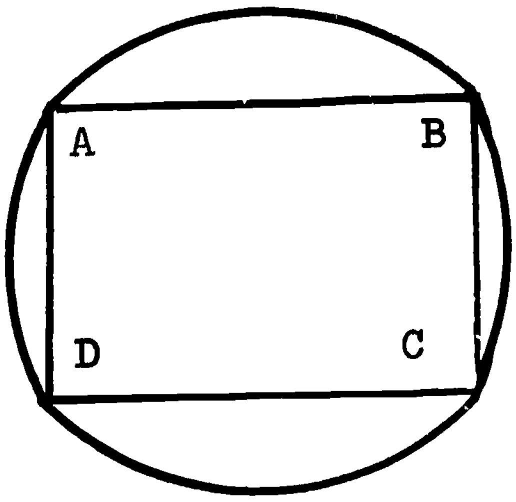

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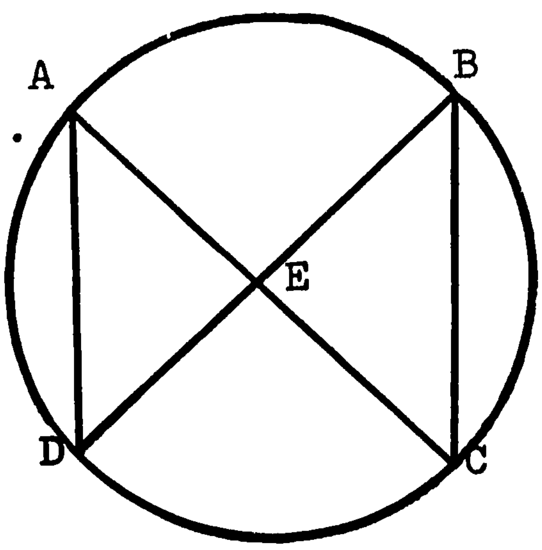

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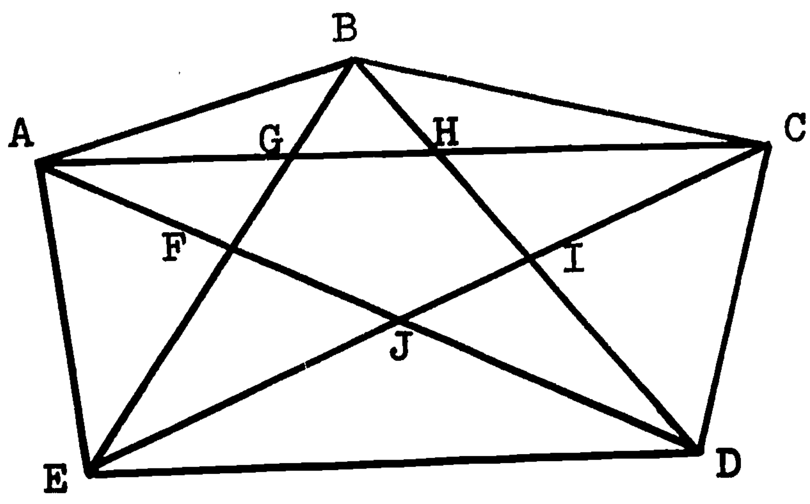

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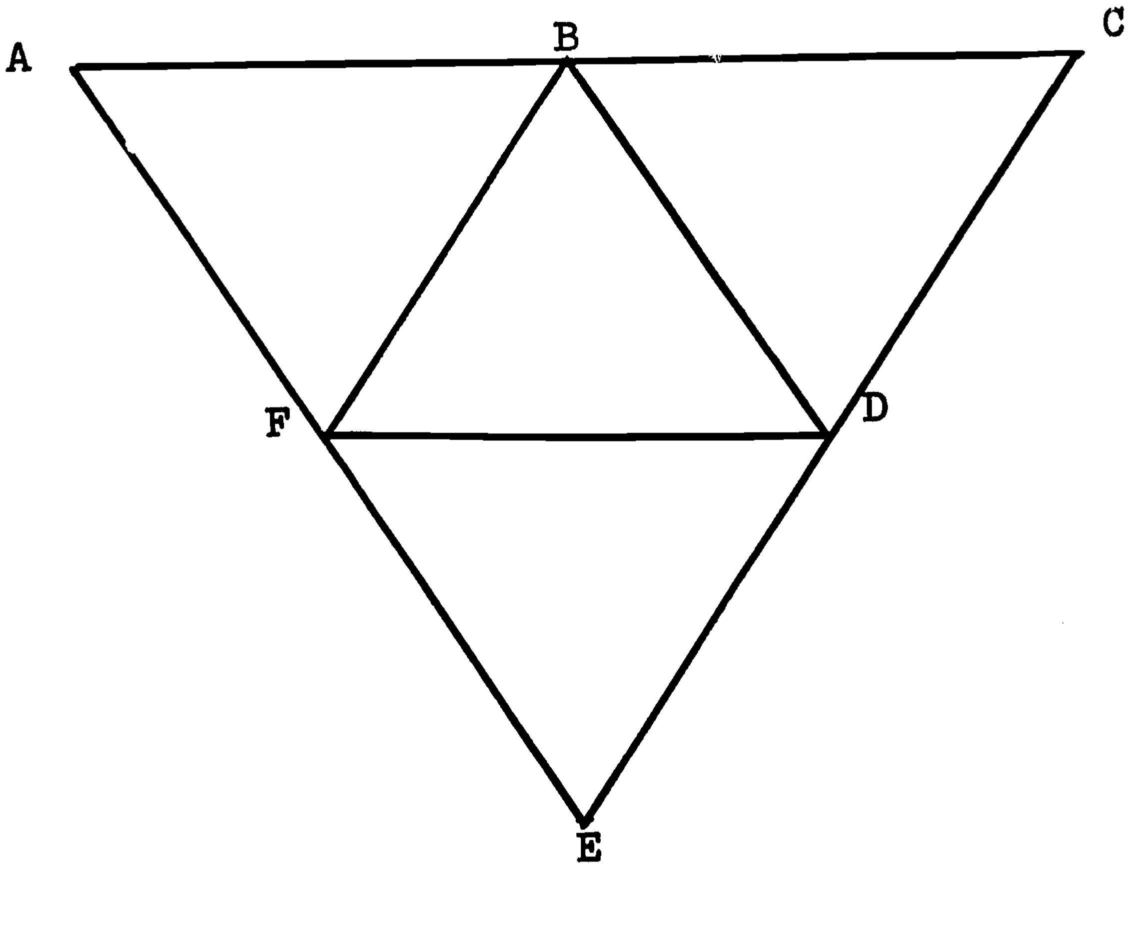

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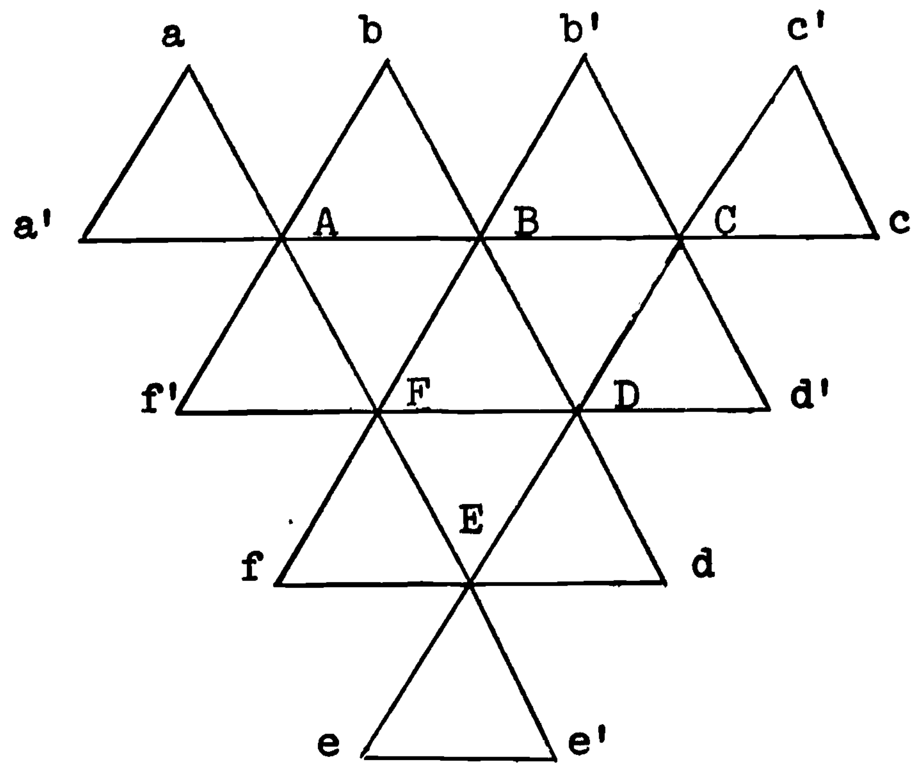
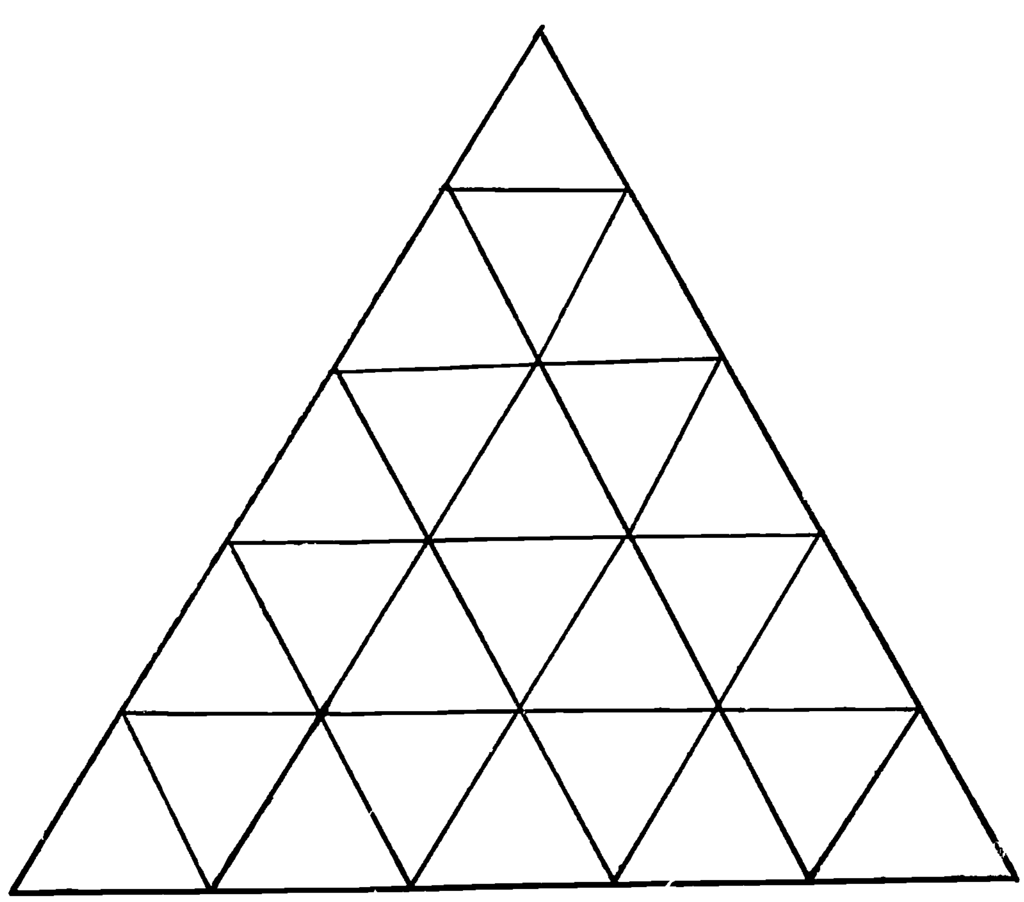


Journey 6: (Journey ABCcc'CDEee'EFAaa'AbBDdEfFBb'Cd'DFf'A)

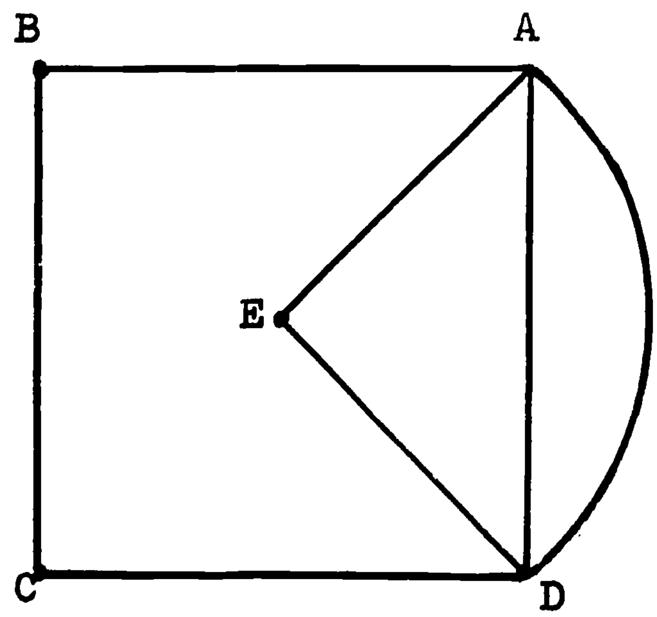
Can you guess why, in figures 1 - 6, you can travel all over and still not double back over your path? In each, count the number of paths (arcs) that meet at each vertex.

Can you travel this without retracing?



Network Example 1

(An arc is the same as a path.)



2 arcs at vertex B
2 arcs at vertex C
4 arcs at vertex D
4 arcs at vertex A
2 arcs at vertex E

To travel without going on the same arc twice, we could follow this path:

vertices A E D C B A D A (or arcs $\overline{AE}$, $\overline{ED}$, $\overline{DC}$, $\overline{CB}$, $\overline{BA}$, $\overline{AD}$, $\overline{DA}$)

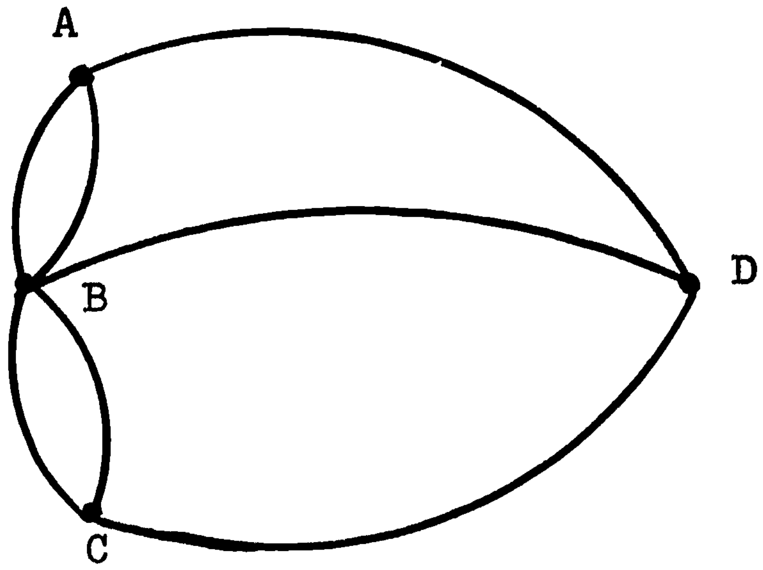
There are an even number of arcs at each vertex (all even vertices).

Try to make up some other paths where it is possible to travel the entire figure covering each arc only once and returning to your starting point.

Do you agree with Euler's discovery that arcs can be traveled without retracing steps if our network contains only <u>even vertices</u>?

Do you suppose there is a possibility of a network where we could travel each arc only once without each vertex having an even number of arcs?

<u>Network Example 2</u>. Try tracing this one.



3 arcs at vertex A

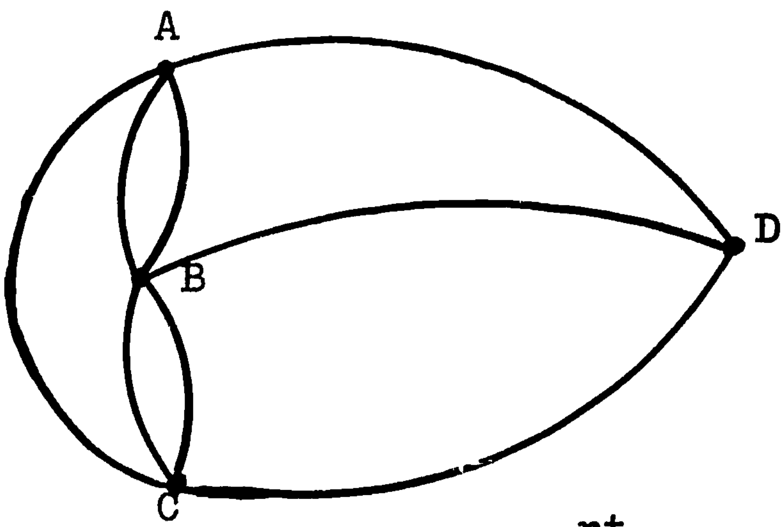
5 arcs at vertex B

3 arcs at vertex C

3 arcs at vertex D

After several trys, did you find Example 2 impossible to follow without retracing?

<u>Network Example 3</u>. Try this one.



4 arcs at vertex A

5 arcs at vertex B

4 arcs at vertex C

3 arcs at vertex D

By starting at D, we can go $\overline{DB}$, $\overline{BA}^{rt}$, $\overline{AD}$, $\overline{DC}$, $\overline{CB}^{rt}$, $\overline{BA}^{left}$, $\overline{AC}$, $\overline{CB}^{left}$. There are 8 arcs, and we have traveled each one only once, but we did not return to our starting point. Try some more, but you'll probably agree with Euler again that it is possible to make the journey if our network has <u>two odd vertices, but we will not get back to our starting vertex</u>. Euler also found that for any journey, if there are an <u>even number</u> of <u>odd vertices</u>, the number of different journeys is the answer you get when you divide the number of odd vertices by 2.

Suppose Phil, who lives in Boca Raton, plans to go to Paris. He finds there are two flights (#1 and #2) to New York, and from New York to Paris he has a choice of two flights (A and B) or an ocean liner (C).

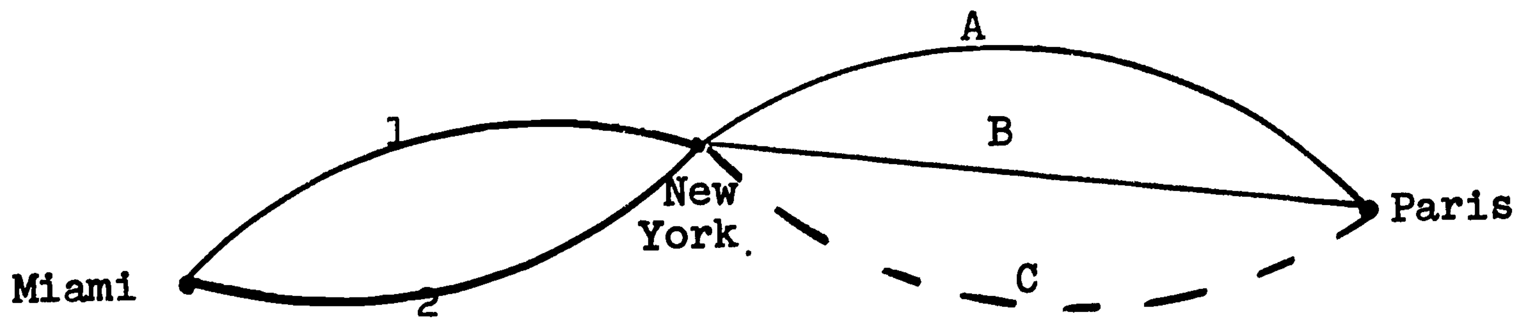


Figure 1

Phil met a man in New York who said he had traveled each one of the routes Phil had to consider. Could the man travel each route exactly once? Could he have started in New York? (Hint: The man spoke French very well.)

How many vertices are there? _______

How many odd vertices? _______

Suppose Phil thinks this would be fun to try, but by now he has found 3 flights to New York.

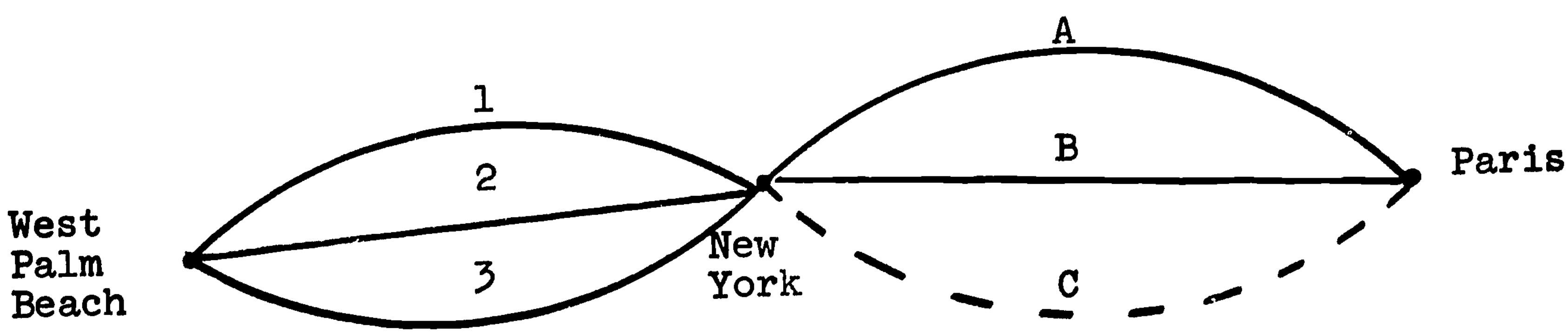


Figure 2

Can Phil travel each path (arc) exactly once? There are three vertices (West Palm Beach, New York, Paris). Which of these are odd vertices? _______

Will Phil ever be able to come back to his starting point without going on the same route twice? ______

Suppose we had this situation:

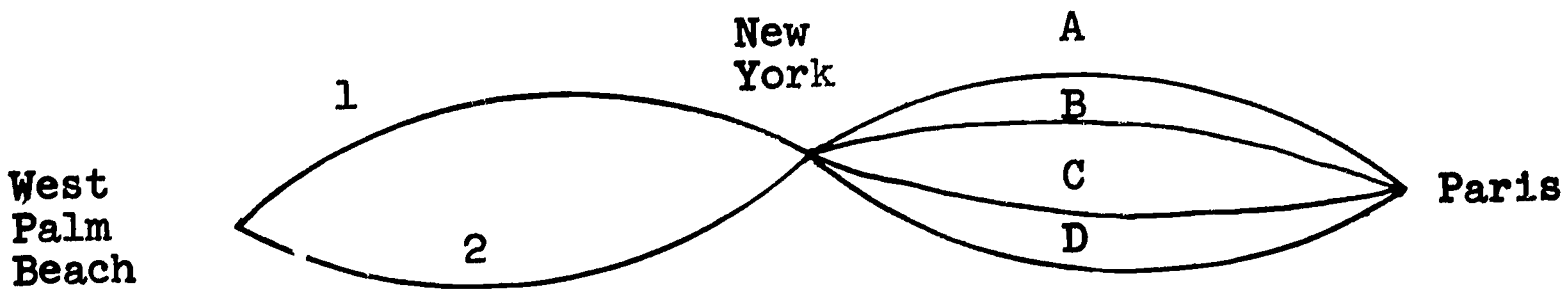


Figure 3

Can Phil travel over every path just once? _______

Can he finish back at his starting place? _______

How many odd vertices are there? _______

Let's extend Phil's trip:

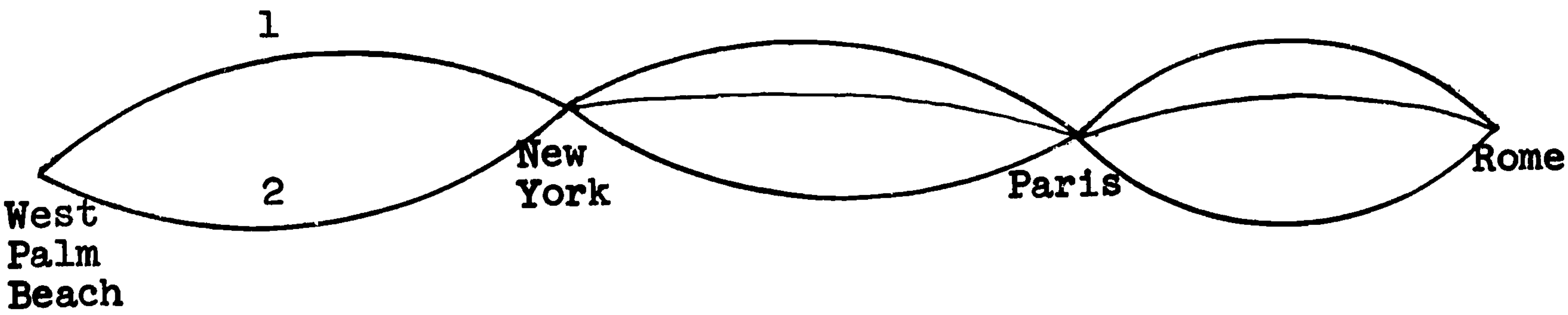


Figure 4

Can he travel every path just once? _______

Can he return to his starting place? _______

How many odd vertices are there? _______

Try these trips:

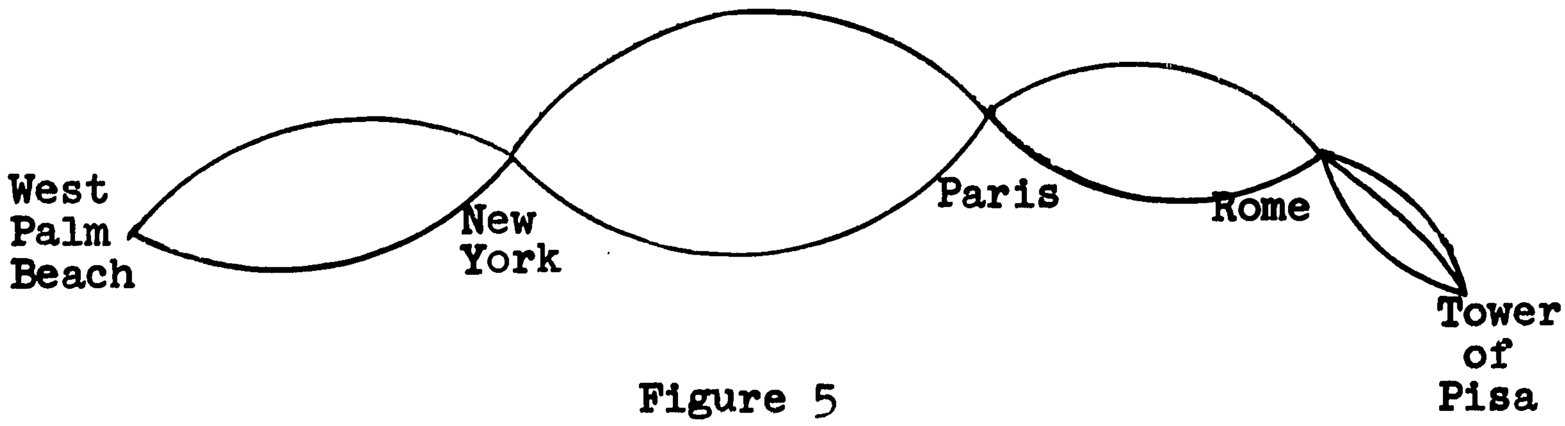


Figure 5

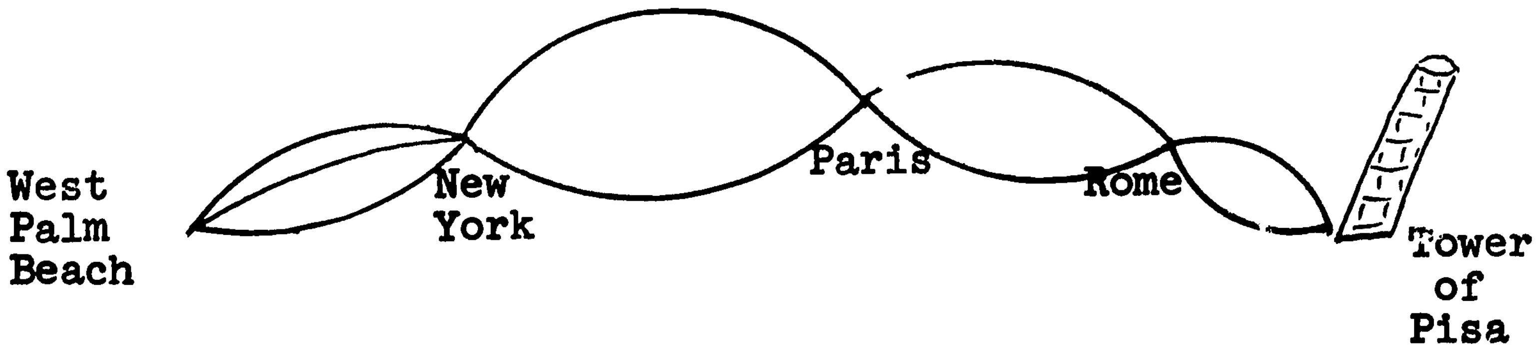


Figure 6

Let's consider Euler's conclusions:

1. A closed network always has an even number of odd vertices.

2. A closed network of all even vertices can always be traced without traveling any arc twice. (We may always go through a vertex more than once.)

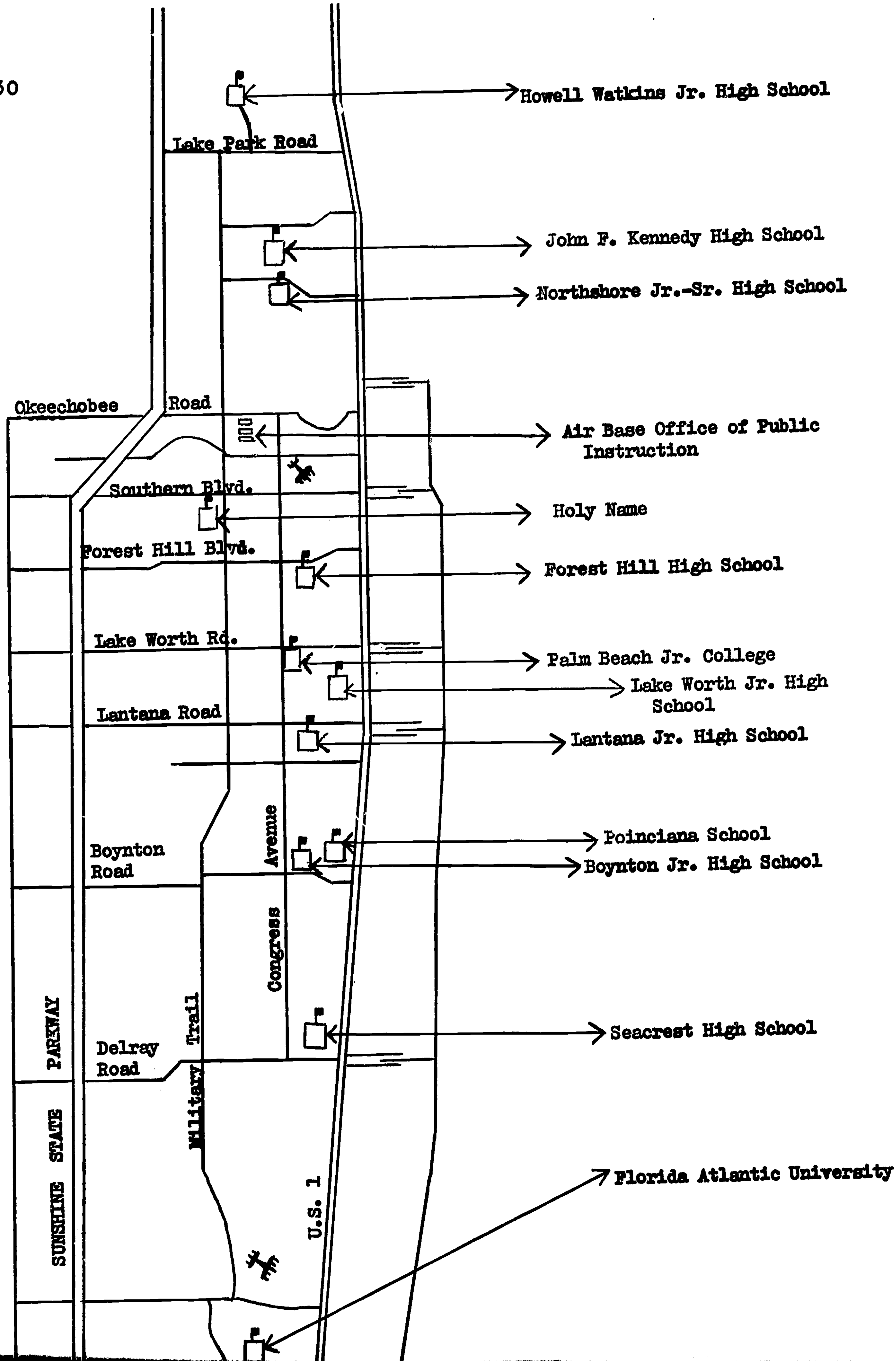
3. If a closed network contains two and only two odd vertices, it can be traced without traveling any arc twice by starting at an odd vertex. (You will not end up at your starting point--you will end at the other odd vertex.)

4. If a closed network contains more than two odd vertices, it cannot be traveled in one journey without retracing an arc.

Let's make up a Table of our facts.

Figure	Total Vertices	Total Arcs	Odd Vertices	Complete Trip Possible	Ending at Starting Point	Ending at Different Point	* Regions Made By Path	Vertices + Regions - Arcs (V + R - A)
1	3	5	2	Yes	No	Yes	4	2
2								
3								
4	4	8					6	2
5	5	10	4	No			7	2
6	5	9					6	2

* Remember to call space outside only one region. Thus in Figure 1 we have three regions inside plus one outside, or four total regions.


Howell Watkins Jr. High School
Lake Park Road
John F. Kennedy High School
Northshore Jr.-Sr. High School
Okeechobee
Road
Air Base Office of Public Instruction
Southern Blvd.
Holy Name
Forest Hill Blvd.
Forest Hill High School
Lake Worth Rd.
Palm Beach Jr. College
Lake Worth Jr. High School
Lantana Road
Lantana Jr. High School
Boynton Road
Avenue
Poinciana School
Boynton Jr. High School
Congress
PARKWAY
Trail
Delray Road
Seacrest High School
Military
STATE
SUNSHINE
U.S. 1
Florida Atlantic University

Suppose you need to deliver some material from the air base office of the Board of Public Instruction to the following schools. Time is important, so plan a trip that will require the least time. See if you can go on these trips and end back at the air base without covering the same path twice.

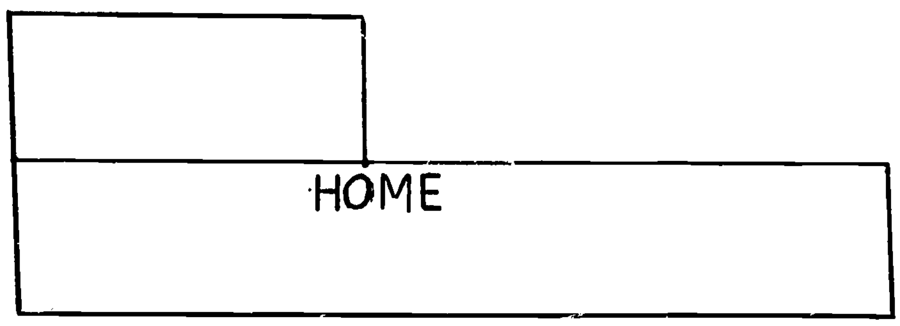
Trip I: Forest Hill High School
Lake Worth Junior High School
Lantana Junior High School
Palm Beach Junior College

Trip II: Howell Watkins Junior High School
John F. Kennedy High School
Holy Name School
Northshore Junior-Senior High School

Trip III: Florida Atlantic University
Your home
Poinciana School
Seacrest High School
Boynton Beach Junior High School

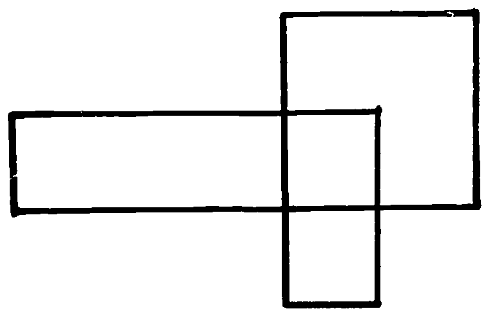
Can you see how important this problem would be to the man who drives the Pony Express for the school system? Wouldn't a good taxi driver have to solve the same problems many times?

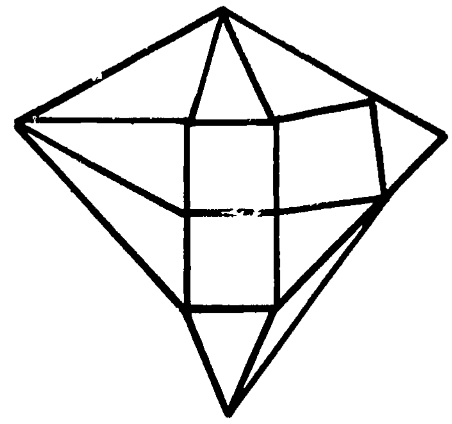
Suppose you are a paper boy and your route looks like this:



Can you cover this route without retracing one or more streets?

How about these paper routes?



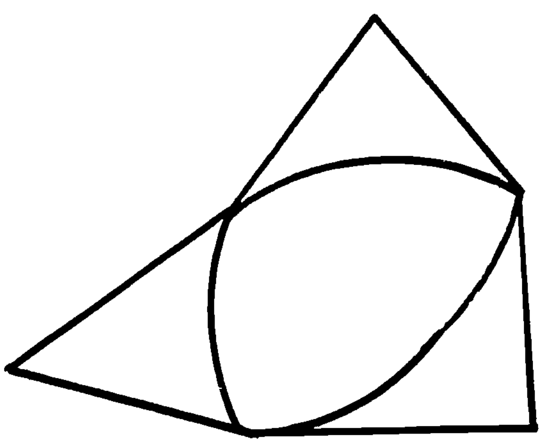


Does it make any difference where you start?
Try several starts and see.
Would a paper boy be wise to use some knowledge of topology in planning his route?

Suppose you are a highway inspector. If you need to drive along each highway only once, can you find a route?

Can you predict your answer by counting vertices, arcs, and regions?

Here is a map of your route.



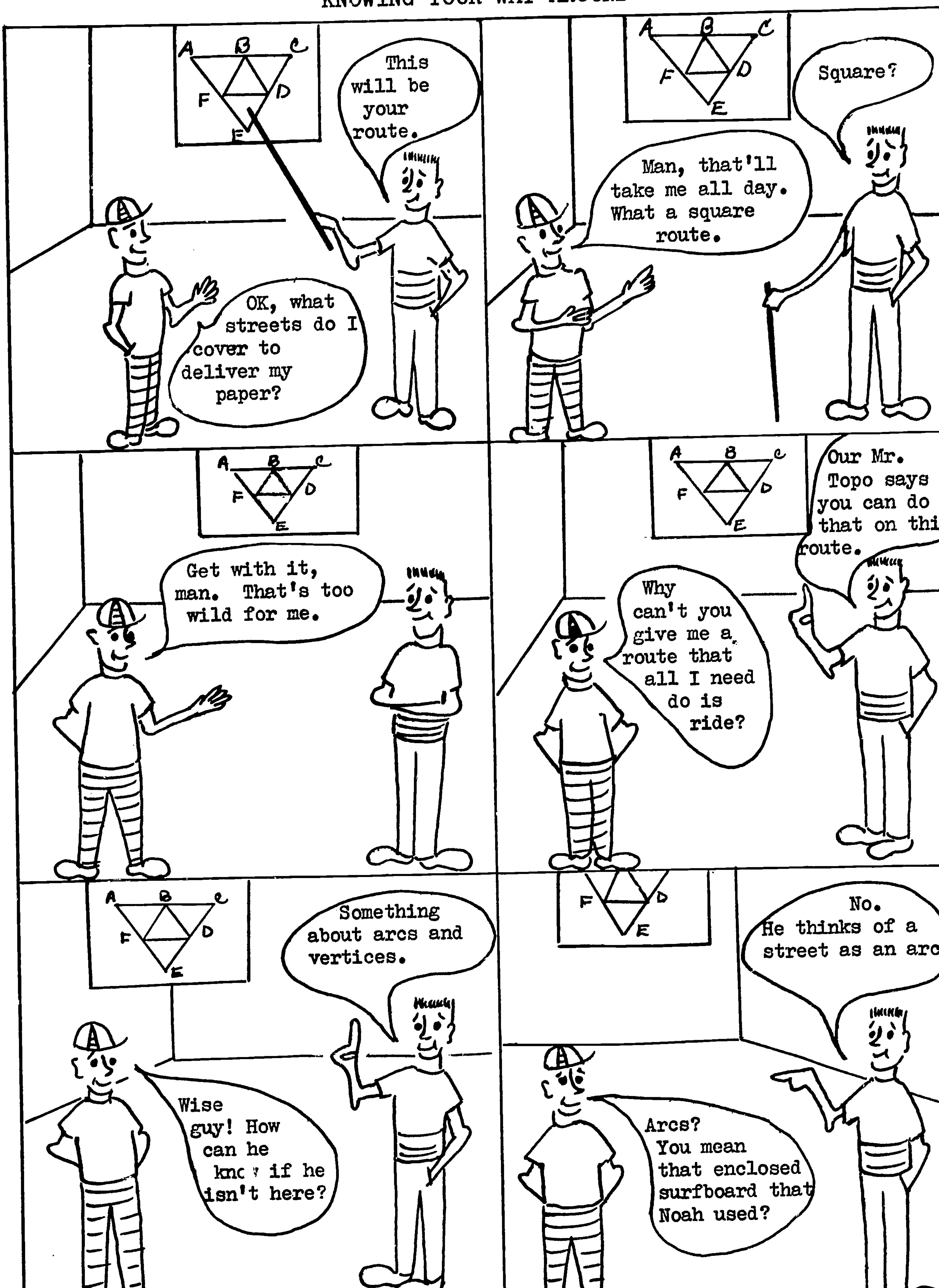
Vertices + Regions - Arcs =

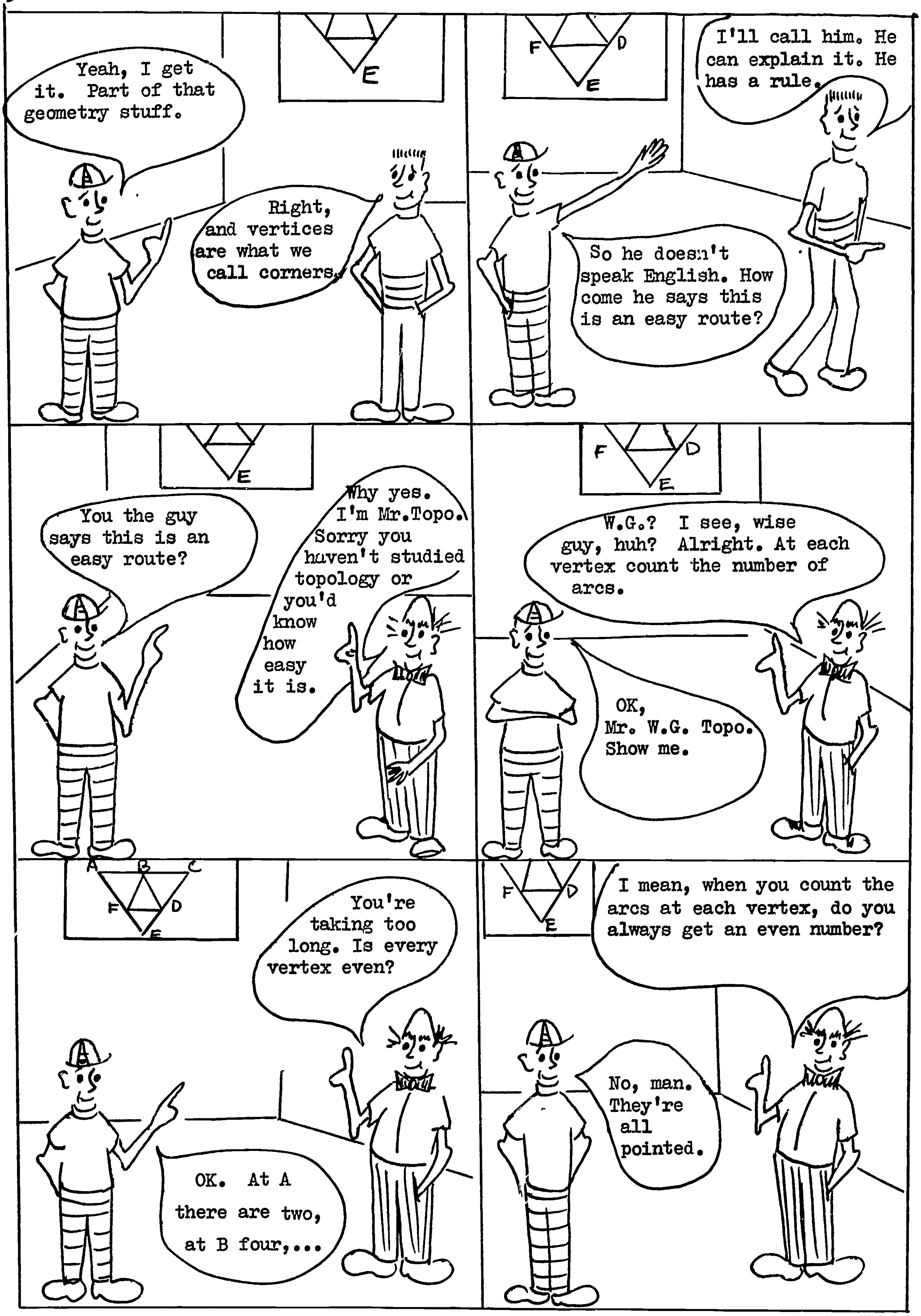
To summarize some of the ideas we know about routes or networks:

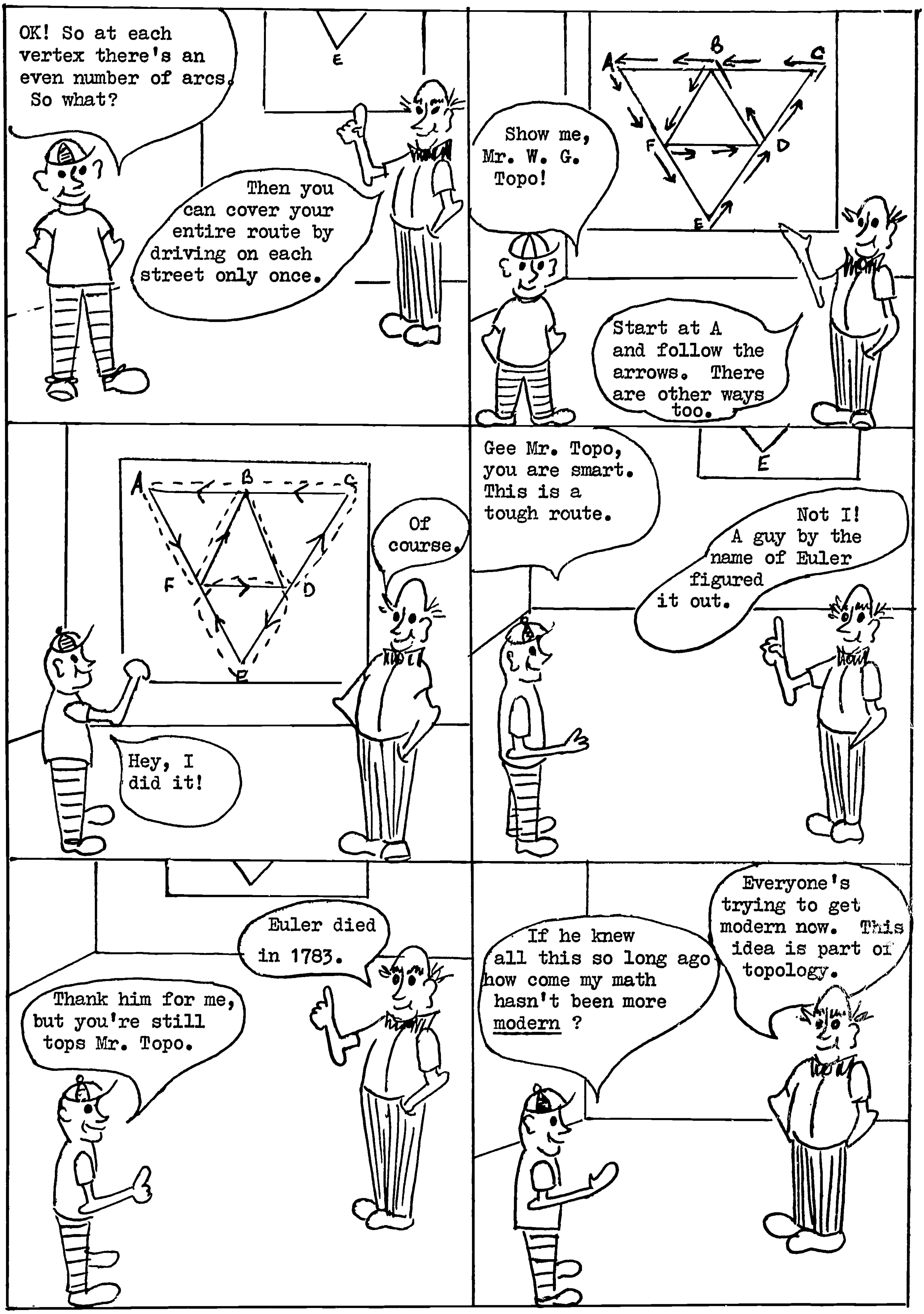
1. If a network has all even vertices, then we can find a path starting at any vertex to cover each arc once and only once and end up at the original vertex.

2. If a network has only two odd vertices, then it is possible to find a route that will cover each arc but not possible to return to the starting point. The starting point must be an odd vertex, and the end will be at the other odd vertex.

KNOWING YOUR WAY AROUND




E
Yeah, I get it. Part of that geometry stuff.
Right, and vertices are what we call corners.
F
D
E
I'll call him. He can explain it. He has a rule.
So he doesn't speak English. How come he says this is an easy route?
E
You the guy says this is an easy route?
Why yes. I'm Mr.Topo. Sorry you haven't studied topology or you'd know how easy it is.
F
D
E
W.G.? I see, wise guy, huh? Alright. At each vertex count the number of arcs.
OK, Mr. W.G. Topo. Show me.
A
B
C
F
D
E
You're taking too long. Is every vertex even?
OK. At A there are two, at B four,...
F
D
E
I mean, when you count the arcs at each vertex, do you always get an even number?
No, man. They're all pointed.


OK! So at each vertex there's an even number of arcs. So what?
E
Then you can cover your entire route by driving on each street only once.
Show me, Mr. W. G. Topo!
A
B
C
F
D
E
Start at A and follow the arrows. There are other ways too.
A
B
C
F
D
E
Of course.
Hey, I did it!
Gee Mr. Topo, you are smart. This is a tough route.
E
Not I! A guy by the name of Euler figured it out.
Euler died in 1783.
Thank him for me, but you're still tops Mr. Topo.
If he knew all this so long ago how come my math hasn't been more modern ?
Everyone's trying to get modern now. This idea is part of topology.

Map Coloring

Map makers find they have to color any countries that border on each other different colors so that people can see they are separate countries.

Clever John

John lived many years ago. In fact, it was on June 10, 1842, that he started to work. John was a very clever boy who liked to figure things out. His first job was with a company that made maps. Now, back in those days coloring material was very expensive and hard to find or make. So, each person who worked for the map company was required to bring his own paints.

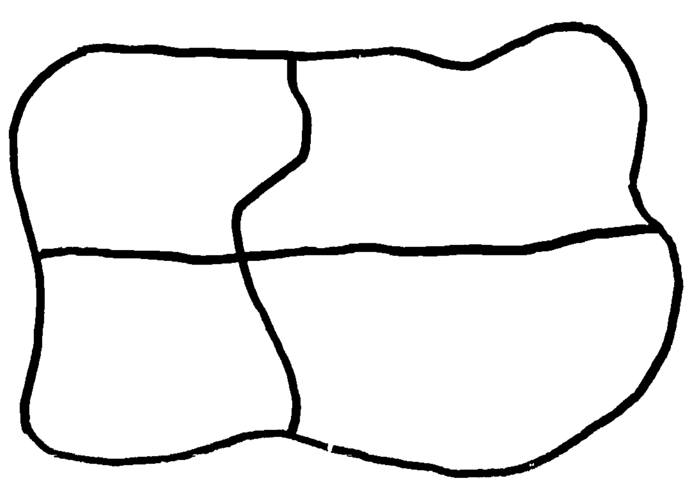
The first day when John came to work he brought only four colors of paint. The boss took one look at John holding the four cans of paint and became very angry.

"Do you think we make all our maps with only four countries?" he sputtered. John tried to answer, but the boss continued, "If you want to work for us, you must have enough colors so that no states that are beside each other are the same color. I told you that when I hired you. How dare you come not ready to work?"

This was all John could stand, so he shouted, "I have all the colors I need!"

The boss was so angry he just picked up a map that needed coloring and threw it at John saying, "All right, you smart young punk, color this, and it had better be right!"

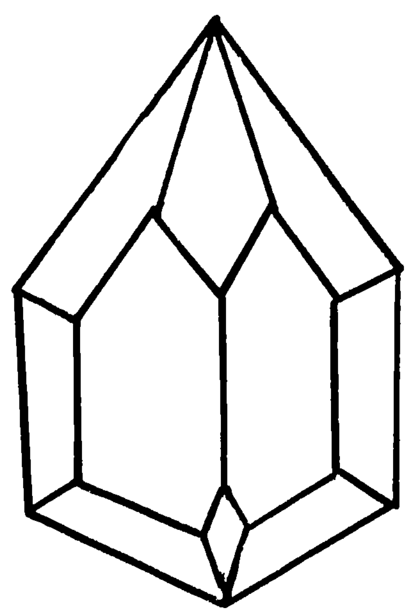
This is the map John had to color:



John used only 2 of his cans of paint to do it. (Can you see how?) The boss had to admit that John's work was satisfactory.

"But," said the boss, "this was a simple map. I'll have one of the men give you a more difficult one."

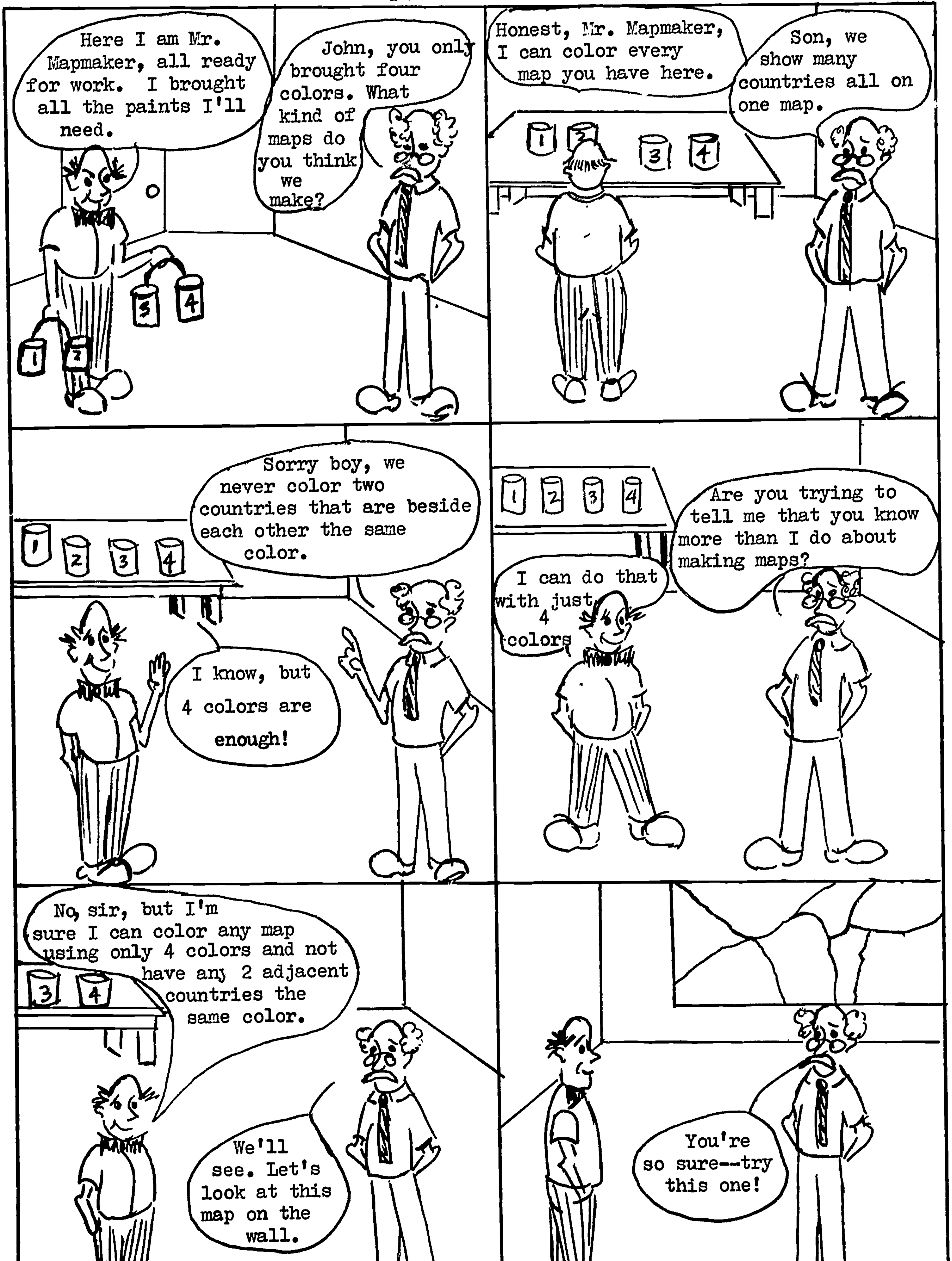
This is the new map John received.

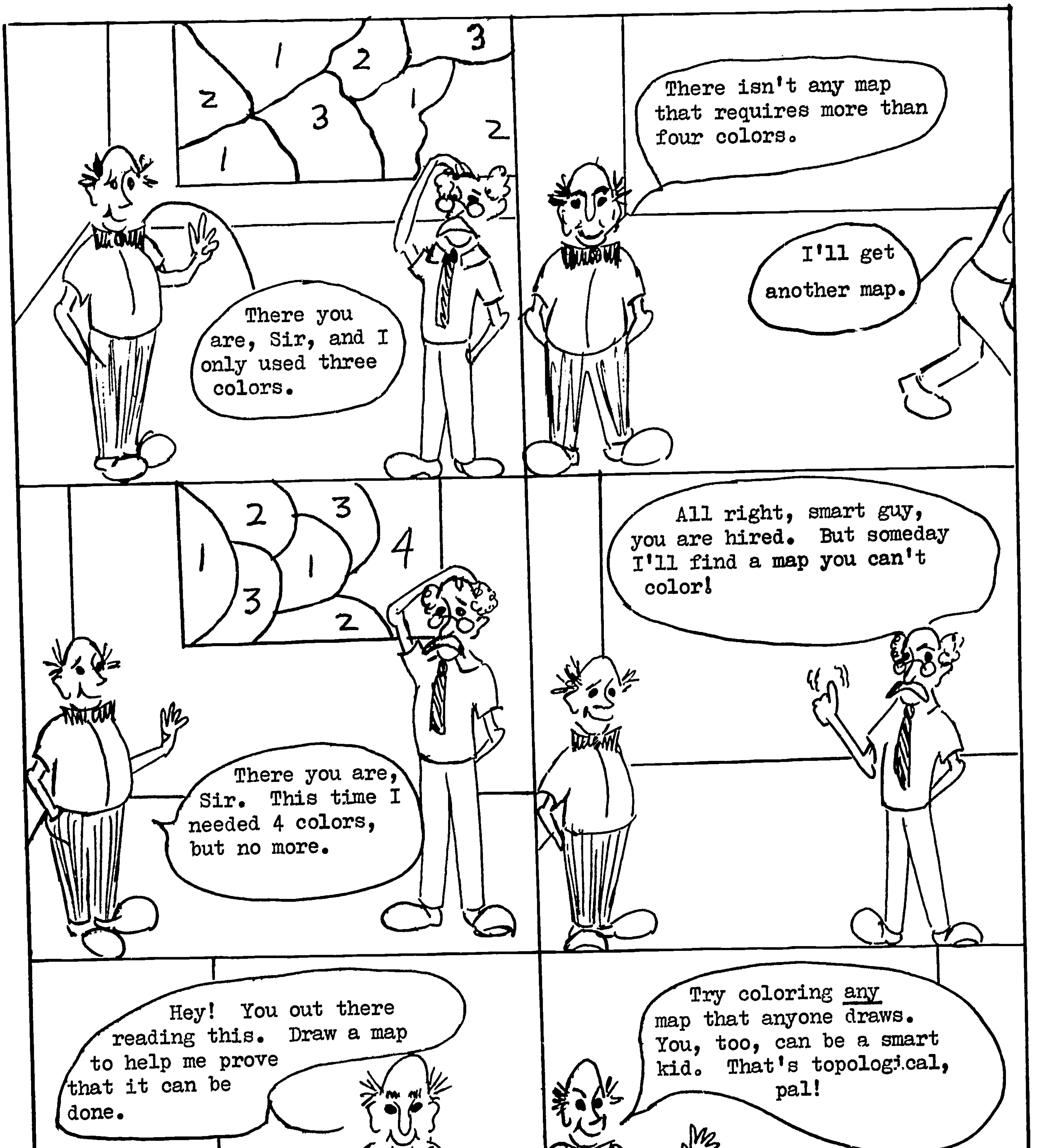


This time John had to use 3 colors. As he showed it to the boss John said, "You cannot draw a map that requires more than 4 colors."

The boss was very angry. He intended to show John he was wrong. The boss offered anyone in the shop who could draw a map that needed more than 4 colors a $100 raise. No one ever got the raise. Could you?

No one has ever been able to draw a map that requires more than 4 colors. Can you? Try some.


Here I am Mr. Mapmaker, all ready for work. I brought all the paints I'll need.
John, you only brought four colors. What kind of maps do you think we make?
Honest, Mr. Mapmaker, I can color every map you have here.
Son, we show many countries all on one map.
Sorry boy, we never color two countries that are beside each other the same color.
I know, but 4 colors are enough!
I can do that with just 4 colors
Are you trying to tell me that you know more than I do about making maps?
No, sir, but I'm sure I can color any map using only 4 colors and not have any 2 adjacent countries the same color.
We'll see. Let's look at this map on the wall.
You're so sure--try this one!

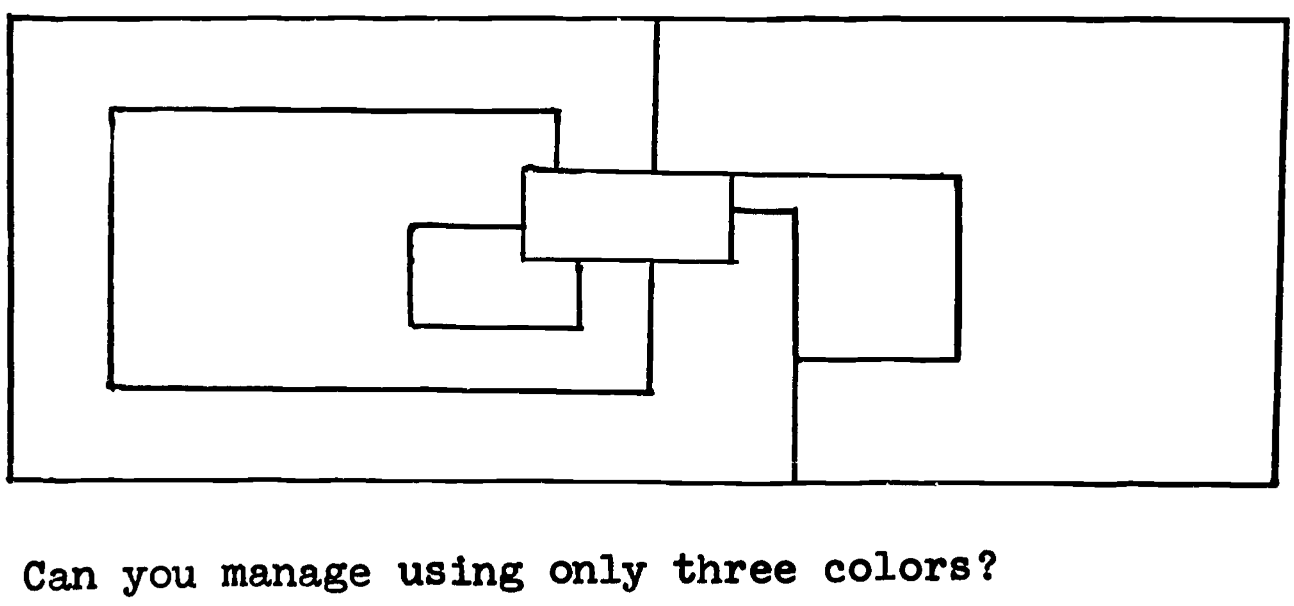

2
1
2
3
3
1
2
1
There you are, Sir, and I only used three colors.
There isn't any map that requires more than four colors.
I'll get another map.
2
3
1
4
1
3
2
There you are, Sir. This time I needed 4 colors, but no more.
All right, smart guy, you are hired. But someday I'll find a map you can't color!
Hey! You out there reading this. Draw a map to help me prove that it can be done.
Try coloring any map that anyone draws. You, too, can be a smart kid. That's topological, pal!

A wealthy landowner died and left his land to his four sons. There was only one provision in the distribution of this land. The father said the sons had to divide this land so that each son could visit each of the other three sons without crossing any land other than his own and the land of the one he was to visit. If the sons could not figure out a way to do this, they had to give the land to charity. Since there was uranium under all the lands, the sons were very anxious to keep the property.

Can you sketch a possible division of the land? The shape of the land does not make any difference. Show your finished sketch here. (There are several ways this can be done.)

Suppose the wealthy landowner had five sons. Could they still divide the land according to his instructions?

Can you color this map in four colors or less?



Can you manage using only three colors?

Topological Transformations

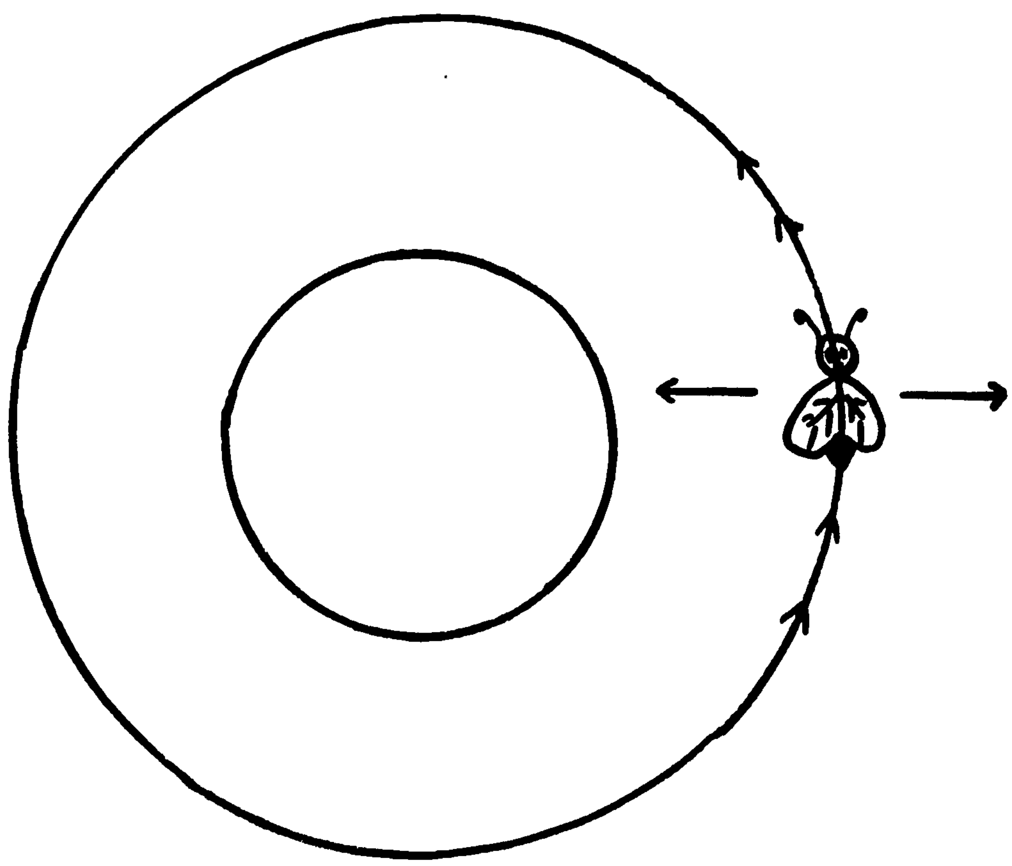
Inside and Outside

The Modern Hiawatha

He killed the noble Mudjokinis.
Of the skin he made him mittens;
Made them with the fur side inside,
Made them with the skin side outside.
He, to get the warm side inside,
Put the inside skin side outside;
He, to get the cold side outside,
Put the warm side fur side inside.
That's why he put the fur side inside,
Why he put the skin side outside,
Why he turned them inside outside.

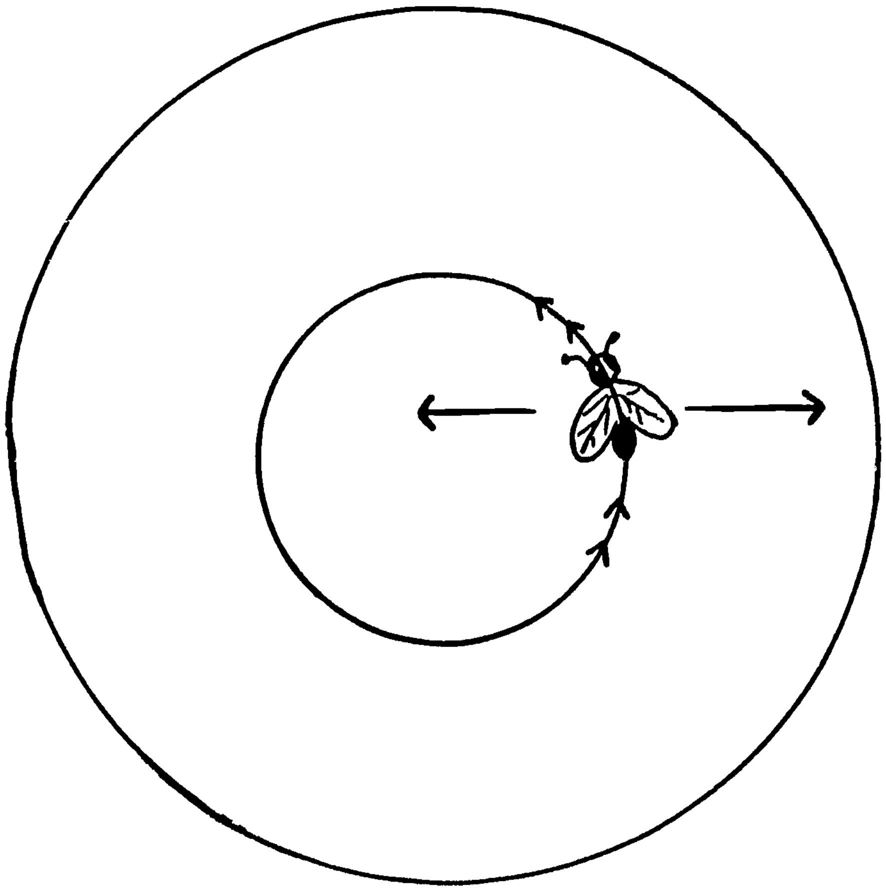
--Author unknown

Inside -- outside. Do you know what these two words mean?



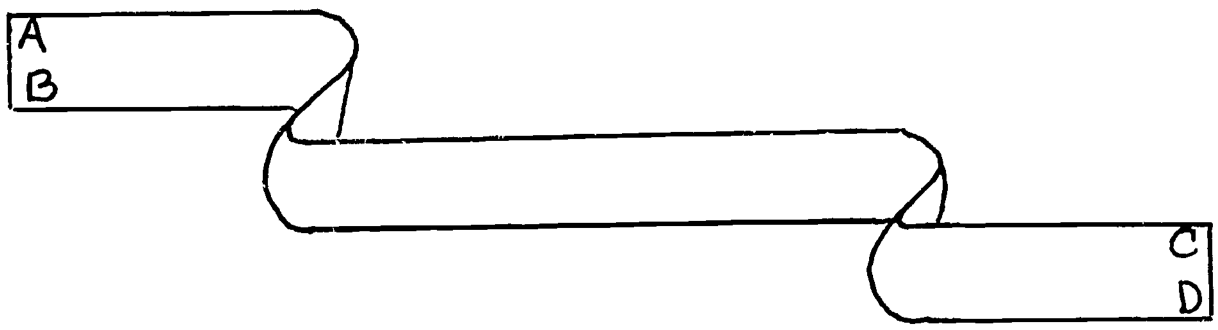
If a fly were walking, as shown, to his right would be the outside of the doughnut and to his left the inside.

Now, the fly is walking the same direction but at a different place on the doughnut. Is it still outside the doughnut to his right and inside to his left? Are you sure you know what you mean by "inside" and "outside"?

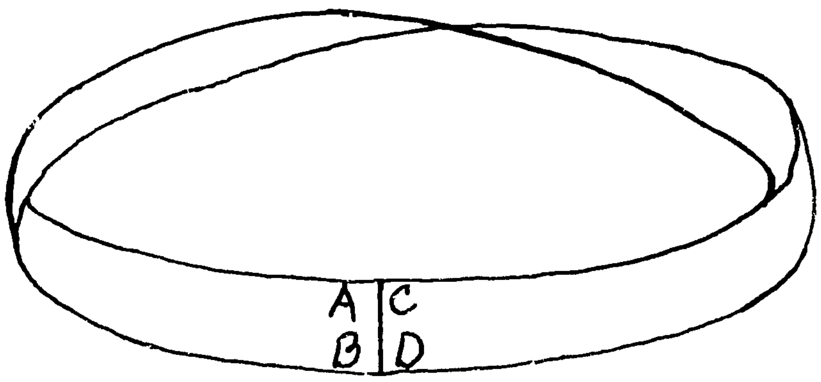


Suppose a very intoxicated man while walking home came to a large tree--a tree so large he couldn't reach around it. So, he put both hands on the tree and started feeling for an opening so he could proceed on his way. He went 'round and 'round without finding any opening or, to his mind, a way out. Finally, in desperation, he cried, "Help, I'm walled in!" Poor man. He really was mixed up about outside and inside.

This sheet of paper obviously has two sides--page 43 and page 44. Suppose you join this side somehow to the other side. Then you would have only one side. This idea is demonstrated by the Moebius strip. Take a strip of paper--it works best if it is at least 18" ($1\frac{1}{2}$') long and about 3" wide.

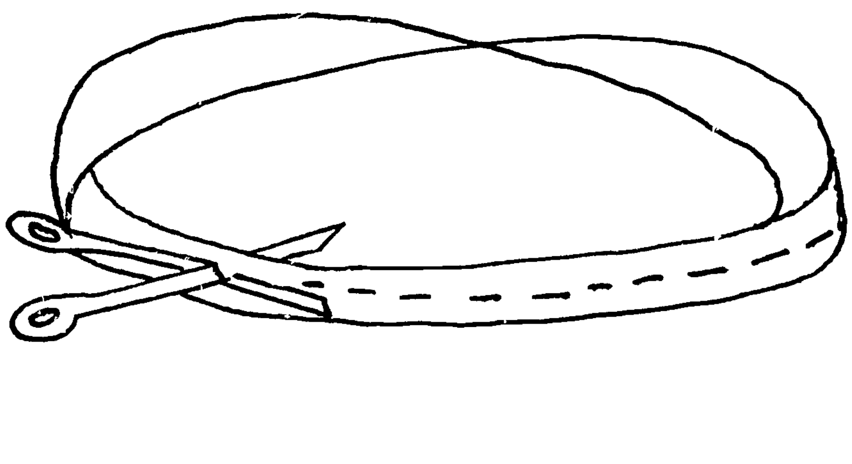


Now join it together so that C is glued to B and A to D. To do this, it is necessary to have a half twist in the paper. Use an old newspaper to make some Moebius strips.



To have some fun, ask a friend to draw a line on only one side of the strip. (It is impossible because you have a surface joined into itself; with the Moebius strip there is only one side.)

Try cutting your Moebius strip so you will have two Moebius strips each $1\frac{1}{2}$" wide. Did you get two strips? Try cutting only 1" in from the side of your Moebius strip.



Paul Bunyan Versus The Conveyor Belt

By William Hazlett Upson

One of Paul Bunyan's most brilliant successes came about not because of brilliant thinking, but because of Paul's caution and carefulness. This was the famous affair of the conveyor belt.

Paul and his mechanic, Ford Fordsen, had started to work an uranium mine in Colorado. The ore was brought out on an endless belt which ran half a mile going into the mine and another half mile coming out--giving it a total length of one mile. It was four feet wide. It ran on a series of rollers, and was driven by a pulley mounted on the transmission of Paul's big blue truck "Babe." The manufacturers of the belt had made it all in one piece, without any splice or lacing, and they had put a half-twist in the return part so that the wear would be the same on both sides.

After several months' operation, the mine gallery had become twice as long, but the amount of material coming out was less. Paul decided he needed a belt twice as long and half as wide. He told Ford Fordsen to take his chain saw and cut the belt in two lengthwise.

"That will give us two belts," said Ford Fordsen. "We'll have to cut them in two crosswise and splice them together. That means I'll have to go to town and buy the materials for two splices."

"No," said Paul. "This belt has a half-twist--which makes it what is known in geometry as a Moebius strip."

"What difference does that make?" asked Ford Fordsen.

"A Moebius strip," said Paul Bunyan, "has only one side, and one edge, and if we cut it in two lengthwise, it will still be in one piece. We'll have one belt twice as long and half as wide."

"How can you cut something in two and have it still in one piece?" asked Ford Fordsen.

Paul was modest. He was never opinionated. "Let's try this thing out," he said.

They went into Paul's office. Paul took a strip of gummed paper about two inches wide and a yard long. He laid it on his desk with the gummed side up. He lifted the two ends and brought them together in front of him with the gummed sides down. Then he turned one of the ends over, licked it, slid it under the other end, and stuck the two gummed sides together. He had made himself an endless paper belt with a half-twist in it just like the big belt on the conveyor.

"This," said Paul, "is a Moebius strip. It will perform just the way I said--I hope."

Paul took a pair of scissors, dug the point in the center of the paper and cut the paper strip in two lengthwise. And when he had finished--sure enough--he had one strip twice as long, half as wide, and with a double twist in it.

Ford Fordsen was convinced. He went out and started cutting the big belt in two. And, at this point, a man called Loud Mouth Johnson arrived to see how Paul's enterprise was coming along, and to offer any destructive criticism that might occur to him. Loud Mouth Johnson, being Public Blow-Hard Number One, found plenty to find fault with.

"If you cut that belt in two lengthwise, you will end up with two belts, each the same length as the original belt, but only half as wide."

"No," said Ford Fordsen, "this is a very special belt known as a Moebius strip. If I cut it in two lengthwise, I will end up with one belt twice as long and half as wide."

"Want to bet?" said Loud Mouth Johnson.

"Sure," said Ford Fordsen.

They bet a thousand dollars. And, of course, Ford Fordsen won. Loud Mouth Johnson was so astounded that he slunk off and stayed away for six months. When he finally came back he found Paul Bunyan just starting to cut the belt in two lengthwise for the second time.

"What's the idea?" asked Loud Mouth Johnson.

Paul Bunyan said, "The tunnel has progressed much farther and the material coming out is not as bulky as it was. So I am lengthening the belt again and making it narrower."

"Where is Ford Fordsen?"

Paul Bunyan said, "I have sent him to town to get some materials to splice the belt. When I get through cutting it in two lengthwise pieces I will have two belts of the same length but only half the width of this one. So I will have to do some splicing."

Loud Mouth Johnson could hardly believe his ears. Here was a chance to get his thousand dollars back and show up Paul Bunyan as a boob besides. "Listen," said Loud Mouth Johnson, "when you get through you will have only one belt twice as long and half as wide."

"Want to bet?"

"Sure."

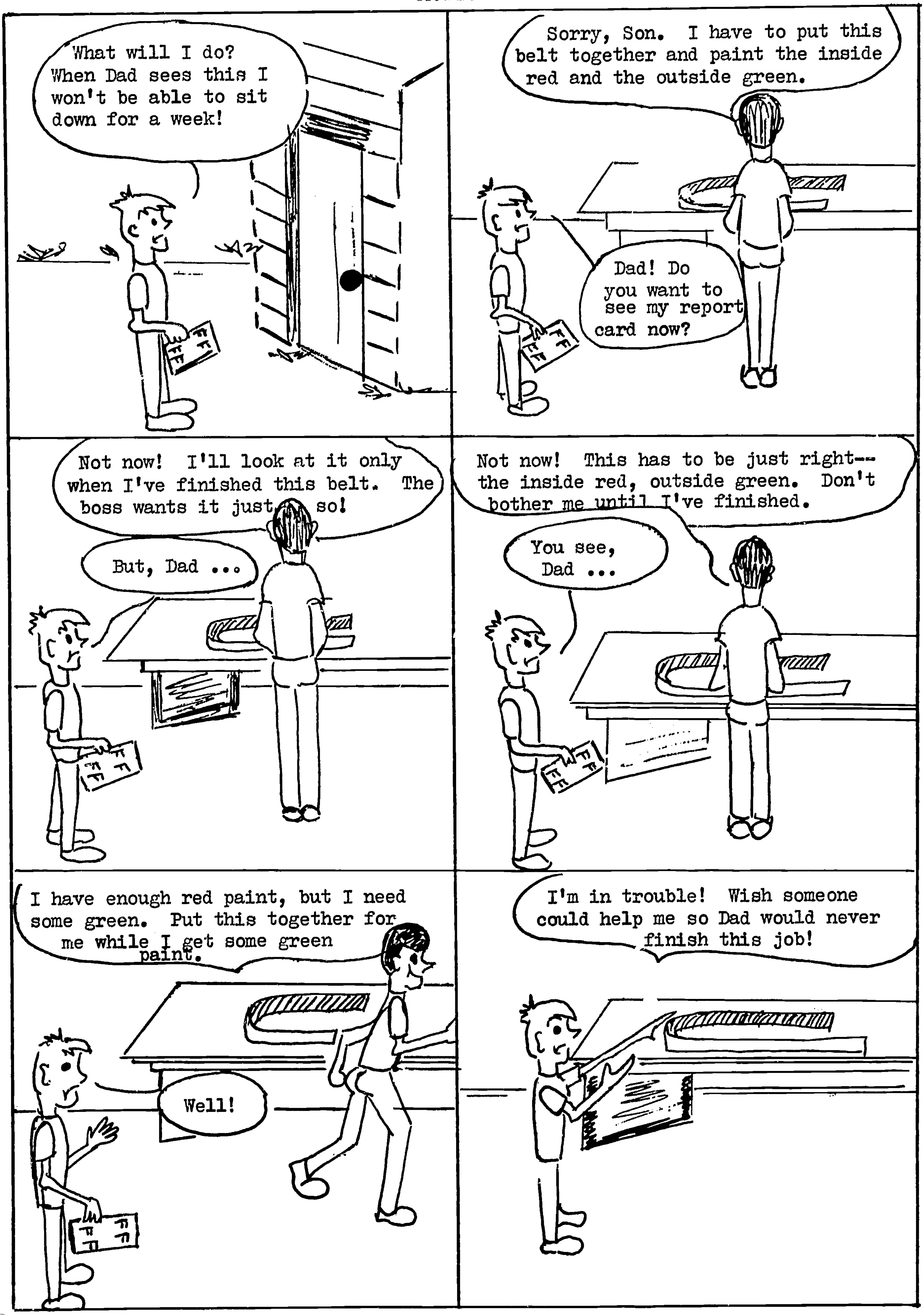
So they bet a thousand dollars and, of course, Loud Mouth Johnson lost again. It wasn't so much that Paul Bunyan was brilliant. It was just that he was methodical. He had tried it out with that strip of gummed paper, and he knew that the second time you slice a Moebius strip you get two pieces--linked together like an old-fashioned watch chain.

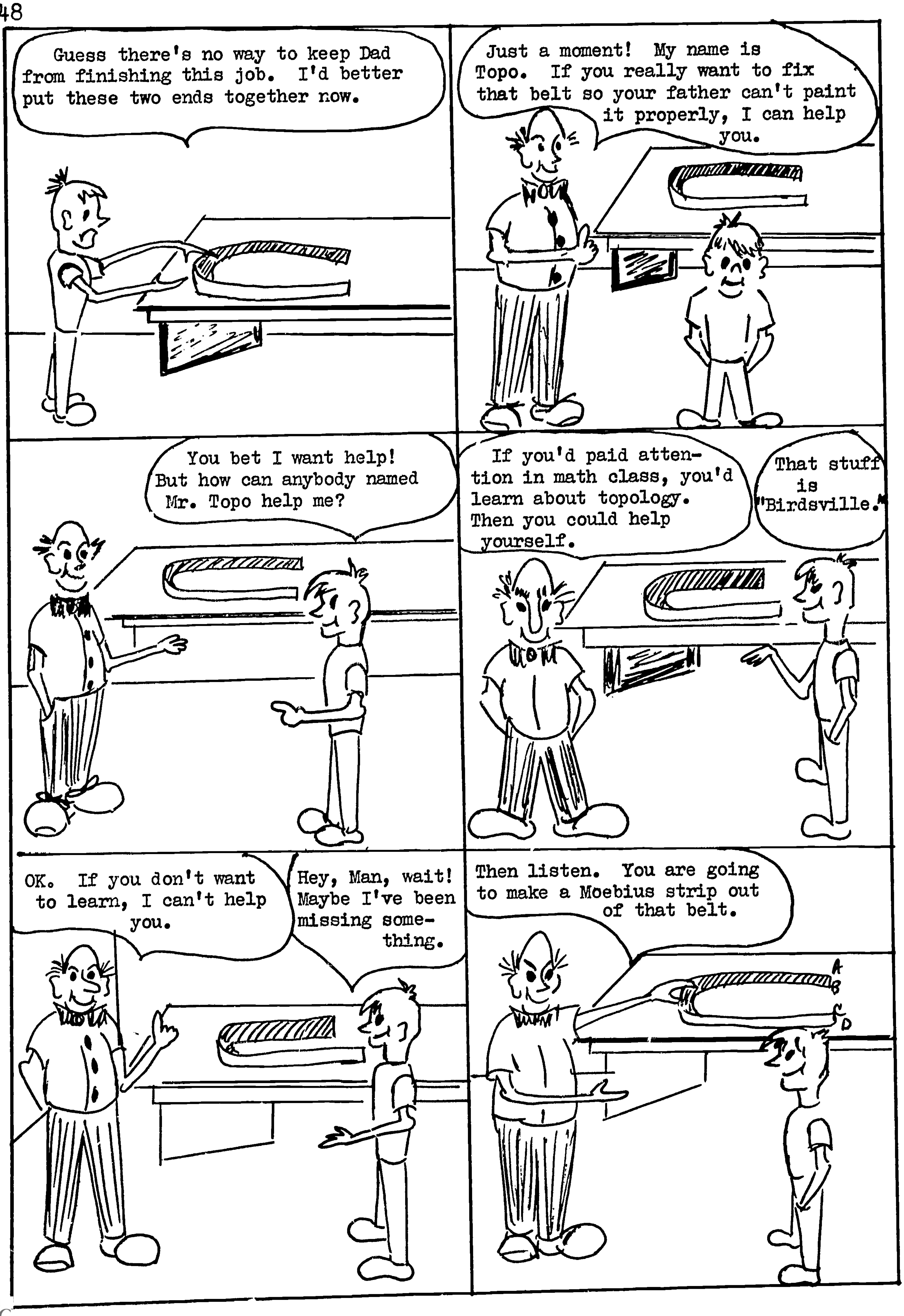
Make several Moebius strips in the ways indicated; then list your findings in the chart.

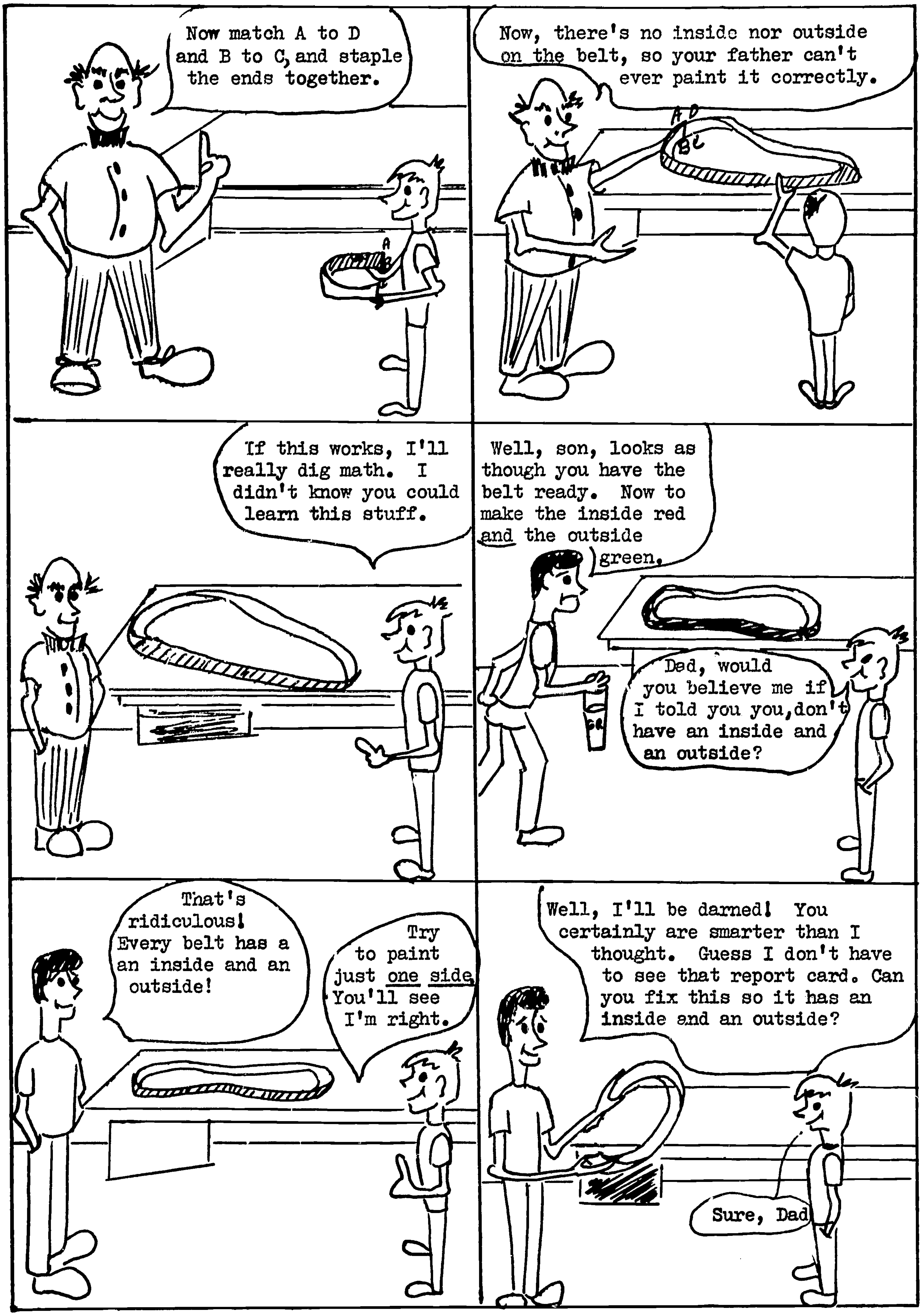
Number of Half-twists	Number of Sides and Edges	Kind of Cut	Number of Loops	Number of Knots
0	2 sides 2 edges	Center		
1	1 side 1 edge	Center		
1	1 side 1 edge	One-third		
2	2 sides 2 edges	Center		
2	2 sides 2 edges	One-third		
3	1 side 1 edge	Center		
3	1 side 1 edge	One-third		

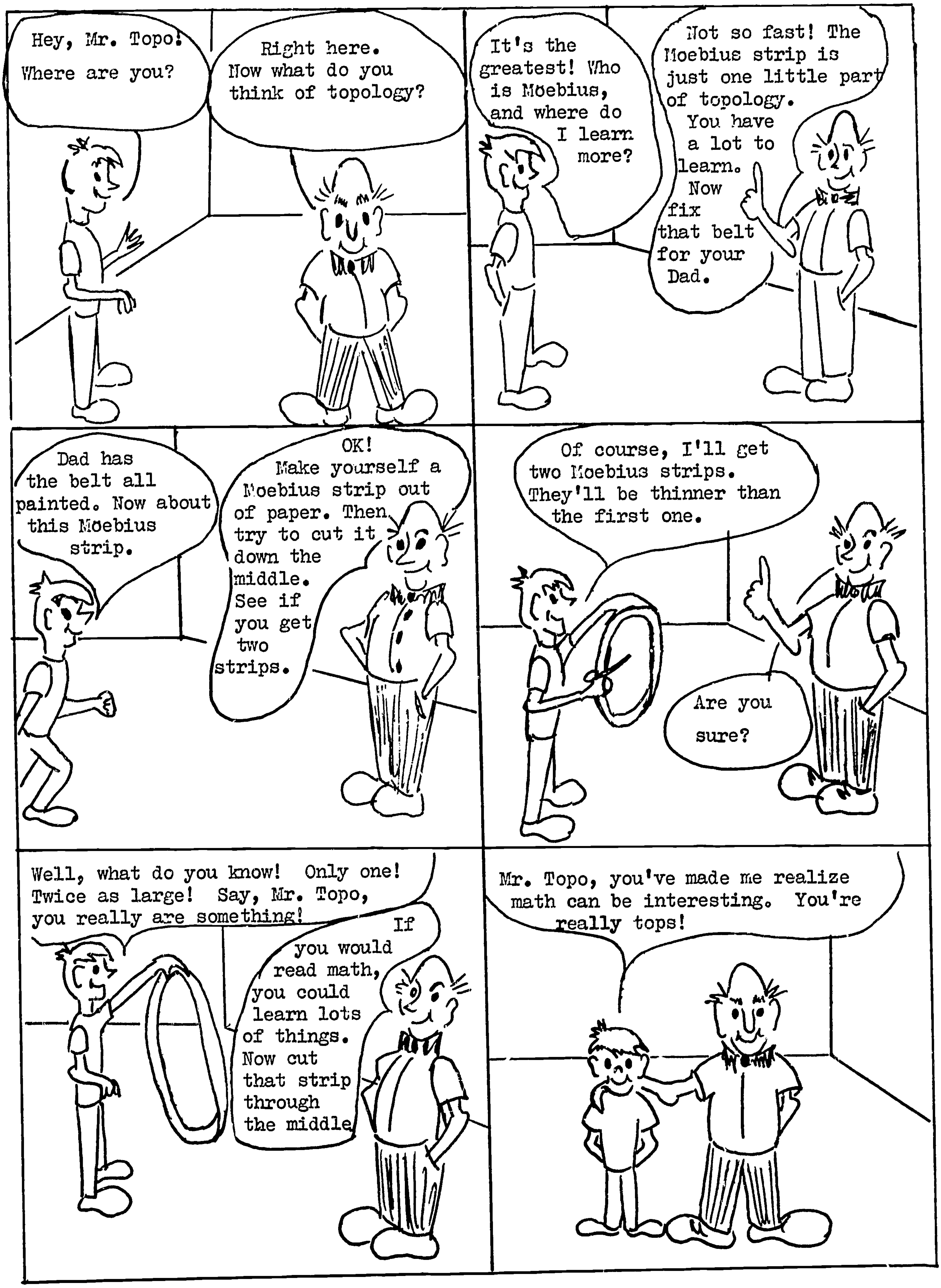
A mathematician confided
That a Moebius band is one-sided,
And you'll get quite a laugh
If you cut one in half,
For it stays in one piece when divided.

MOEBIUS STRIP




Guess there's no way to keep Dad from finishing this job. I'd better put these two ends together now.
Just a moment! My name is Topo. If you really want to fix that belt so your father can't paint it properly, I can help you.
You bet I want help! But how can anybody named Mr. Topo help me?
If you'd paid attention in math class, you'd learn about topology. Then you could help yourself.
That stuff is "Birdsville."
OK. If you don't want to learn, I can't help you.
Hey, Man, wait! Maybe I've been missing something.
Then listen. You are going to make a Moebius strip out of that belt.
A
B
C
D

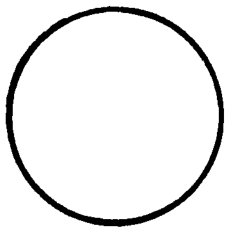

Now match A to D and B to C, and staple the ends together.
A
Now, there's no inside nor outside on the belt, so your father can't ever paint it correctly.
A D
B C
If this works, I'll really dig math. I didn't know you could learn this stuff.
Well, son, looks as though you have the belt ready. Now to make the inside red and the outside green.
Dad, would you believe me if I told you you don't have an inside and an outside?
That's ridiculous! Every belt has a an inside and an outside!
Try to paint just one side. You'll see I'm right.
Well, I'll be darned! You certainly are smarter than I thought. Guess I don't have to see that report card. Can you fix this so it has an inside and an outside?
Sure, Dad


Hey, Mr. Topo! Where are you?
Right here. Now what do you think of topology?
It's the greatest! Who is Moebius, and where do I learn more?
Not so fast! The Moebius strip is just one little part of topology. You have a lot to learn. Now fix that belt for your Dad.
Dad has the belt all painted. Now about this Moebius strip.
OK! Make yourself a Moebius strip out of paper. Then try to cut it down the middle. See if you get two strips.
Of course, I'll get two Moebius strips. They'll be thinner than the first one.
Are you sure?
Well, what do you know! Only one! Twice as large! Say, Mr. Topo, you really are something!
If you would read math, you could learn lots of things. Now cut that strip through the middle
Mr. Topo, you've made me realize math can be interesting. You're really tops!

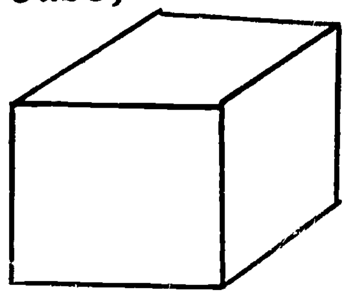
THREE DIMENSIONAL TOPOLOGY

If a child has some modeling clay rolled into the shape of

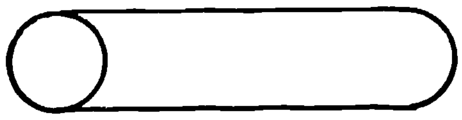
a ball,



he can change it to a cube,

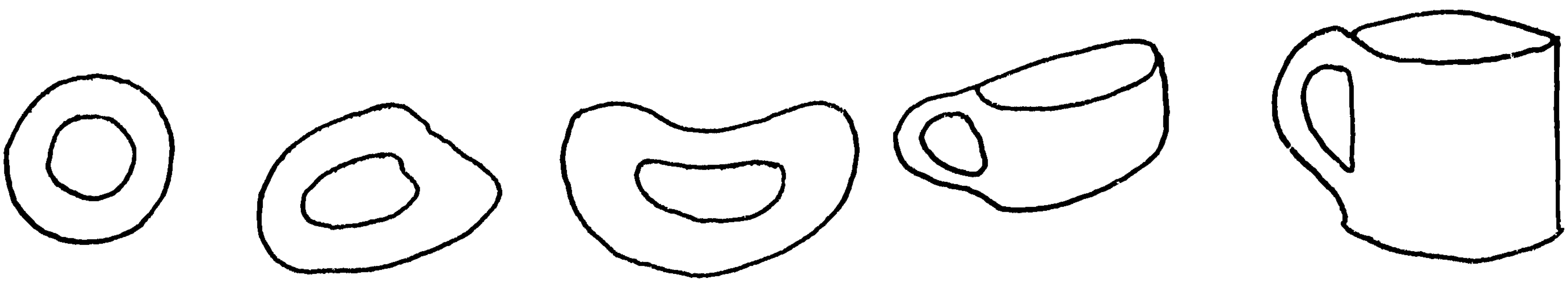


or a sausage,

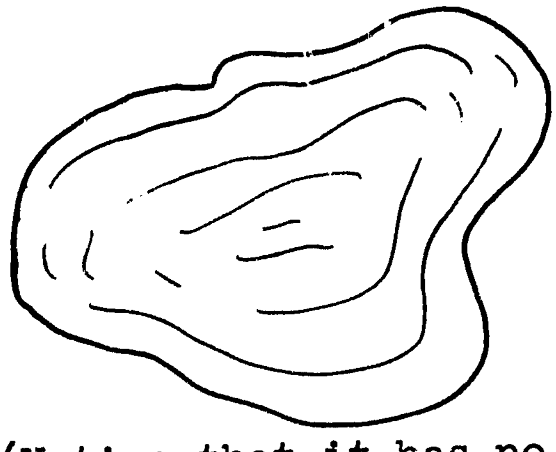


and it will be the same topologically.

This gets us back to the first problem we had in this book. Is a doughnut the same in topology as a coffee cup?



All topological transformations involve a property called the "genus" of a surface. Genus is defined according to the number of holes an object has. Topologists say genus is defined by the number of non-intersecting closed or completely circular cuts that can be made on the surface without cutting it into two pieces. Thus the ball is a genus, so is the cube, disk or even something that looks like this.

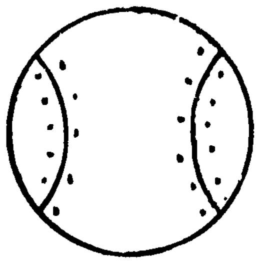


(Notice that it has no holes.)

The doughnut and the coffee cup have only one hole and are classified as "genus 1."

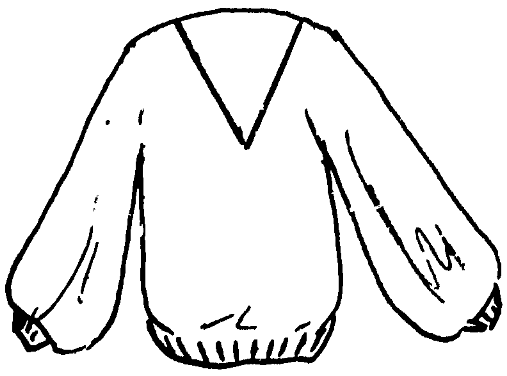
A sugar bowl with two handles would be "genus 2."

Classify each of the following objects:



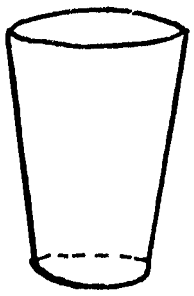
Baseball

Genus ______



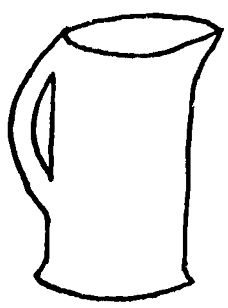
Sweater

Genus ______



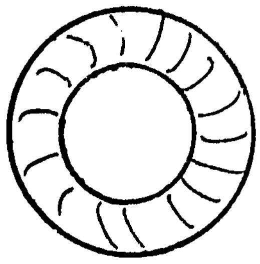
Drinking
Glass

Genus ______



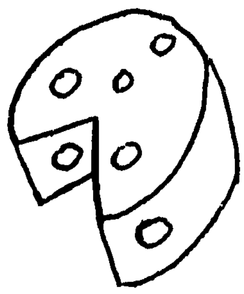
Pitcher

Genus ______



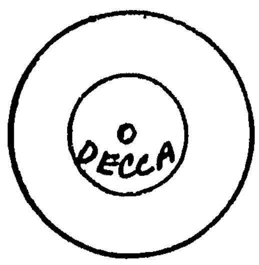
Paper Plate

Genus _______



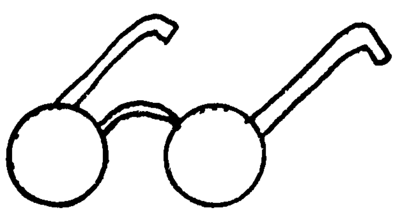
Mouse's Delight
Swiss Cheese

Genus _______



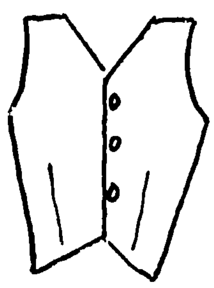
Record

Genus _______



Glasses

Genus _______



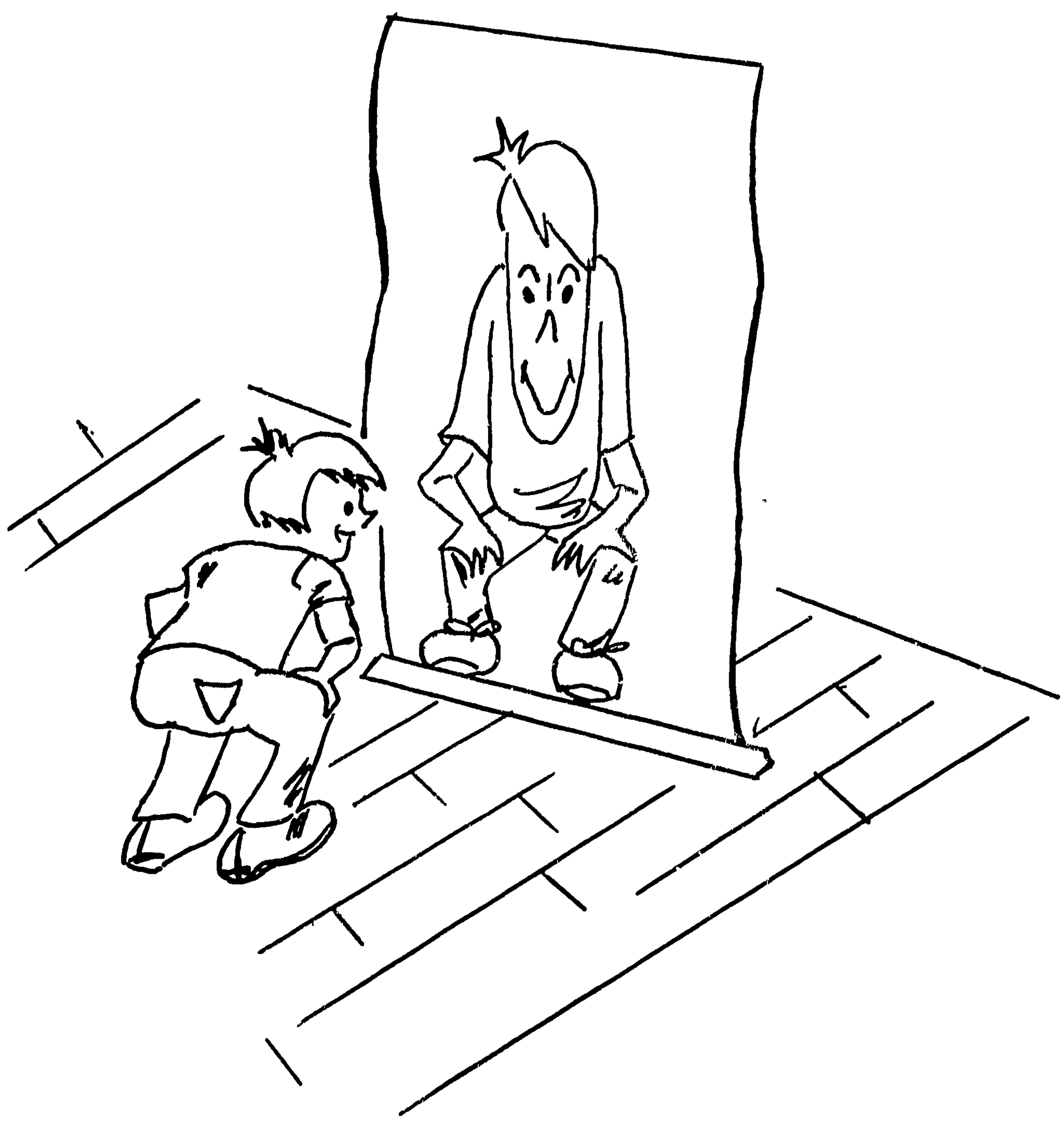
Vest

Genus _______

Any solid that fits a given genus is considered to be topologically equivalent to any other solid in the same genus.

A <u>topological transformation</u> means that an object has the same <u>set of points, but that they</u> can be distorted.

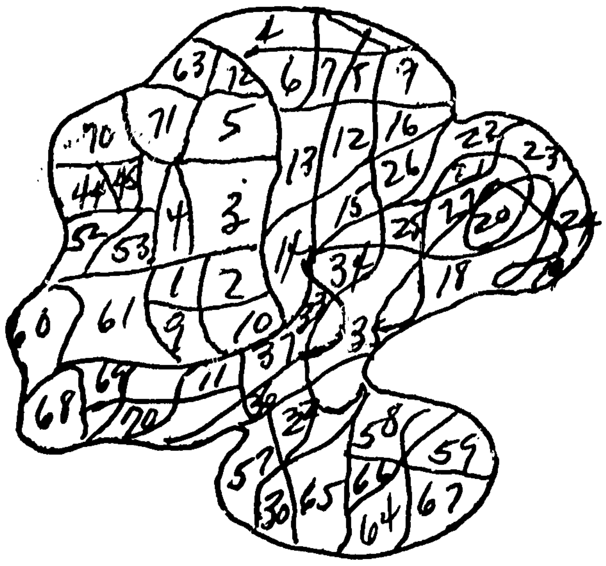
Have you ever looked at yourself in a wavy mirror?



Is the image in the mirror still you? Actually your nose is still in the middle of your face. Your head is still above your shoulders; everything is in the same order.

This is an example of a topological transformation.

Is this



the same sheet of paper as this?

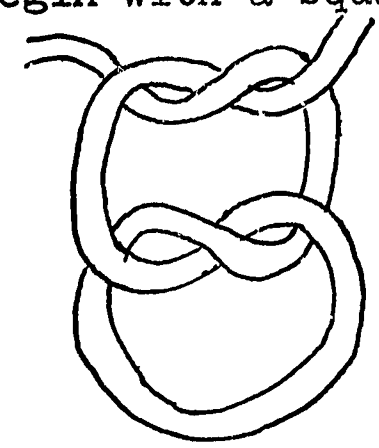
1	2	3	4	5	6	7	8
9	10	11	12	13	14	15	16
17	18	19	20	21	22	23	24
25	26	27	28	29	30	31	32
33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48
49	50	51	52	53	54	55	56
57	58	59	60	61	62	63	64
65	66	67	68	69	70	71	72

It could be--one is lying flat and one is crumpled up. Topologically they are the same.

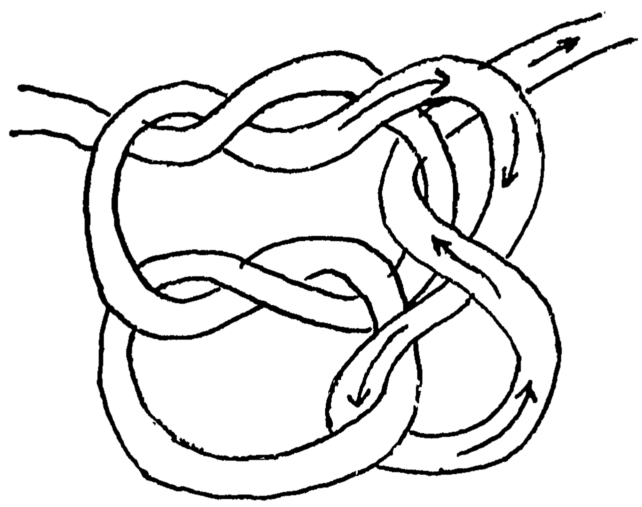
KNOTS AND SUCH

Here are some knots and twists of a topological nature you might like to try.

A. <u>Chefalo Knot</u>. Begin with a square knot.



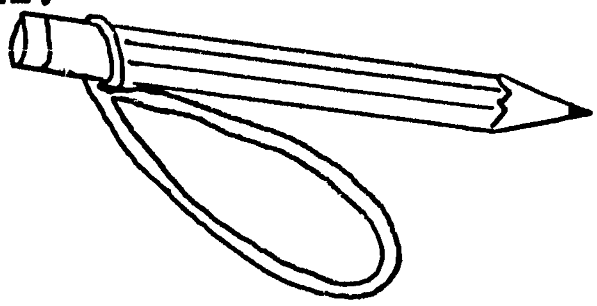
Now, weave one end as shown by arrows.



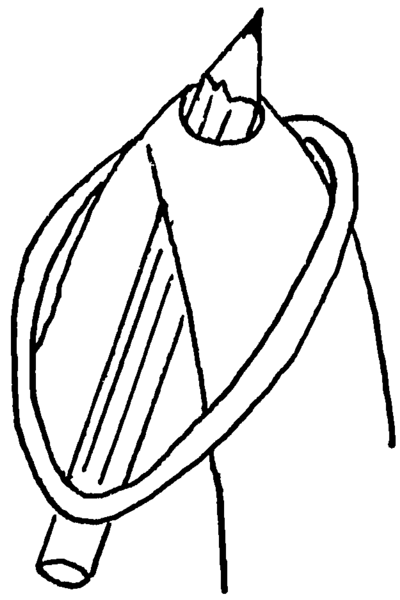
Now, pull both ends and watch the knot disappear.

B. <u>Buttonhole</u> <u>with</u> <u>pencil</u>.

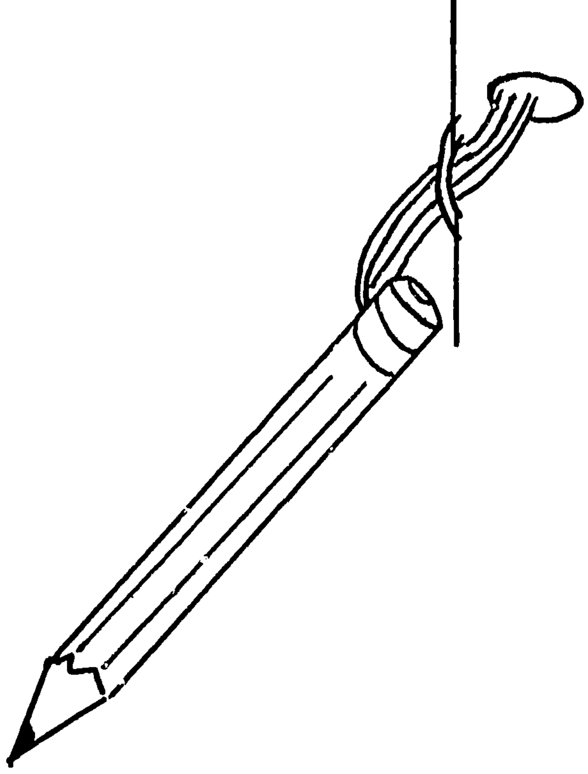
Tie a loop of string to a pencil. Be sure loop is shorter than the pencil.



Now, attach the pencil to the buttonhole of a friend's jacket without untying the loop. Practice this so you can do it rapidly so your friend won't see what you are doing.

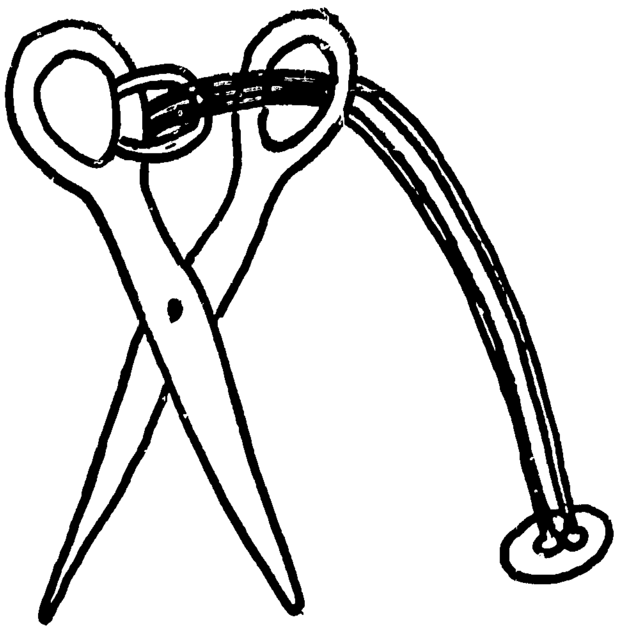


And pull it tight.



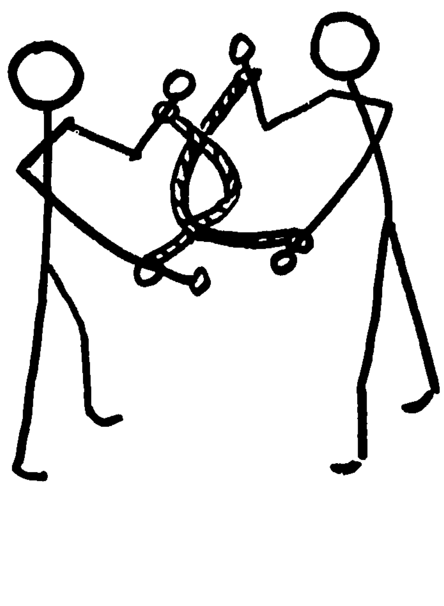
Now, ask him to remove it without untying or cutting the string or breaking the pencil. We assume he won't ruin his coat!

Try this same idea with scissors and a string tied to a button. Be sure the button is too large to fit through the finger holes on the scissors.



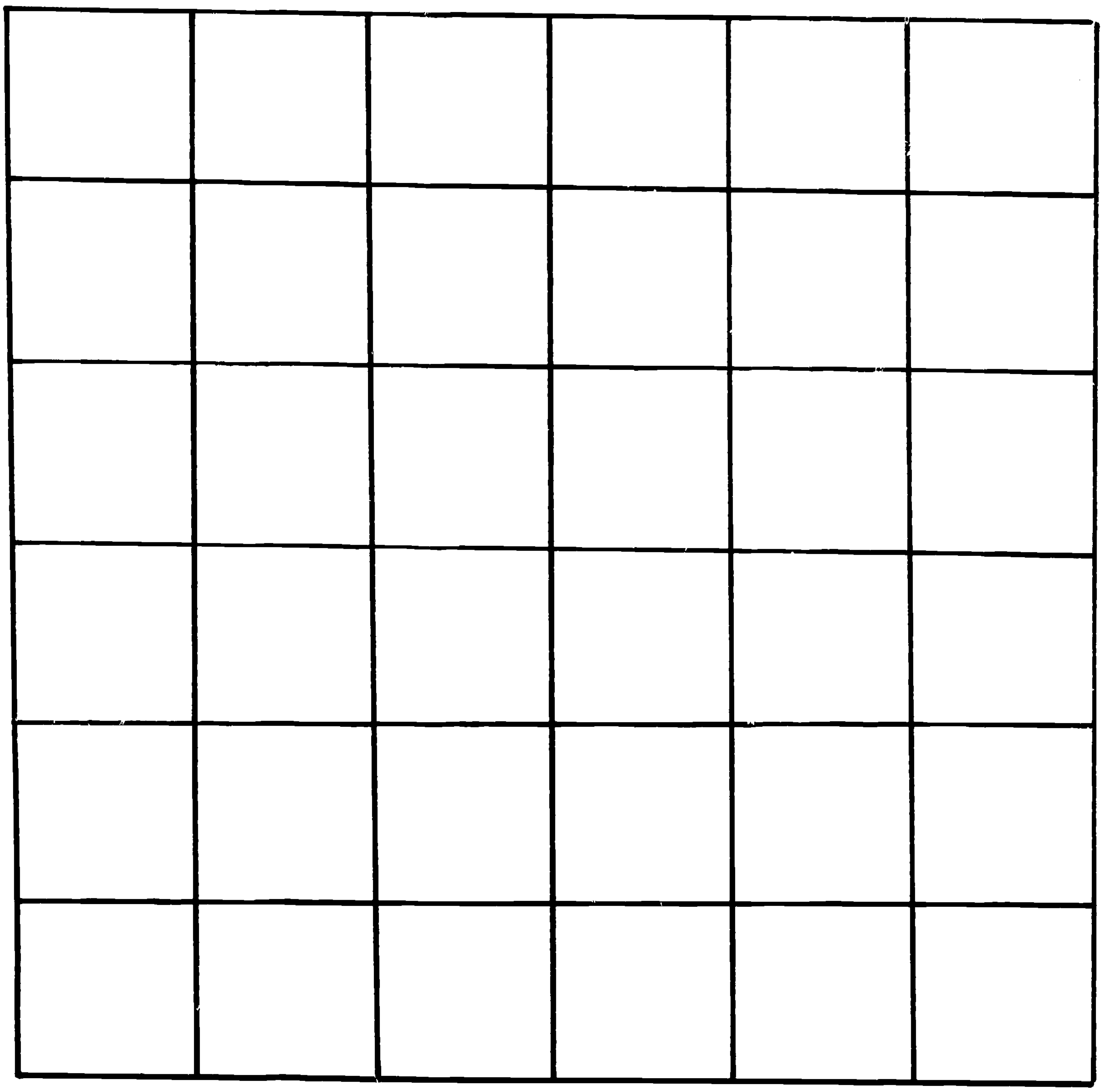
C. Togetherness?

Tie a piece of string to each of your wrists. Tie a second piece of string to each of the wrists of a partner in such a way that the second string loops the first. The object of this stunt is to separate yourself from your partner without cutting the string, untying the knots, or taking the string off your wrists. Try it. You can do it.



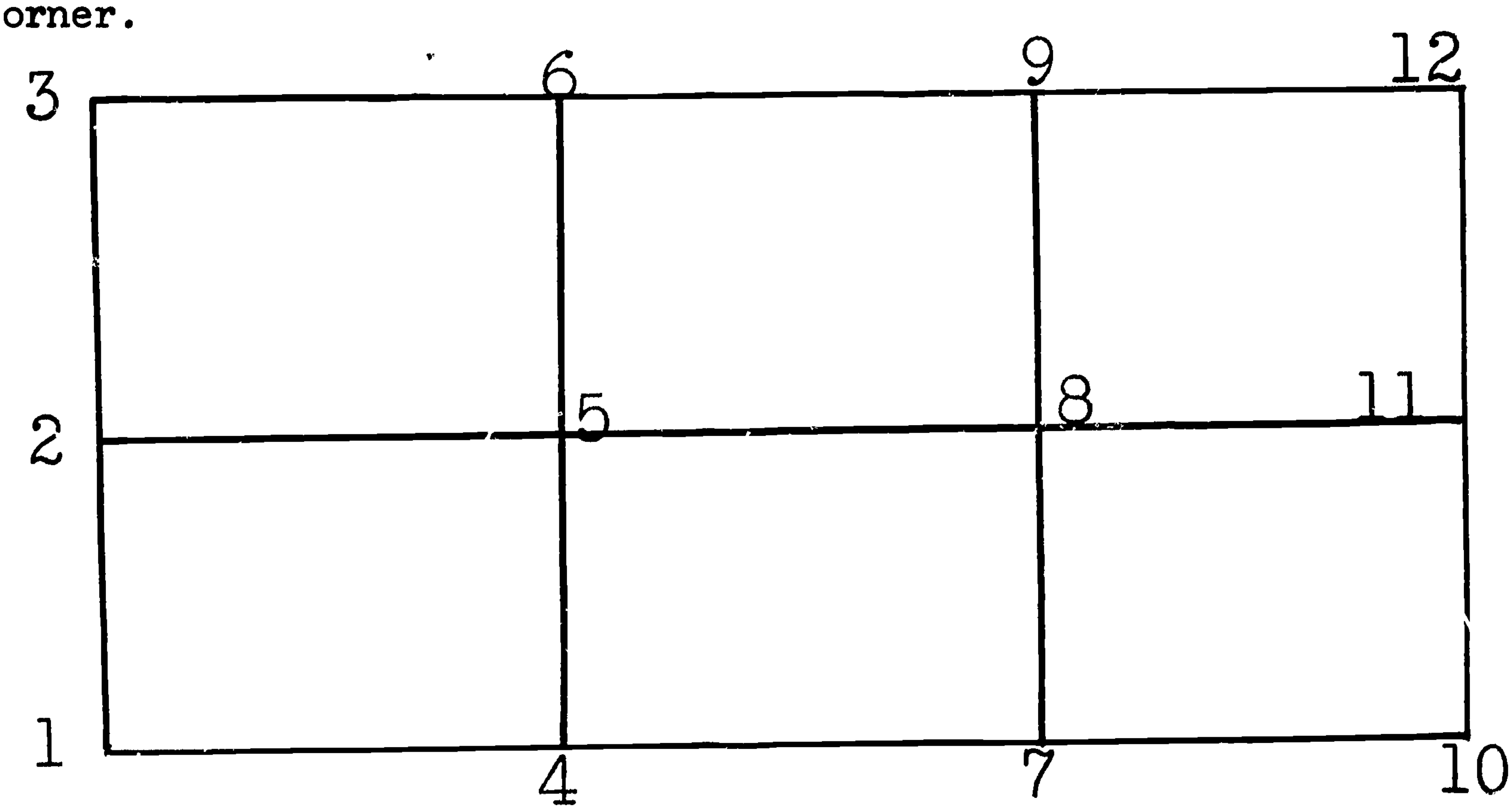
D. Pennies on the square.

Place six pennies on the 6 X 6 checkerboard below so that no penny is in line with another penny horizontally, vertically, or diagonally. No square may have more than one penny.



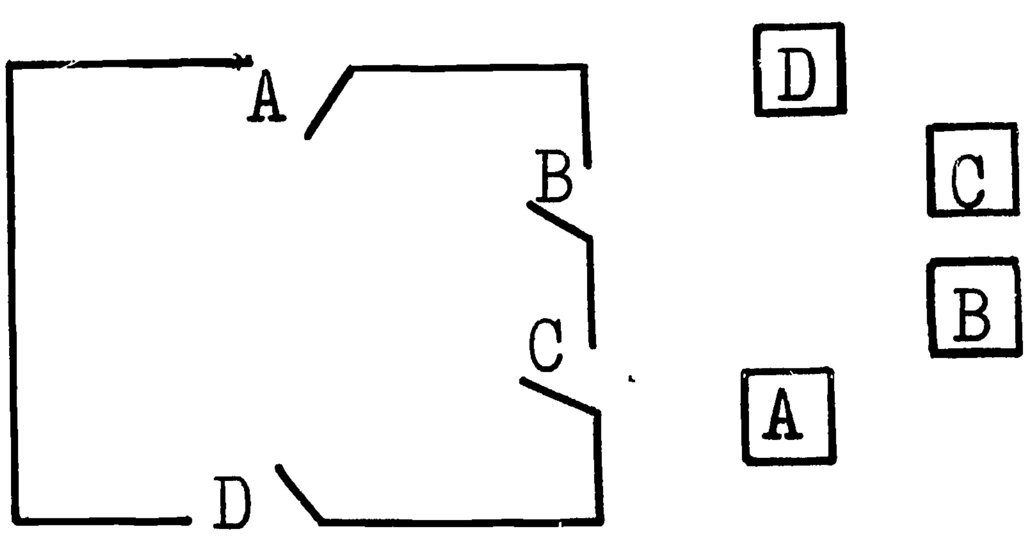
E. Paper boy's route.

A paper boy must deliver papers on each of these streets. To cover the area he must walk (ride?) some blocks twice. How should he plan his route so he can cover it by traveling the least number of blocks possible? He may start at any corner.



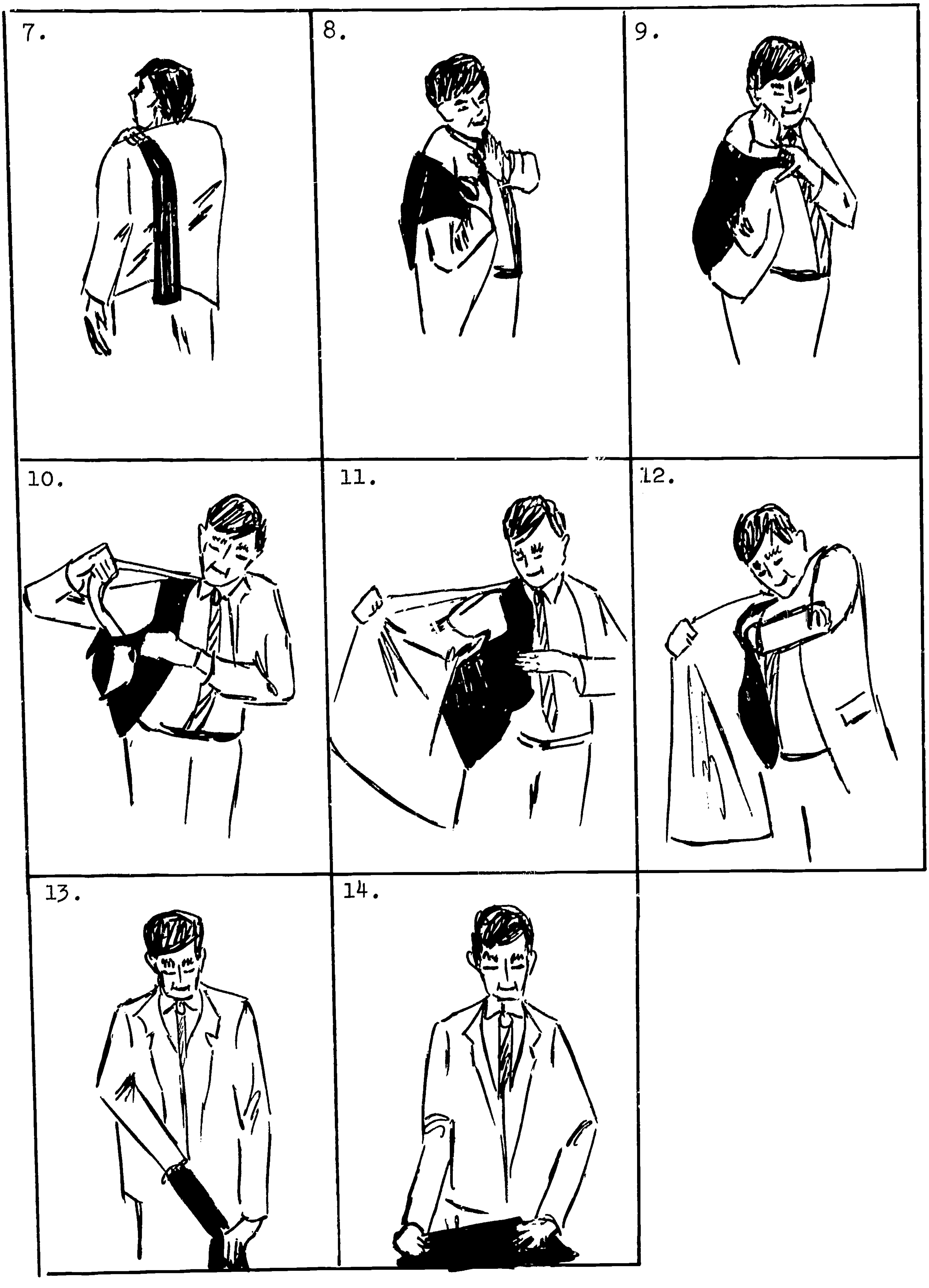
F. How to separate the boys.

Four schoolboys live at homes A, B, C, and D. They go to your school, but, since they fight when they meet, they must enter the school by doors A, B, C, and D. Boy A lives in home A and goes to door A; boy B goes from home B to door B; and so on. How can they go to school without ever crossing the path of one of the other boys?



G. First, take off your vest! But <u>don't</u> take off your coat!

Let's see you take off a vest without removing your coat. Put on a vest and a coat. If the vest is large, the stunt is easier to do. The coat may be unbuttoned, but you are not permitted to let your arms slip out of your coat sleeves. The following pictures show the steps. Practice in front of a mirror until you are good before you try this on your friends. They may be willing to bet you can't do it.


7.
8.
9.
10.
11.
12.
13.
14.