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METRIC GEOMETRY, CONCEPTS OF AREA MEASURE.

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THIS BOOKLET, ONE OF A SERIES, HAS BEEN DEVELOPED FOR THE PROJECT, A PROGRAM FOR MATHEMATICALLY UNDERDEVELOPED PUPILS. A PROJECT TEAM, INCLUDING INSERVICE TEACHERS, IS BEING USED TO WRITE AND DEVELOP THE MATERIALS FOR THIS PROGRAM. THE MATERIALS DEVELOPED IN THIS BOOKLET INCLUDE SUCH CONCEPTS AS (1) VARIOUS UNITS OF MEASURE, (2) FINDING AREAS OF ELEMENTARY CONFIGURATIONS--PARALLELOGRAMS, TRIANGLES, AND TRAPEZOIDS--BY DECOMPOSITION, (3) AREA FORMULAS, AND (4) APPLICATIONS OF THE APPROPRIATE AREA FORMULA TO CIRCLES. ACCOMPANYING THESE BOOKLETS WILL BE A "TEACHING STRATEGY BOOKLET" WHICH WILL INCLUDE A DESCRIPTION OF TEACHER TECHNIQUES, METHODS, SUGGESTED SEQUENCES, ACADEMIC GAMES, AND SUGGESTED VISUAL MATERIALS. (RP)

ESEA Title III

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CONCEPTS OF AREA MEASURE

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CONCEPTS OF AREA MEASURE

Units of Measure

To find the length of a line segment we selected another line segment as our unit of measure. To measure area, we will select a portion of area for our unit.

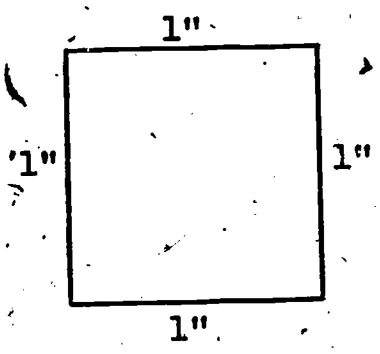
Use your hand, since it is handy, to estimate the area of your desk top. Now estimate the area of the front of your math textbook using your hand. Complete the table below to show how many "hands" it takes to cover the surface of your desk and how many "hands" it takes to cover the front of your textbook. Try the same experiment using the area of your rectangular ruler.

Description of the Unit of Measure	Name of the Unit	Book Front (area)	Desk Top (area)
Area of hand	handarea (ha.)		
Area of ruler	rulerarea (ra.)		

2. Use the square inch grid furnished by your teacher to determine the area of your desk top and the front of your math textbook and then complete the following table.

Description of the Unit of Measure	Name of the Unit	Book Front (area)	Desk Top (area)
a square 1 inch on a side	square inch (sq. in.)		

Just as a unit had to be selected to measure length, there must also be a unit selected to measure area. Usually this unit is a square. A square, of course, is a plane geometric figure with four right angles and with the lengths of all four sides equal. A picture of a square with sides of 1 inch in length is illustrated here. In relation to area, this is termed a square inch.



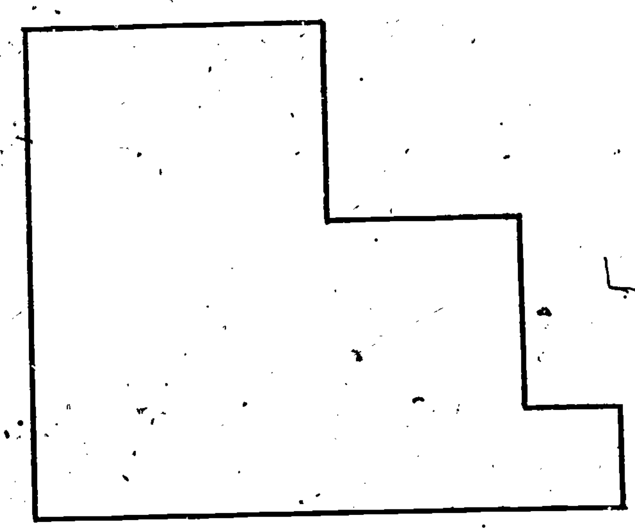
The unit of area may be a square with a side length of one centimeter, one inch, one foot, one mile, 208 feet, or any other measure. The size of the square will depend upon the size of the surface to be measured.

Squares were probably selected as units for measuring area because they "fit together". Try a circle as a unit and see what happens.

Activities

Use the <u>square centimeter</u> grid to determine the area enclosed within the following closed curves.

1.



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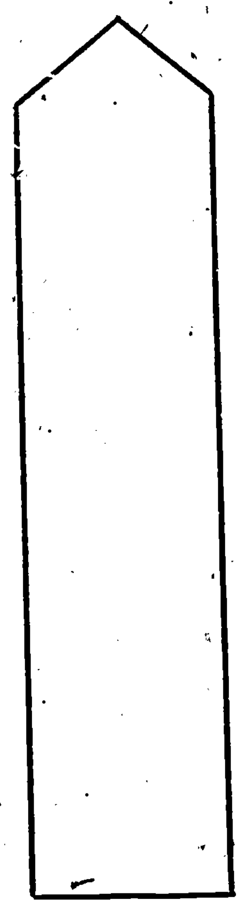

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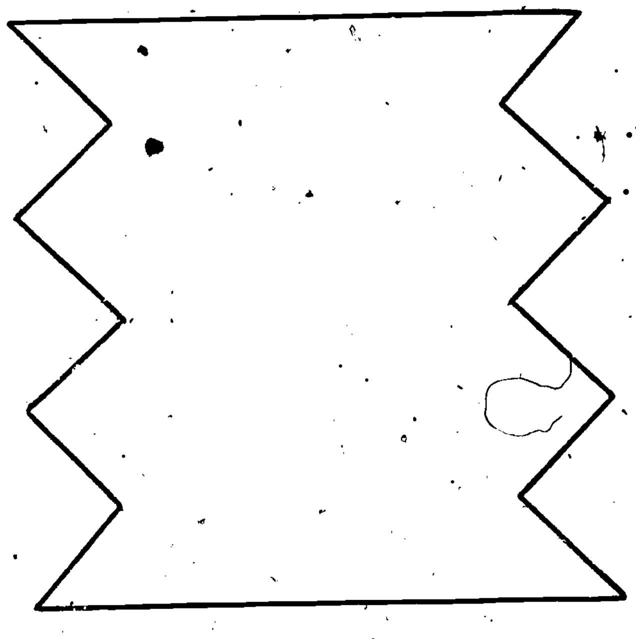

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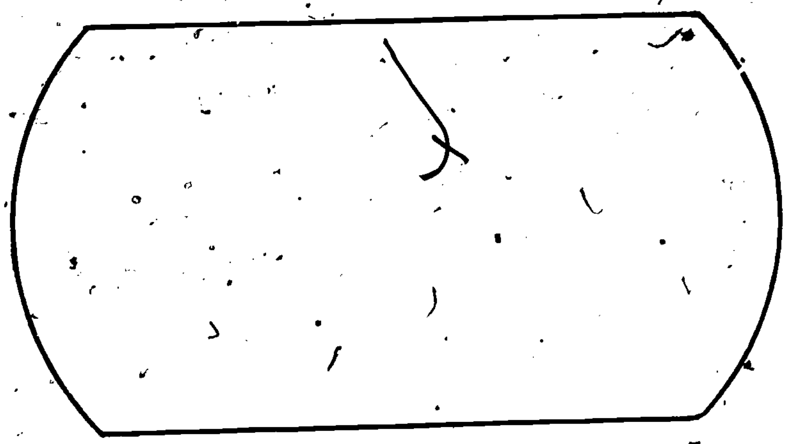

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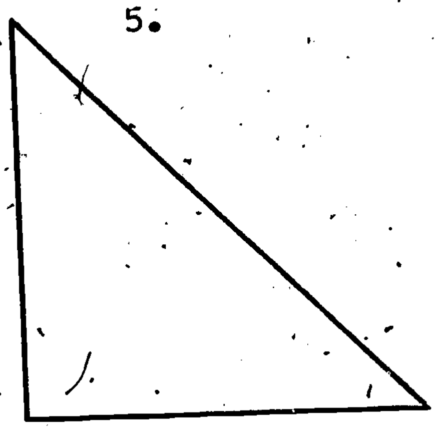

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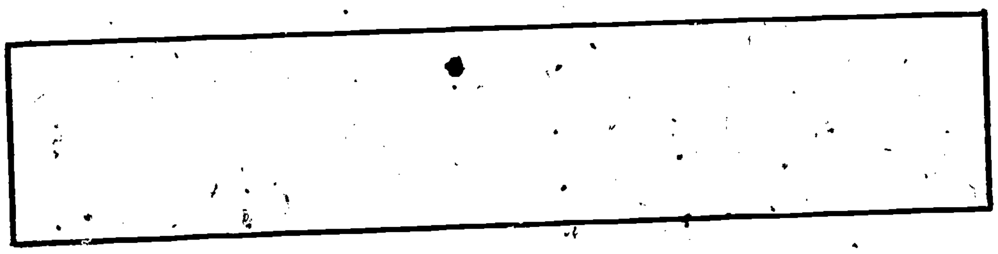

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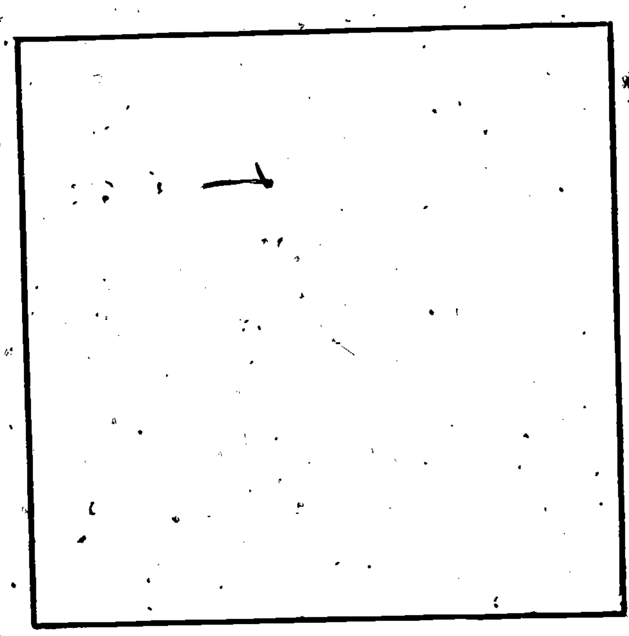
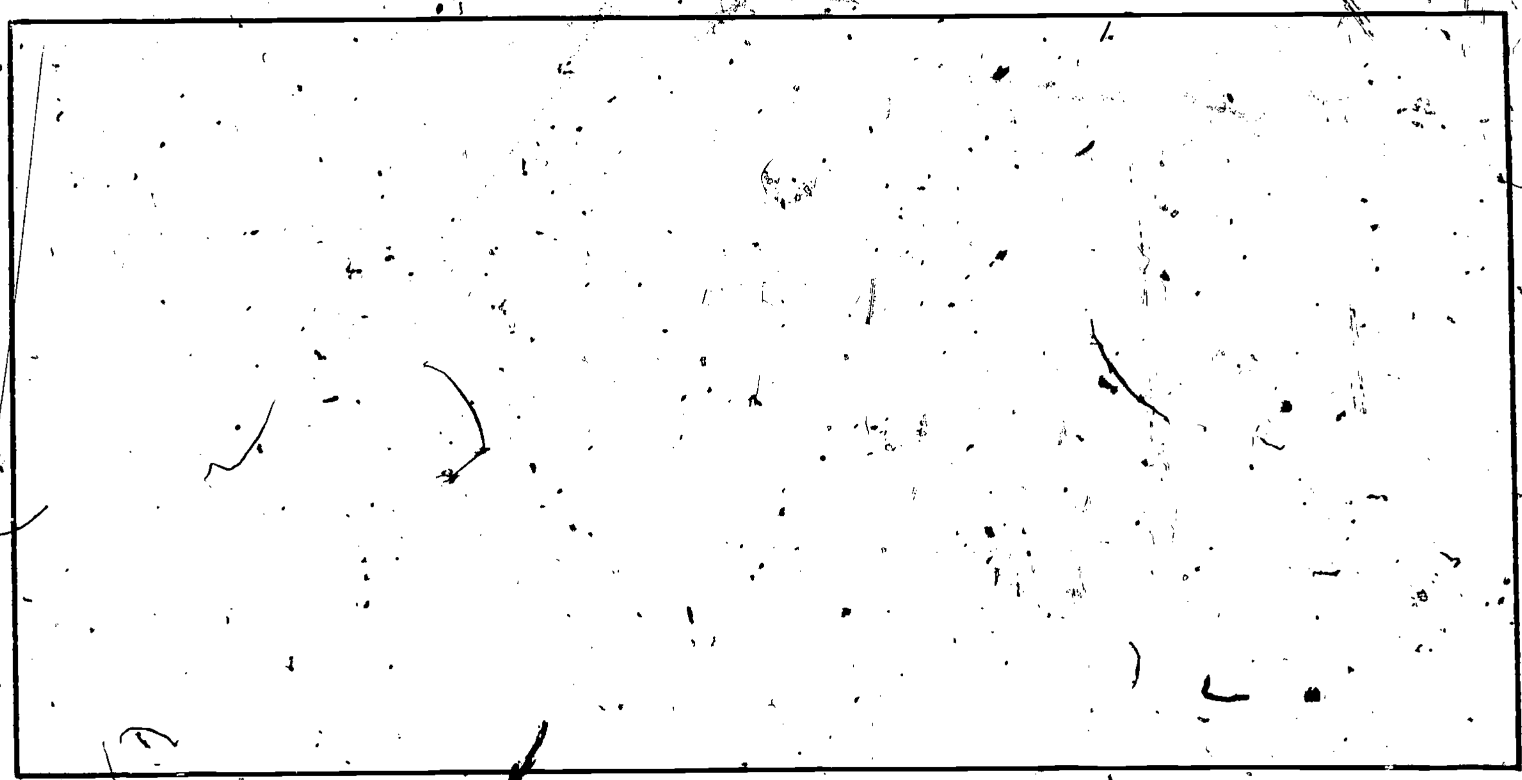


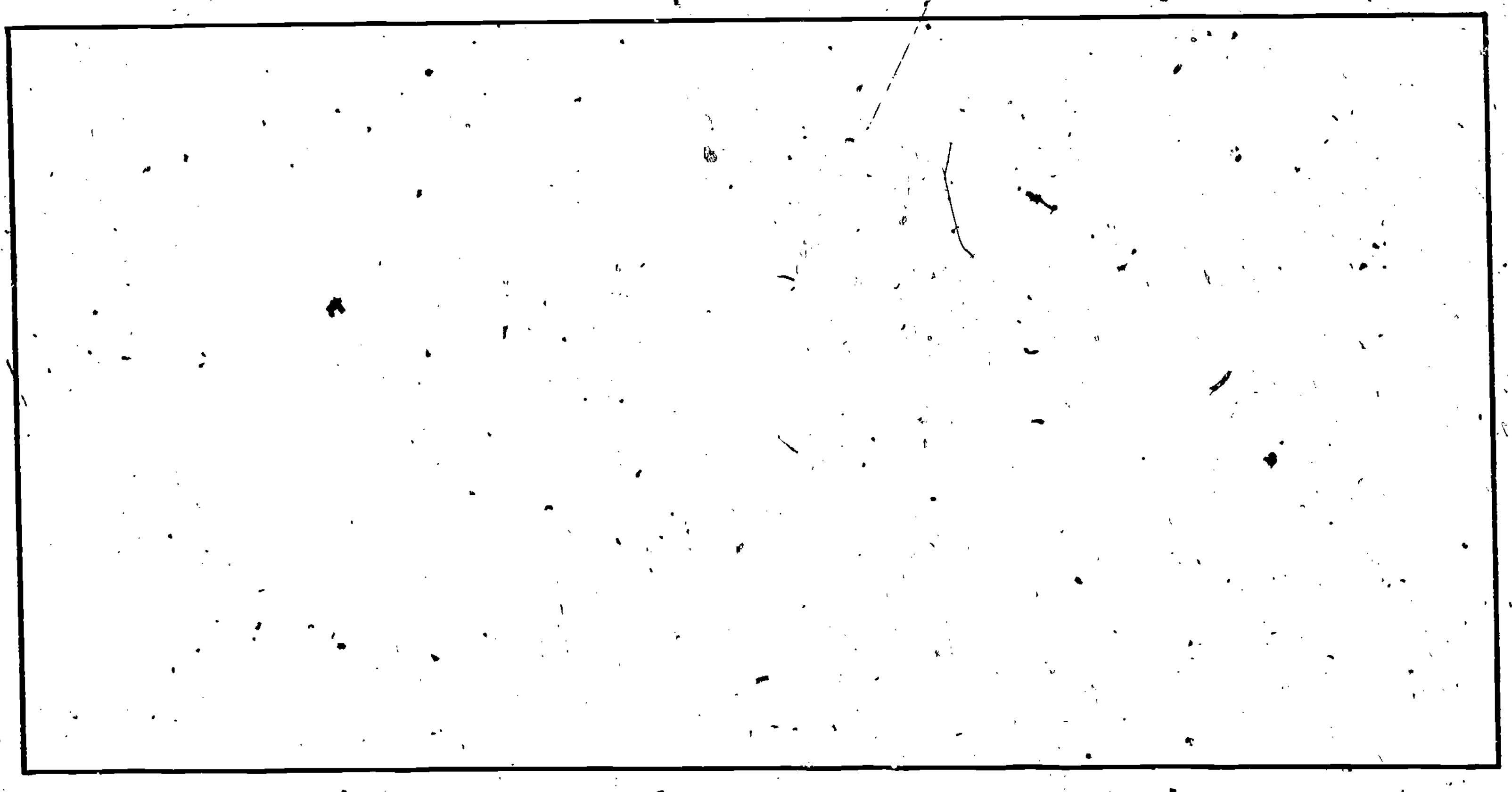
8.

In problems 10 - 11 use the square inch grid to determine the area within the plane geometric figures.

10.



11.



12. If a rectangle is 9 inches in length and 5 inches in height, what is the area enclosed within the rectangle? Give your answer in square inches.

____

13. Of the answers given to problems 8 and 11, which do you think is closer to the exact area? __________ Why? ________________

____

Each of the problems 14 - 15 is "solved" for you. However, the answer is incorrect. Correct it.

14. What is the area enclosed by a rectangle with length 15 inches and height 5 inches? Answer: 75 inches.

15. What is the area enclosed within a rectangle with length 9 feet and height 11 feet. Answer: 99 square inches.

In each of the problems 16 - 21 determine the area enclosed within the rectangle with the given dimensions:

16. Length: 12 inches

Height: 9 inches

17. Length: 16 feet

Height: 12 feet

18. Length: 25 inches

Height: 3 feet

19. Length: 12 inches

Height: 12 inches

20. Length: 1 foot

Height: 1 foot

21. Length: b units

Height: h units

22. How many square inches are contained in one square foot? ________

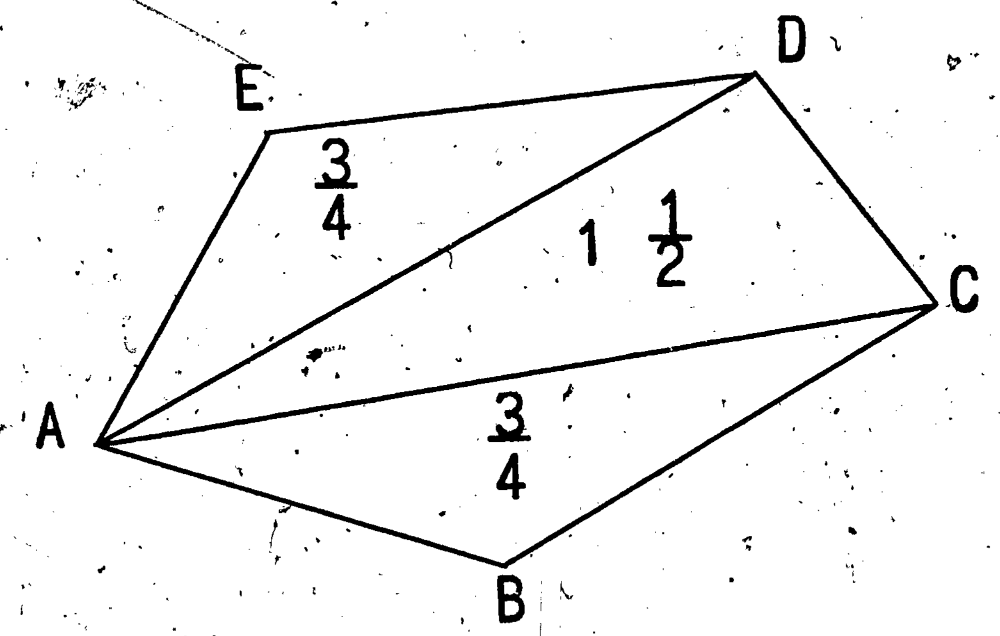
23. How many square feet are contained in one square yard? ________

24. An acre is a measure of area, and it is approximately 208 feet by 208 feet if it is in the shape of a square. Approximately how many square feet are contained in an acre? ________

Suppose a small garden of one acre was planted. One-fourth was planted in beans, one-fourth in turnip greens, one-fourth in tomatoes, and one-fourth in potatoes. What is the size of the garden? Does the fact that the acre of land is divided into four smaller parts change the amount of land in the garden? You can see that it certainly does <u>not</u>. There is one acre of garden here.

Beans	Turnips
Tomatoes	Potatoes

If a plane geometric figure is partitioned into other smaller plane geometric figures, and the areas within these smaller figures are added, this sum is the area of the larger figure.

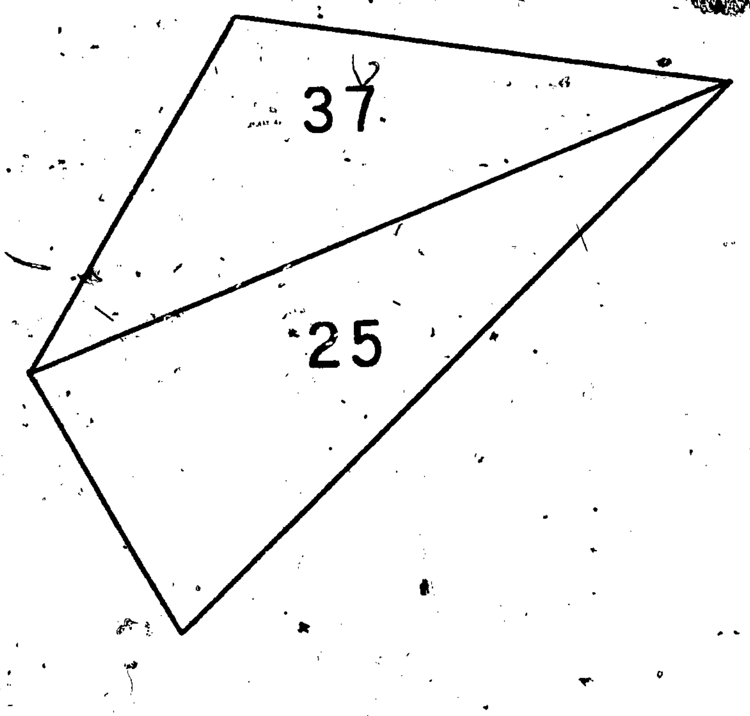


The area within the larger figure is the sum of the areas within each triangle. This area is $\frac{3}{4} + 1\frac{1}{2} + \frac{3}{4}$ or 3 square units.

Activities

In the problems 1 - 9 you are to find the area within the larger plane geometric figures by adding the areas within the smaller ones. Your answers should be given in "square units" since the numbers are not necessarily square inches, square centimeters, etc.

1.



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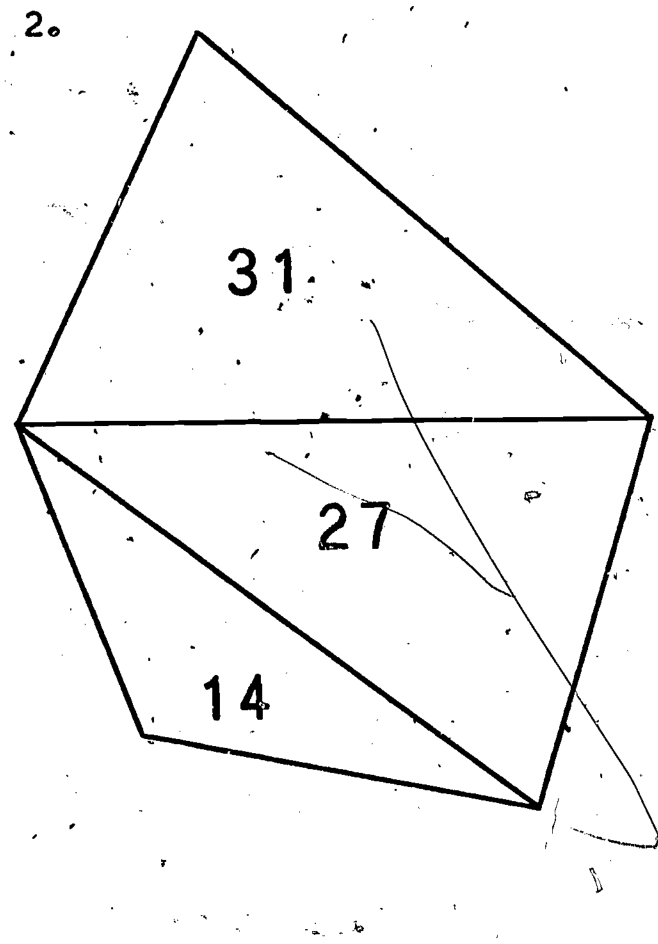

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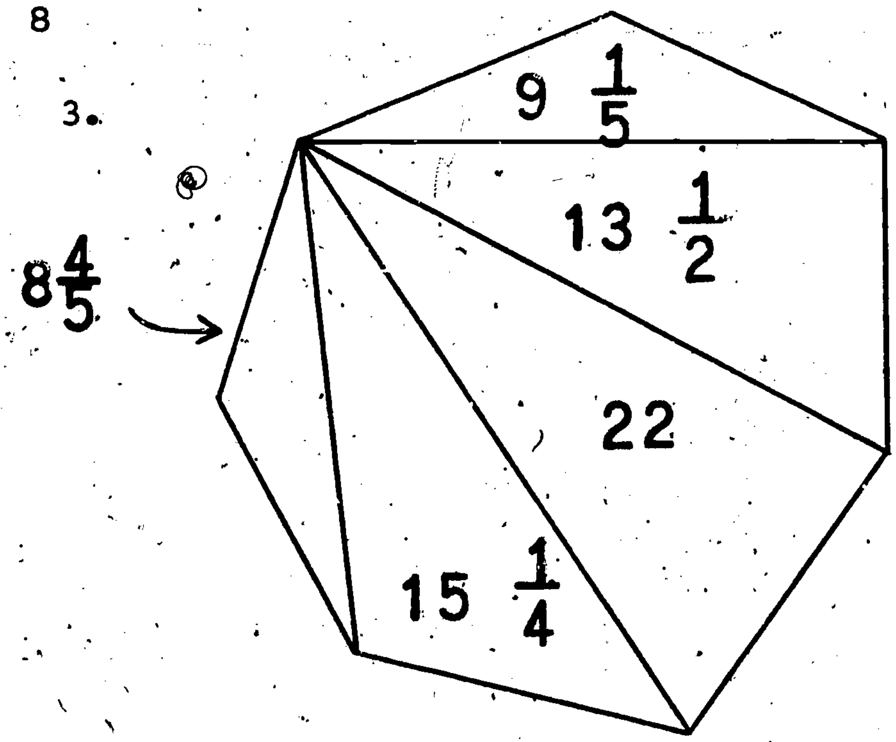
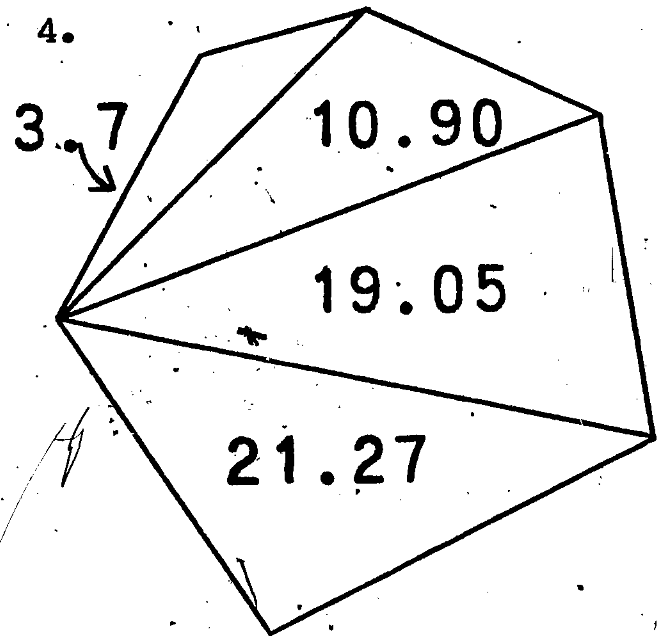
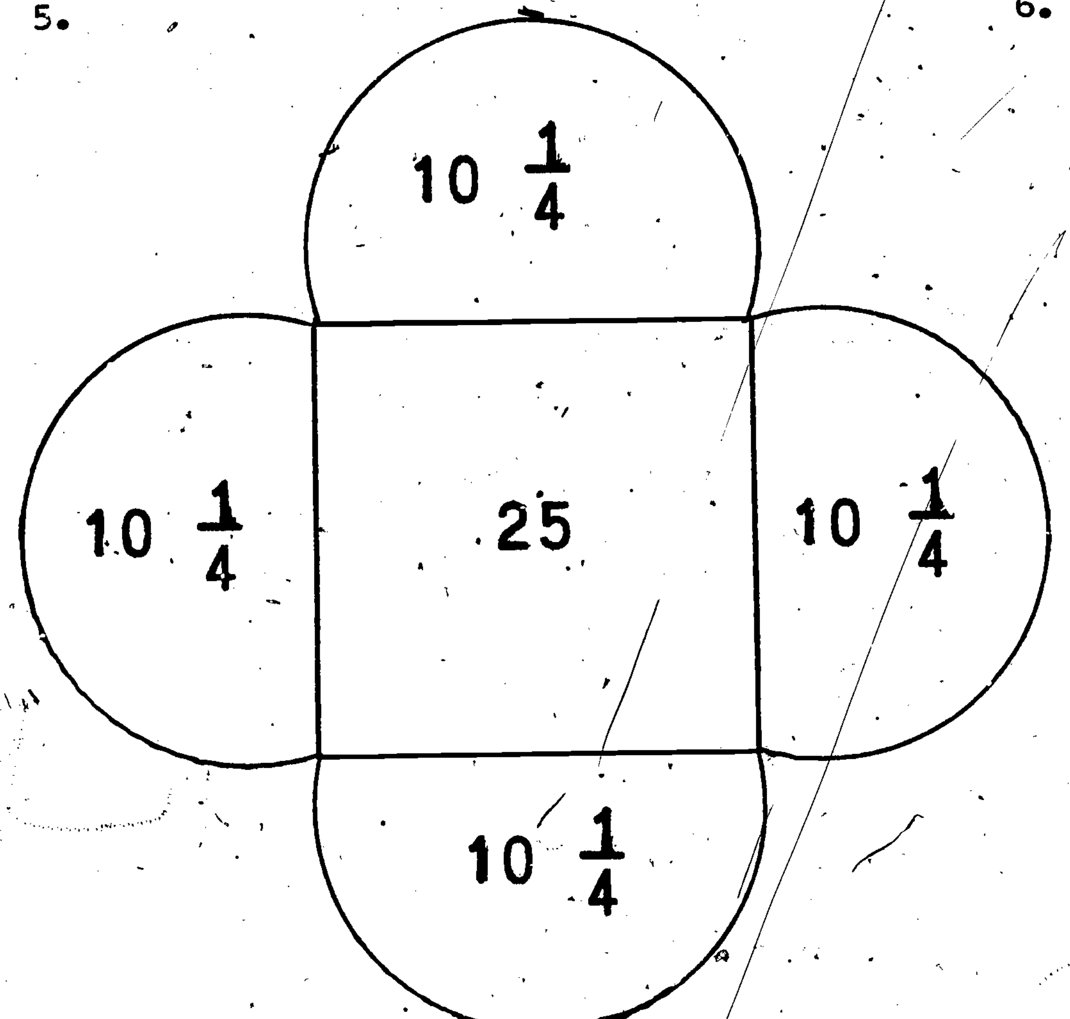

9 1/5
13 1/2
8 4/5
22
15 1/4

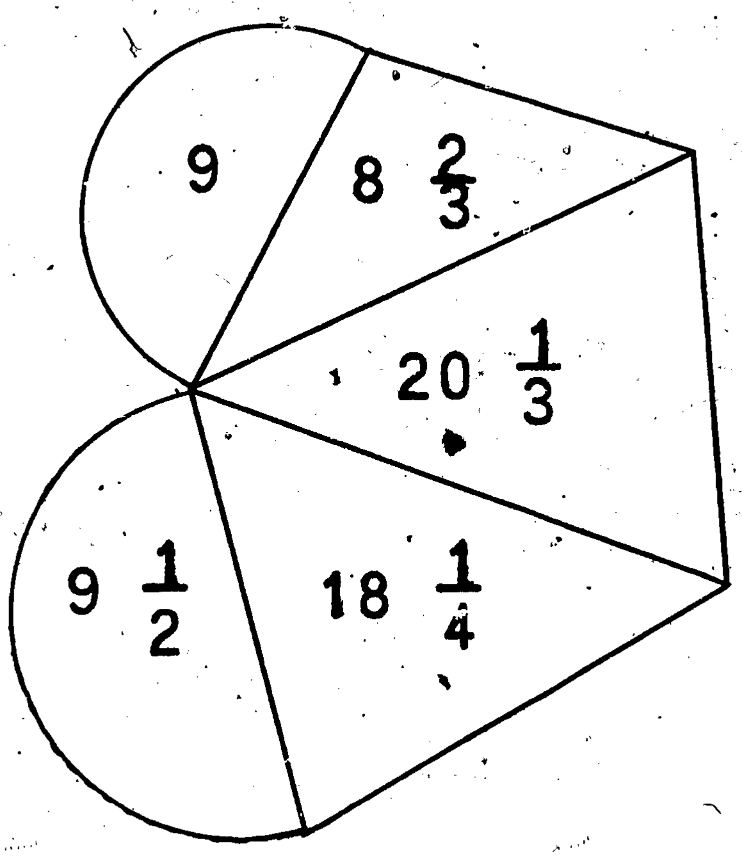
4.


3.7
10.90
19.05
21.27

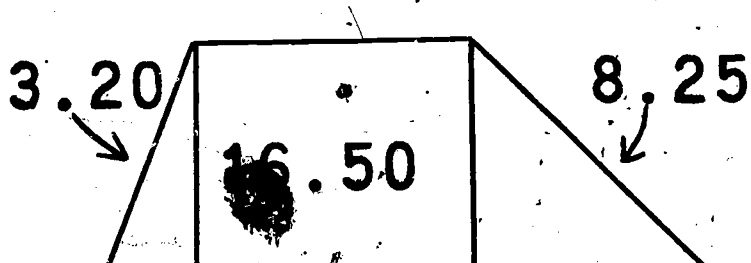
5.


10 1/4
10 1/4
25
10 1/4
10 1/4

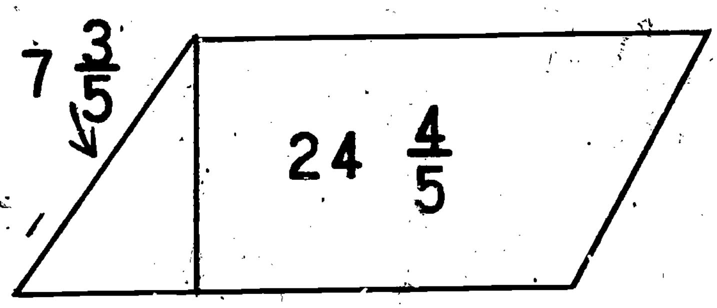
6.

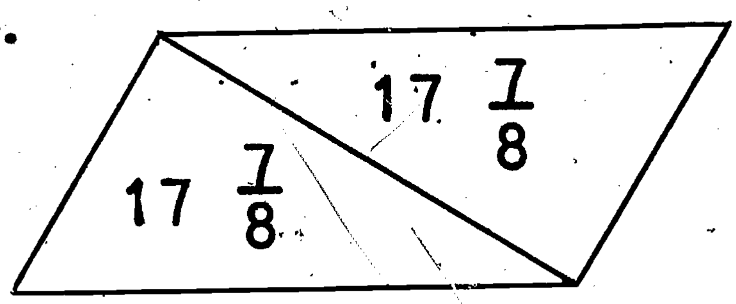

9
8 2/3
20 1/3
9 1/2
18 1/4

7.

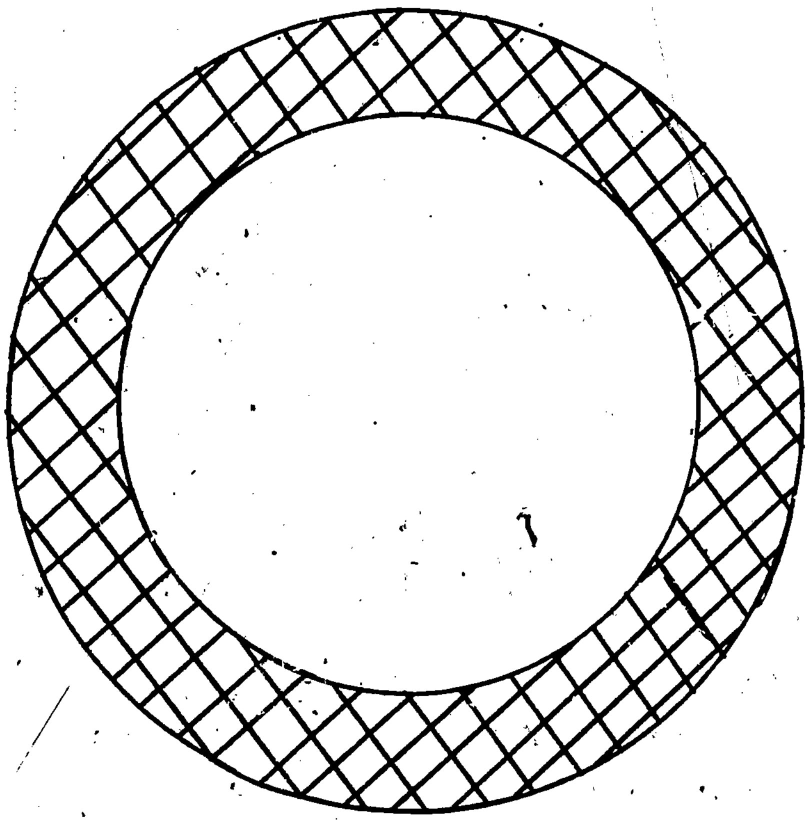

3.20
16.50
8.25

8.


7 3/5
24 4/5

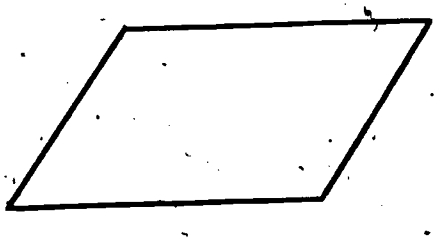
9. 

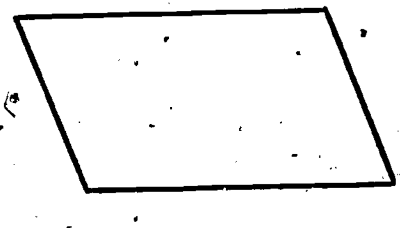
10. The area within the larger circle of problem 10 is 18.84 square units, and the area within the smaller circle is 15.70 square units. What is the area of the cross-hatched portion?

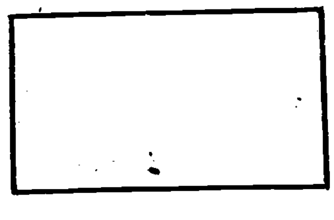


Parallelograms

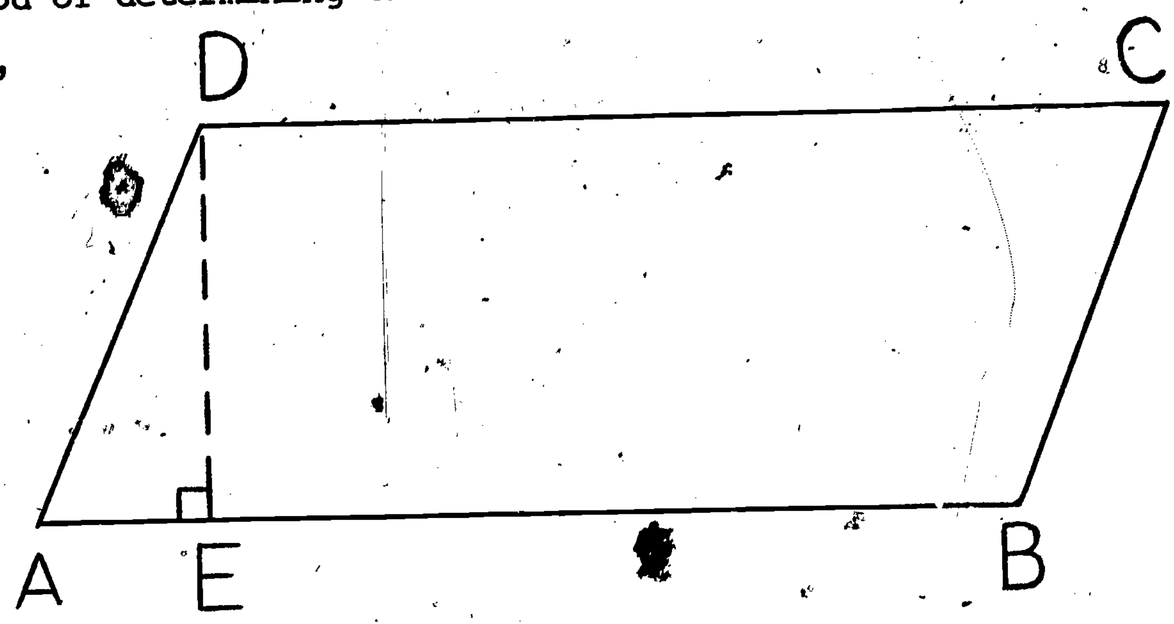
A parallelogram is a four-sided plane geometric figure with both pairs of opposite sides parallel. Several pictures of parallelograms follow.



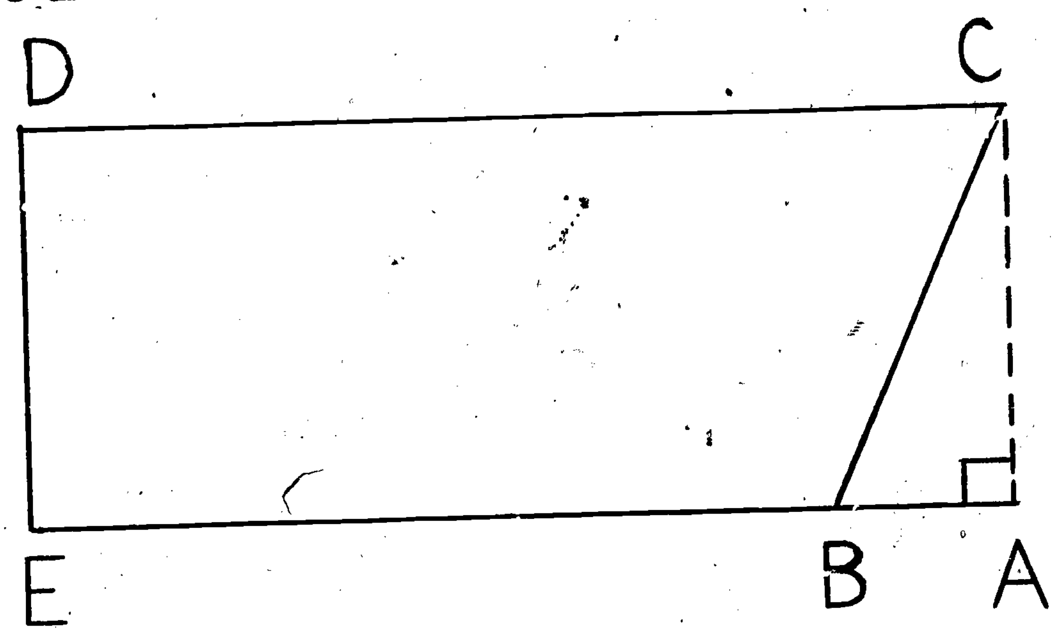




The area enclosed within a parallelogram may be determined by making use of the method of determining the area within a rectangle. Cut along the dotted line,



and then place this triangle in the desired position. The angle AED must be a 90° angle if we are to make use of our knowledge of areas within rectangles.

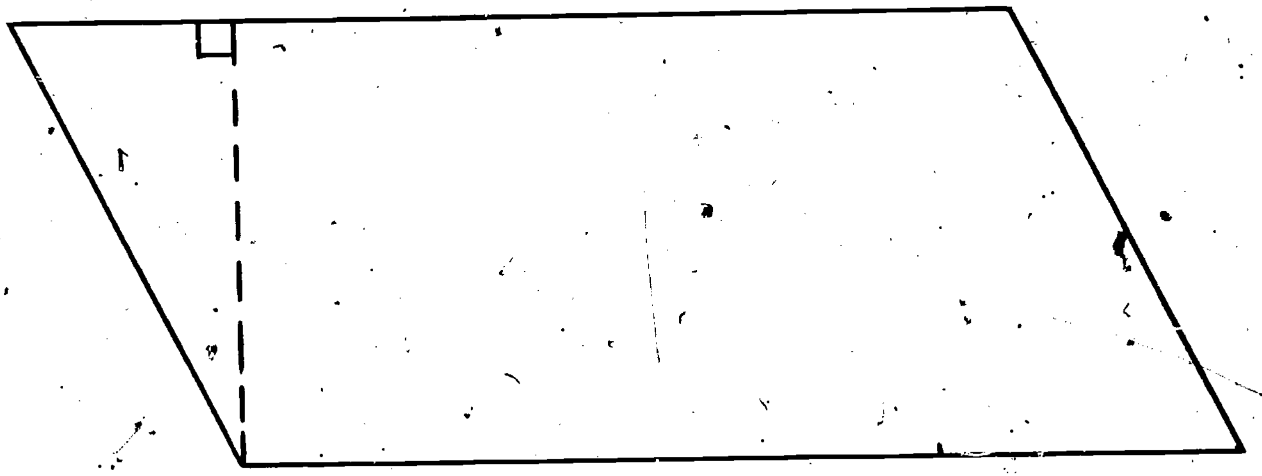


Now the area within the parallelogram ABCD can be determined by finding the area within the rectangle EACD. This area is 40 square centimeters, since the rectangle formed is 10 centimeters in length and 4 centimeters high.

Activities

In the following problems determine the areas by tearing, with the use of your ruler, along the broken line and placing the triangle in position to form a rectangle. (Do this by tracing the figures on another sheet of paper.)

1.



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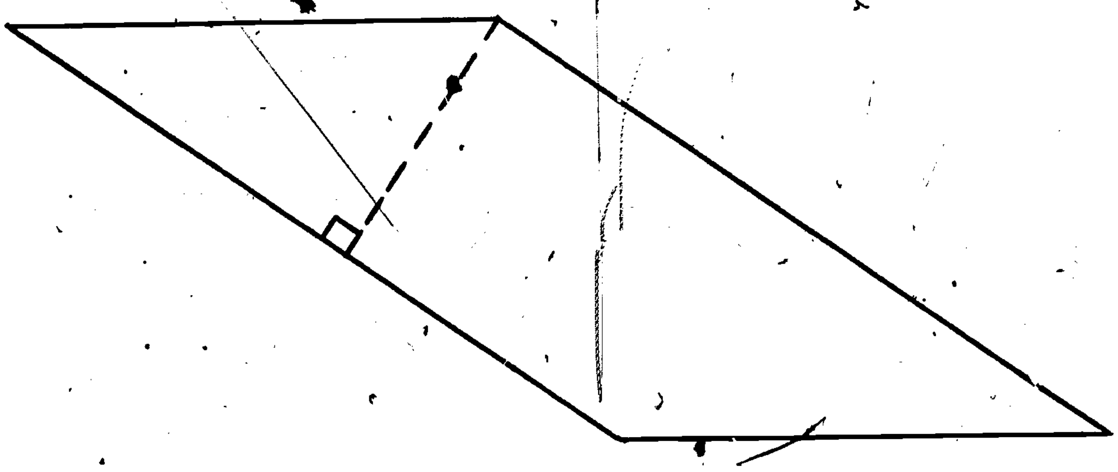


3. The length of the broken line is known as the height of the parallelogram. Instead of length, the parallelogram has a base. The area within a parallelogram is determined by multiplying the height by the ______________.

Find the areas within each of the following parallelograms:

4. height: 15 feet

base: 12 feet

5. height: 9 inches

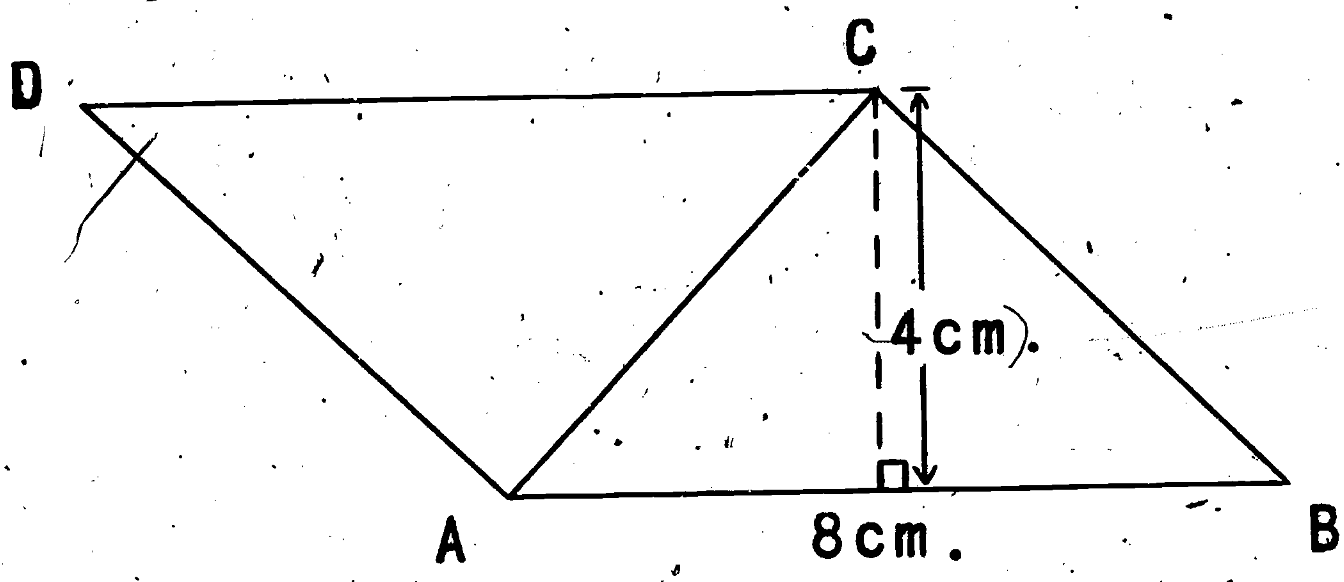
base: 1 foot

6. height: h

base: b

Triangles

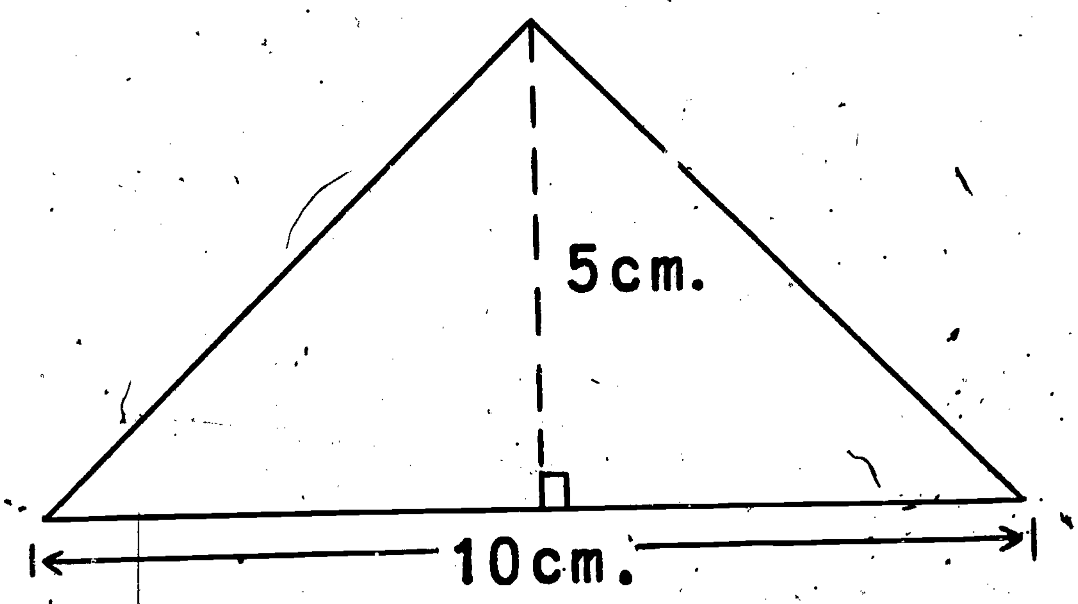
The area within a triangle is found by determining the area within a parallelogram whose base is the same as the base of a triangle and height the same as the height of the triangle and then dividing this parallelogram area by 2.



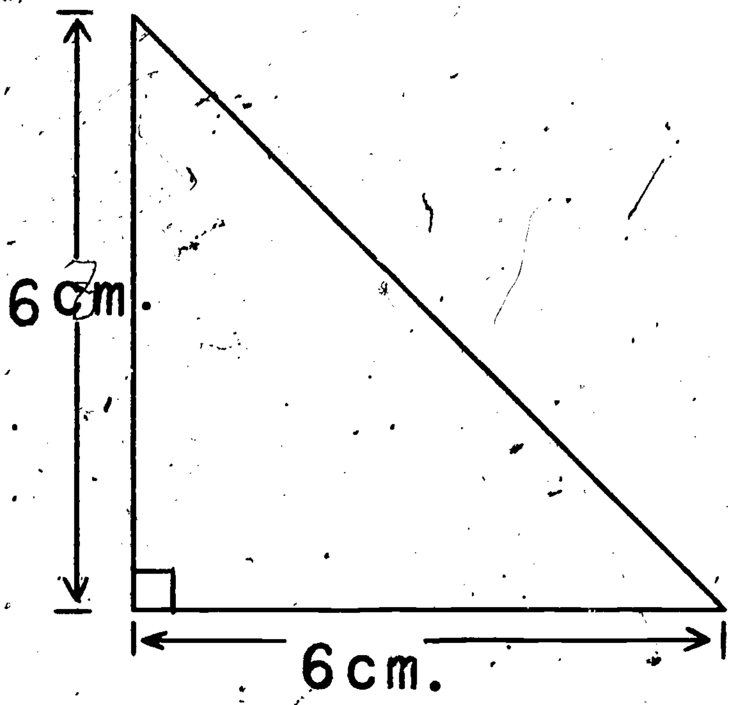
The area within the parallelogram ABCD is 8 x 4 square centimeters or 32 square centimeters. Now, the area within the triangle ABC or triangle ACD is just ½ of 32 square centimeters or 16 square centimeters.

Let us return to the grid method of determining the area within a plane figure.

Activities



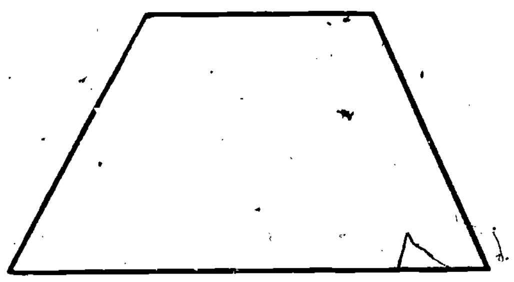
1. Using your square centimeter grid determine the area within the triangle above.
2. How does your answer compare with ½ x 10 x 5?

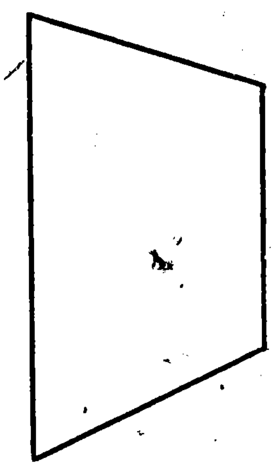


3. Use the same grid to determine the area within the above triangle.
4. How does your answer compare with ½ x 6 x 6?
5. What is the area enclosed within a triangle which has a base of 36 inches and a height of 12 inches?
6. What is the area enclosed within a triangle whose base is b inches in length and height is h inches in length?

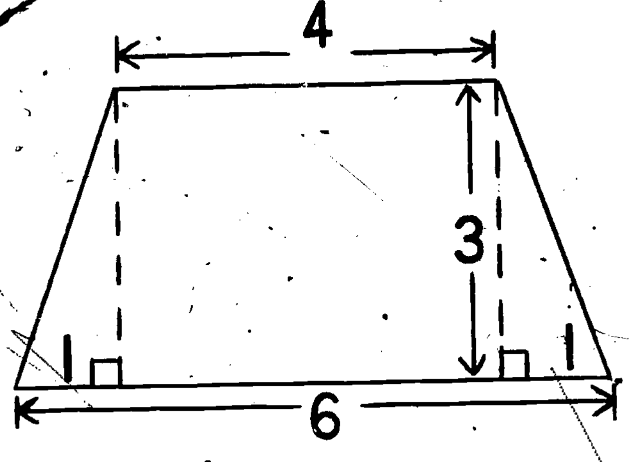
Trapezoids

A trapezoid is a four-sided figure with exactly two sides parallel. The parallel sides are known as the bases. We will not allow a parallelogram to be a trapezoid because it has 2 pairs of parallel sides. The picture of two trapezoids follow.





To begin with, let us find the area within a special kind of trapezoid known as an isosceles trapezoid. The lengths of the two nonparallel sides are equal in an isosceles trapezoid.



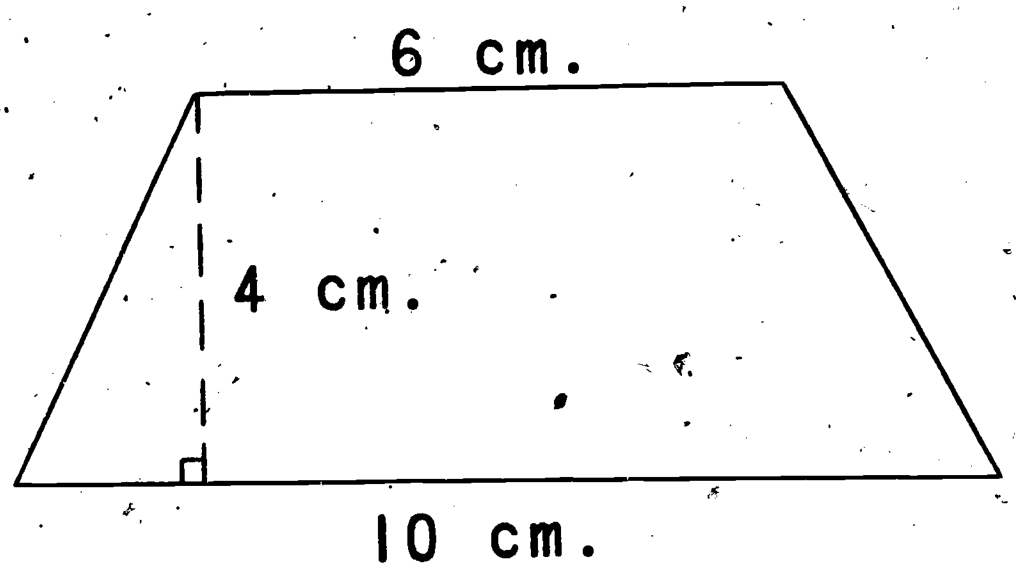
The area within the trapezoid may be determined by adding the areas within the two triangles and the rectangle into which it is divided. Thus we have the area within the trapezoid to be:

$\frac{1}{2}$ (1 x 3) + $\frac{1}{2}$ (1 x 3) + (4 x 3) = 15 square centimeters

<u>Activities</u>

Find the areas within these isosceles trapezoids.

1.



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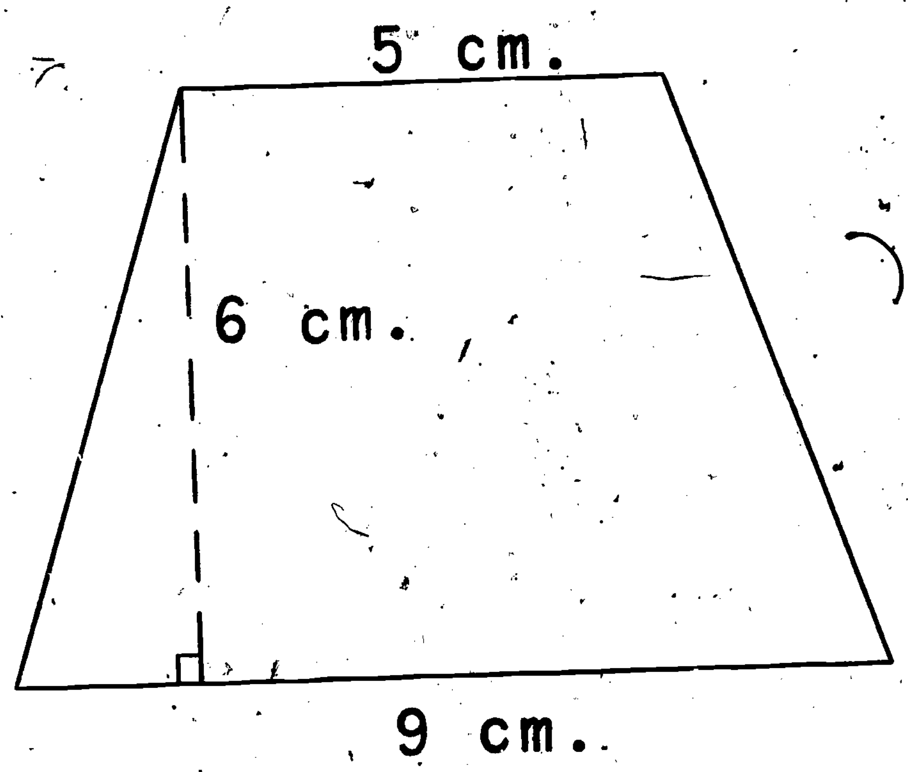

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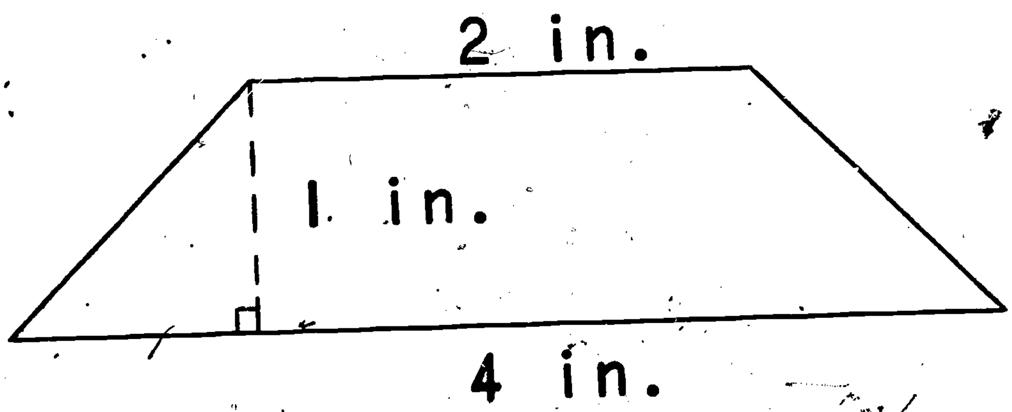
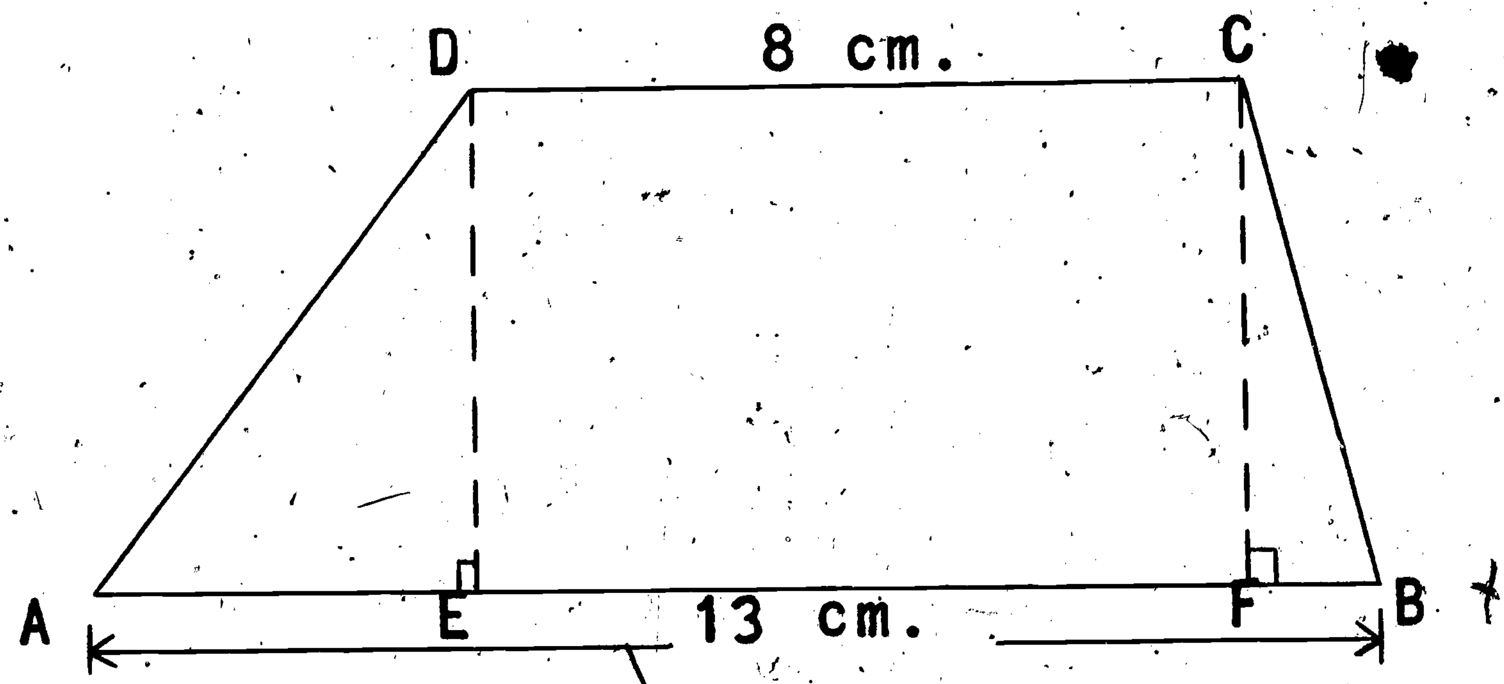


In problems 4 and 5 draw a picture if you need to.

4. Bases: 10 inches and 20 inches
Height: 4 inches

5. Bases: 2 feet and 2 yards
Height: 6 inches

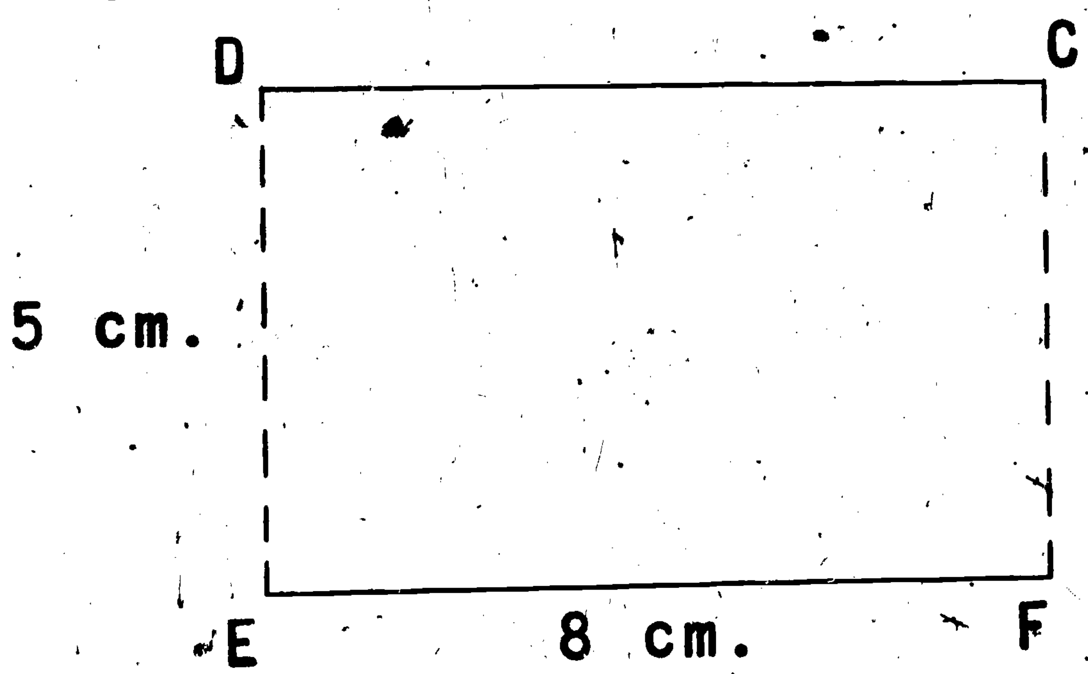
If a trapezoid is not isosceles, then the preceding method for finding the area within a trapezoid is not exactly correct.

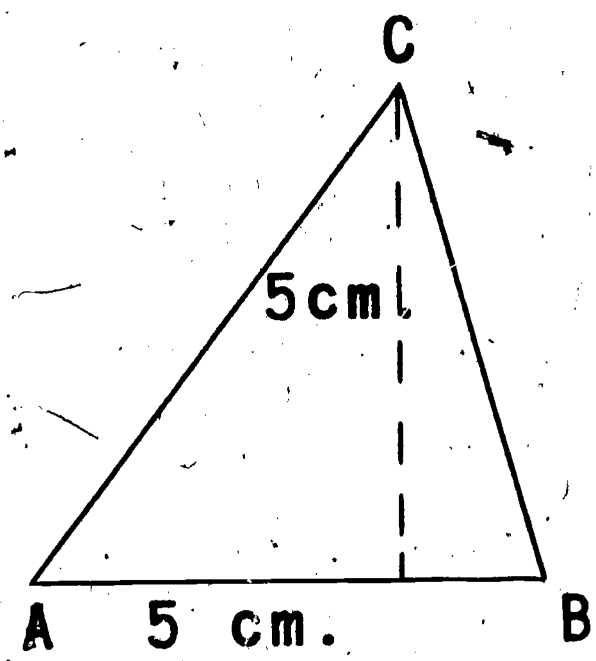


Do you see the difficulty that you will encounter if you try to use the preceding method for finding the area within a trapezoid? The area within the trapezoid is the sum of the area within triangle AED, the area within triangle BCF, and the area within the rectangle DEFC. By letting <u>A</u> represent the word <u>area</u>, just as your initials represent your name, we have:

A of the trapezoid ABCD = A of $\triangle$ AED + A of $\triangle$ FBC + A of rectangle EFCD

As you can see, at this point our procedure is exactly the same as the preceding method. However, we do not know the length of the bases of the two triangles as we did before. The area within the rectangle can be determined. It is 8 x 5 or 40 square centimeters. Take the two triangles AED and FBC and make one out of it. This will be triangle ABC. (Or triangle ABD)

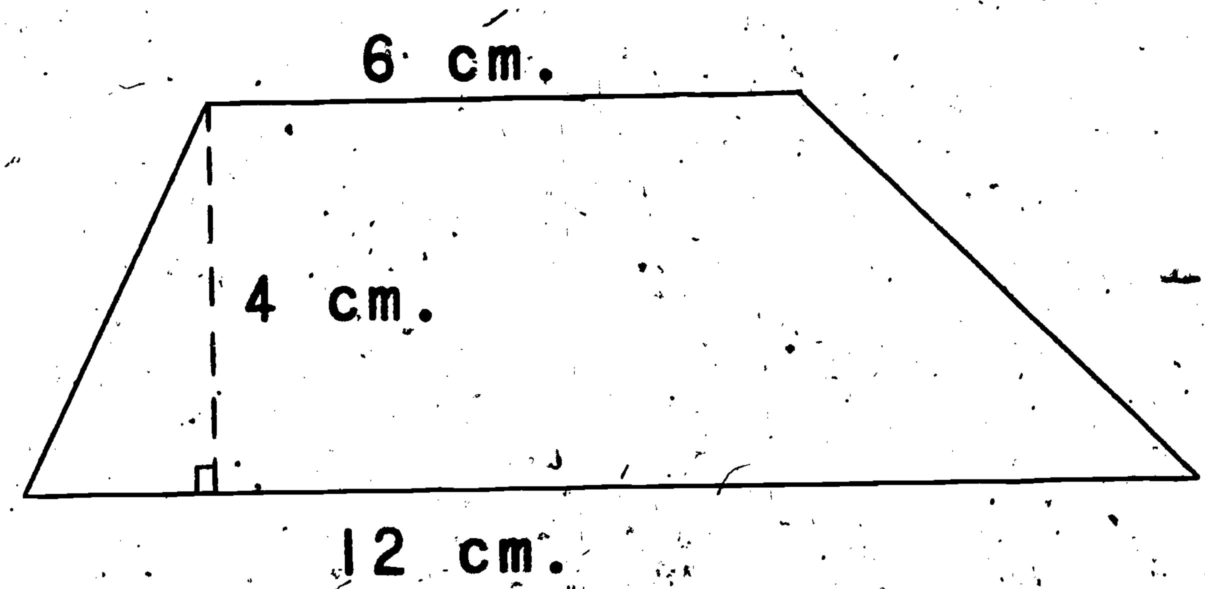




The area of trapezoid ABCD = $(8 \times 5) + \frac{1}{2}(5 \times 5)$ square centimeters or $52\frac{1}{2}$ square centimeters.

Find the area within each trapezoid.

1.



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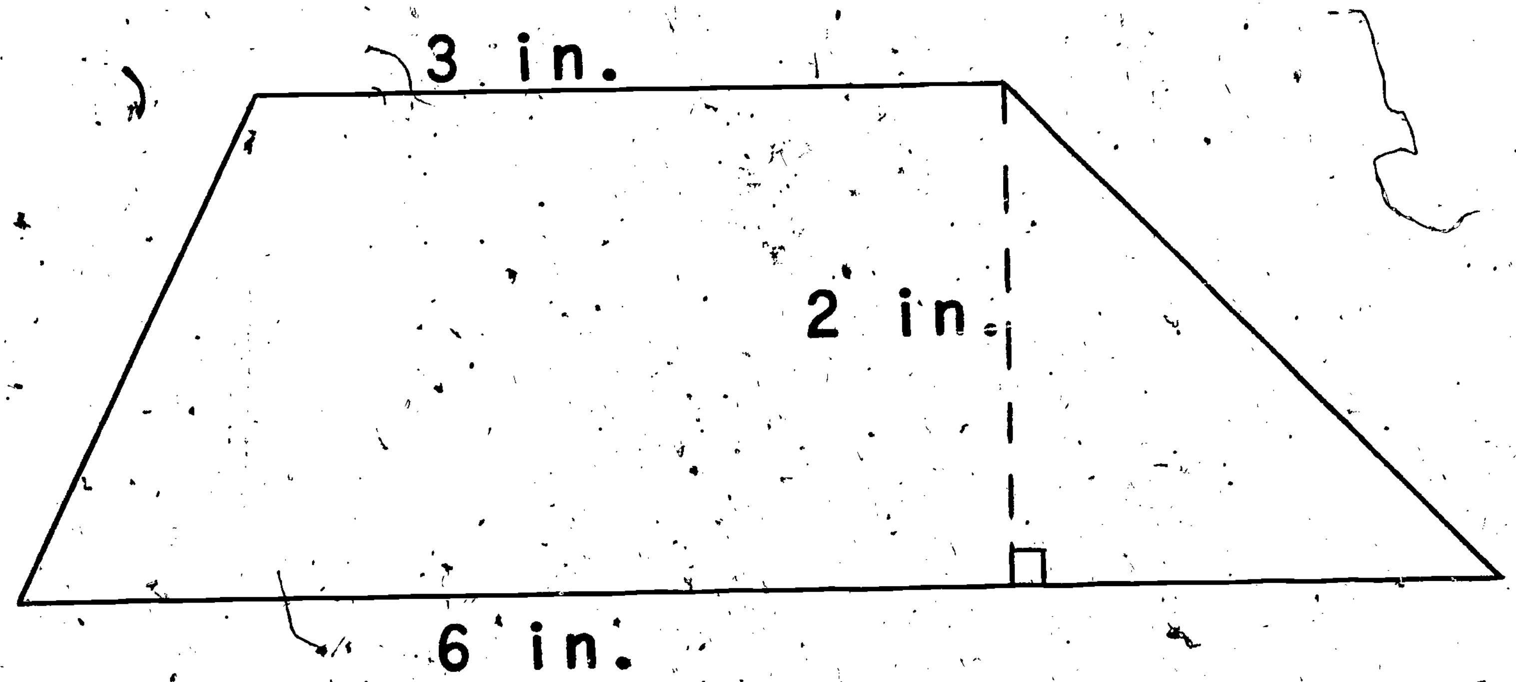


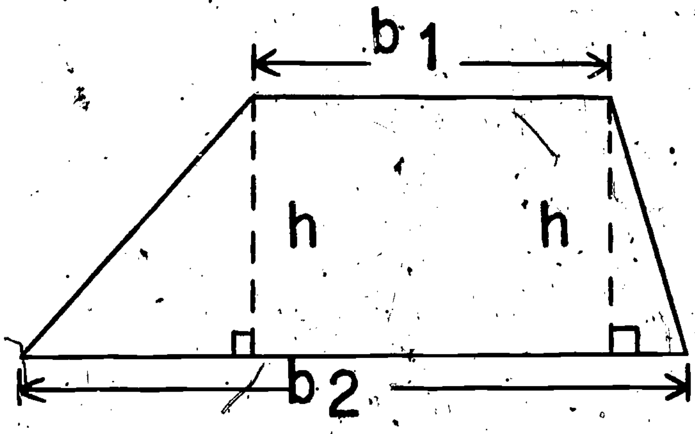
3. Bases: 10 inches and 15 inches
   Height: 5 inches

4. Bases: 15 feet and 20 feet
   Height: 7 feet

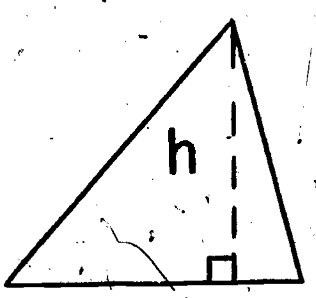
5. Bases: 20 and 35 miles
   Height: 25 miles

6. Bases: b_1 and b_2; $b_2 > b_1$

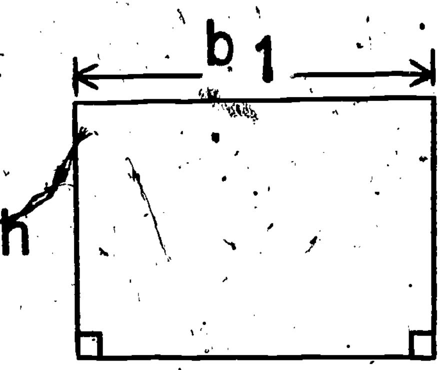
Height: h



(fig. 1)



(fig. 2)



(fig. 3)

In this problem the trapezoid is partitioned into a rectangle and two triangles. The two triangles can be placed together to form one triangle. What is the area within the rectangle of the 3rd figure for example 6?

____

What is the length of the base of the triangle in the 2nd figure for example 6? ________________

What is the area within the triangle that has been formed? __________

What is the sum of the areas within the triangle and the rectangle? _____

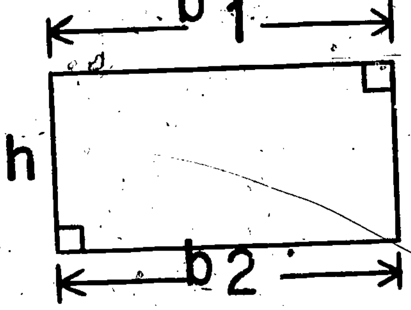
This sum is the area within the trapezoid.

Area Formulas

The following four formulas have been established to find the area within certain plane geometric figures:

Name of Plane Geometric Figure	Method of Finding Area	Formula for Finding Area
Rectangle	base multiplied by height	$A = b \times h$ or $A = bh$
Parallelogram	base multiplied by height	$A = b \times h$ or $A = bh$
Triangle	one-half of the product of the base and the height	$A = \frac{1}{2}(b \times h)$ or $A = \frac{1}{2}bh$
Trapezoid	one-half the product of the height and the sum of the two bases	$A = \frac{1}{2} \times h \times (b_1 + b_2)$ or $A = \frac{1}{2}h(b_1 + b_2)$

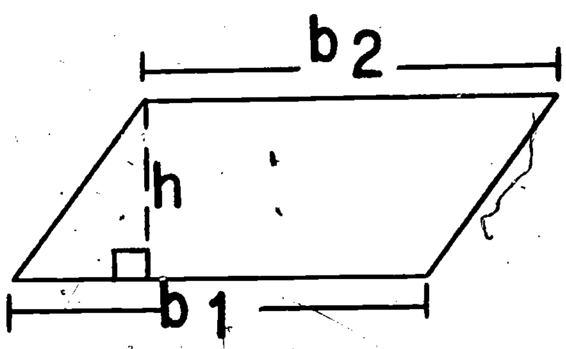
There are four formulas involved here, one for each of the four plane figures. It is possible though to incorporate all four formulas into one, the formula $A = \frac{1}{2}h(b_1 + b_2)$. This formula can be used to find the area within a rectangle because the rectangle has two bases of equal lengths, $b_1 = b$ and $b_2 = b$.



The area within the rectangle:

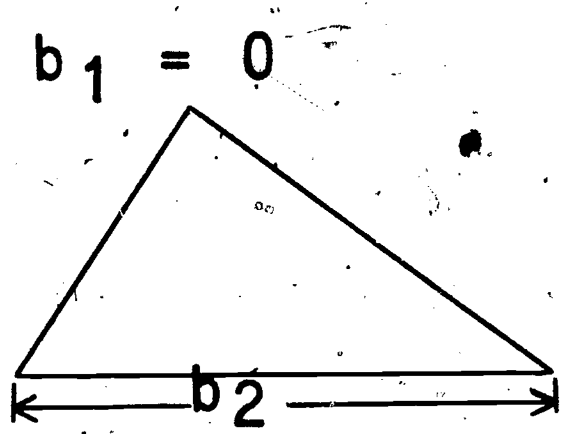
$$
\begin{aligned}
A &= \tfrac{1}{2}h(b_1 + b_2) \\
&= \tfrac{1}{2}h(b + b) \\
&= \tfrac{1}{2}h(2b) \\
&= \tfrac{1}{2}(2b)h \\
&= (\tfrac{1}{2} \times 2)bh \\
&= bh
\end{aligned}
$$

The area within a parallelogram:



Since the area within a parallelogram can be determined by partitioning and forming a rectangle, the area can be determined as in the preceding example.

The important fact to keep in mind when using this formula to find the area within a triangle is that the length of one of the bases is zero. Also we have $b_2 = b$, where b is the length of the base.



The area within the triangle:

$$\begin{aligned} A &= \tfrac{1}{2} h (b_1 + b_2) \\ &= \tfrac{1}{2} h (0 + b) \\ &= \tfrac{1}{2} hb \\ &= \tfrac{1}{2} bh \end{aligned}$$

Using the formula $A = \frac{1}{2} h (b_1 + b_2)$ find the areas within each of the following plane geometric figures:

1. A triangle with a base of 9 inches and a height of 6 inches.
2. A rectangle with a base of 12.5 inches and a height of 7.3 inches.
3. A parallelogram with a base of $5\frac{1}{3}$ inches and a height of $2\frac{1}{4}$ inches.

4. A trapezoid with these dimensions:

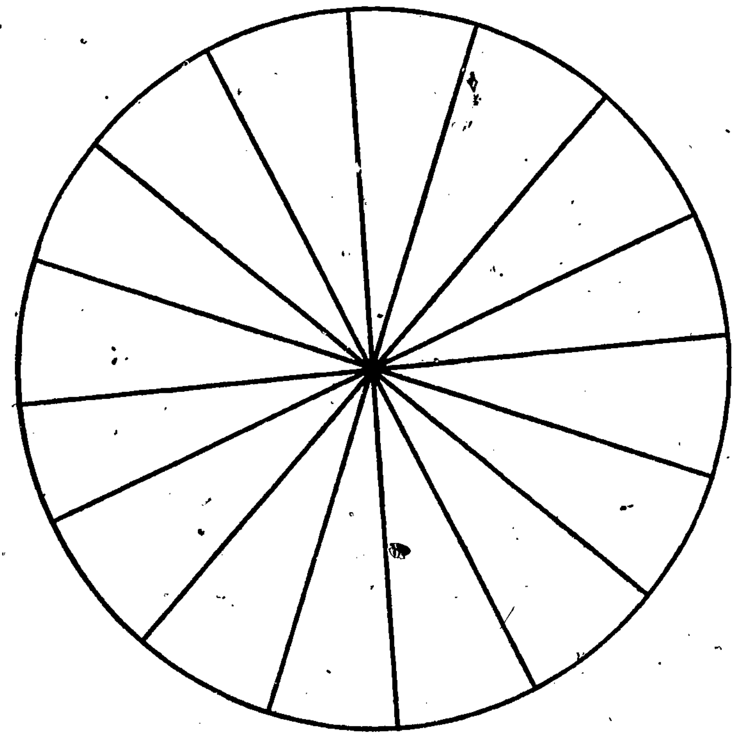
b_1 = 5 2/3 centimeters

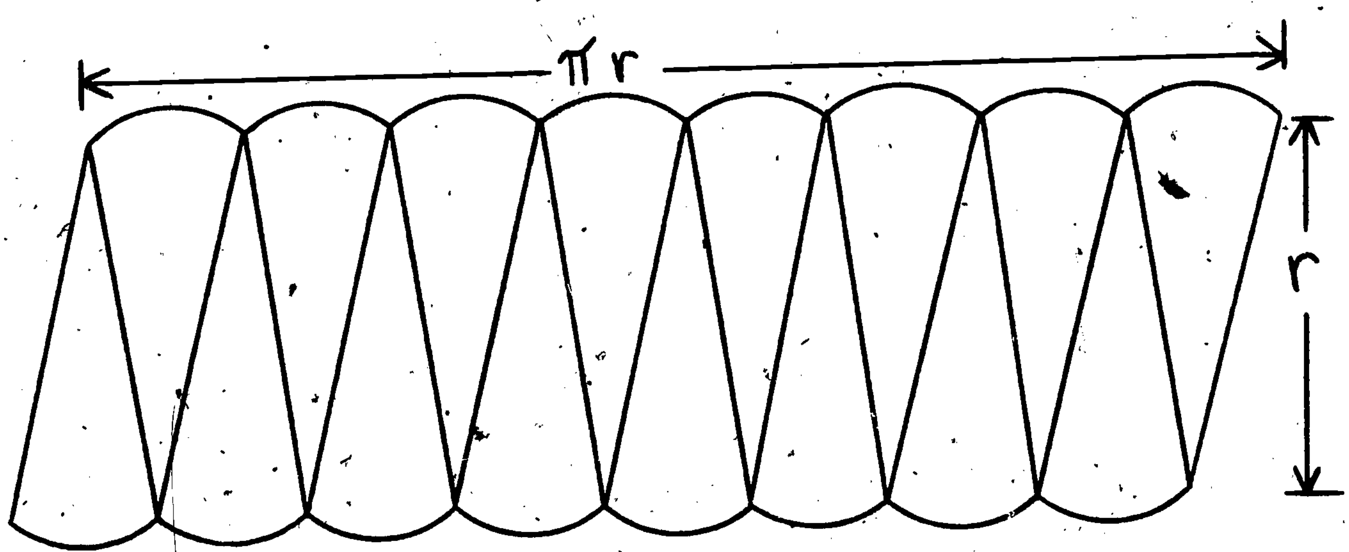
b_2 = 6 5/6 centimeters

h = 4 centimeters

Circles

To determine the area within a circle, divide the interior into pie-shaped parts, and place them together to form a "rectangular"-shaped figure. This is illustrated.



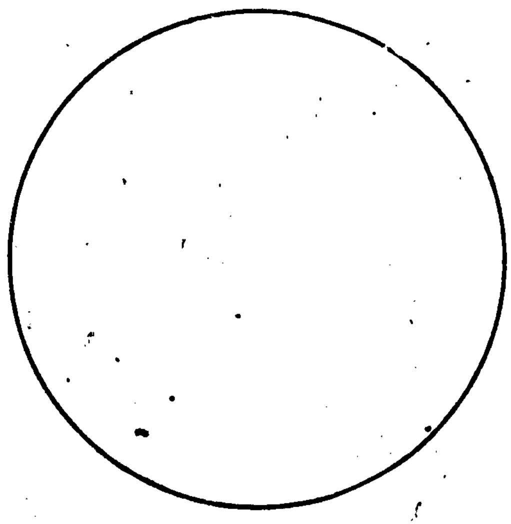


The "length" of the figure formed is about ½ the circumference of the circle. Since the circumference of the circle is $2\pi r$, the "length" is πr. The "width" is just the radius of the circle. If the figure formed is treated as a rectangle, the area of the interior of the circle is seen to be $\frac{1}{2} \times 2\pi r \times r$ which is the same as $\pi r \times r$ or πr^2.

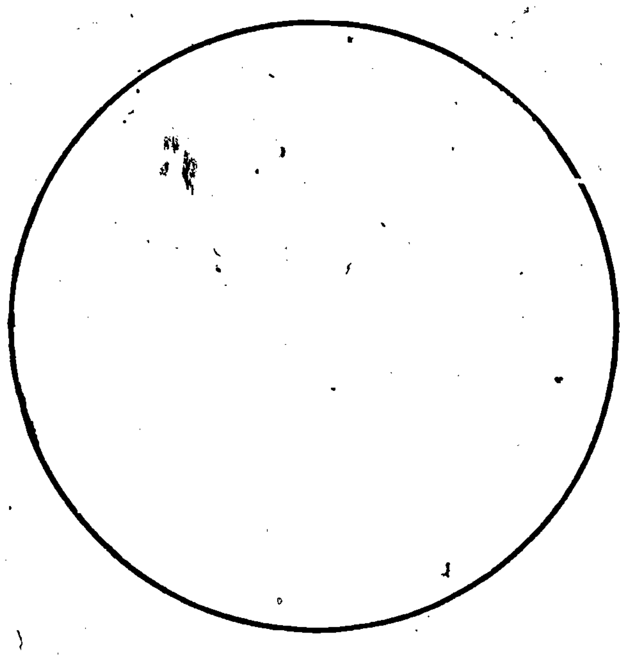
Activities

Find the area of the interior of the circles with these radii. Use 3.14 as the value of π.

1. 4 inches
2. 12 inches
3. 21 inches
4. 14 meters
5. 27 centimeters
6. Using your square inch grid determine the area within a circle which has a radius of 1 inch.



7. Check your answer to problem 6 by using the formula $A = \pi r^2$.
8. Using your square centimeter grid, determine the area within a circle which has a radius of 3 centimeters.

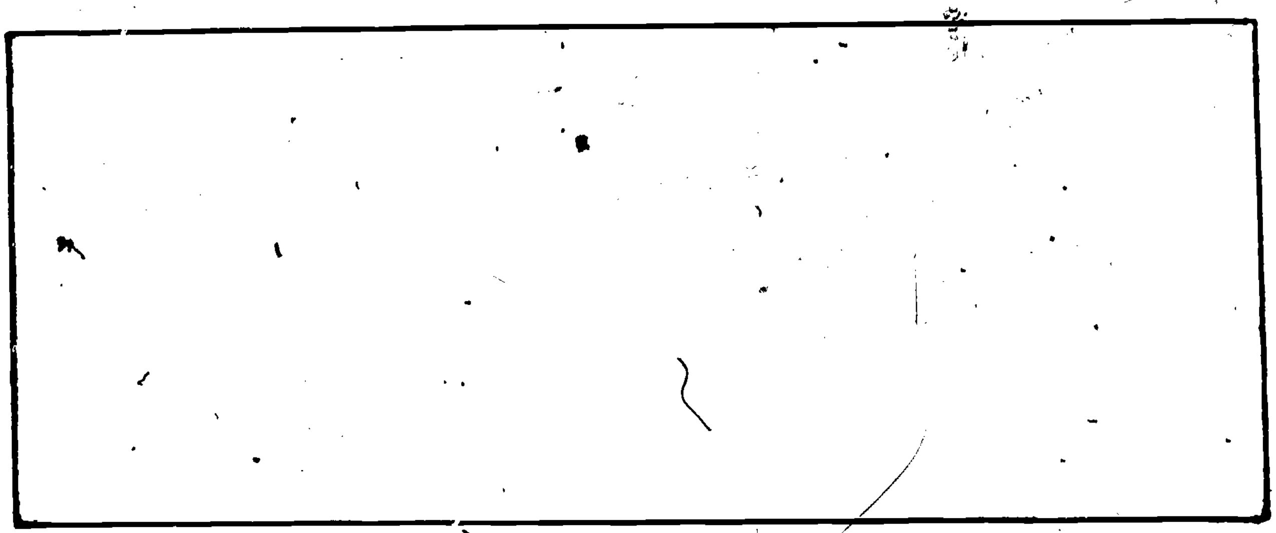


9. Check your answer to problem 8 by using the formula $A = \pi r^2$.

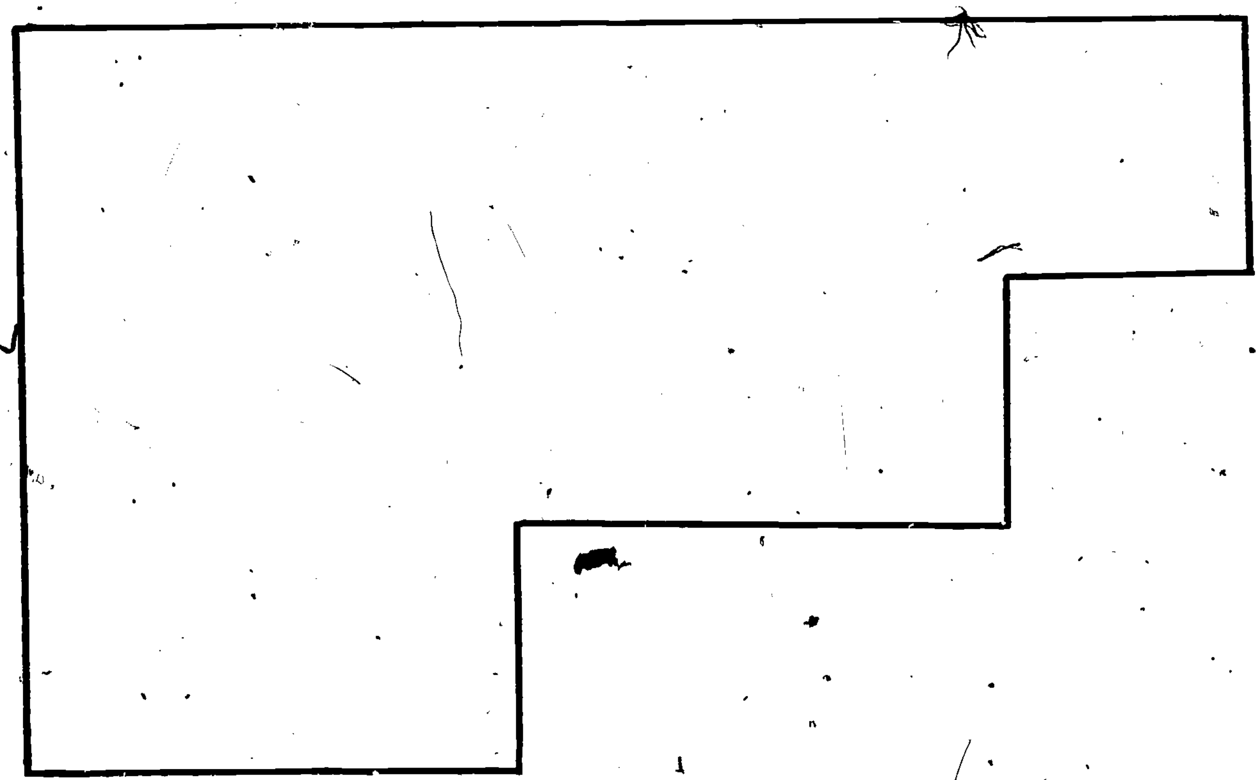
Activities

Find the area in square inches within each of these plane geometric figures. Use the one inch squares.

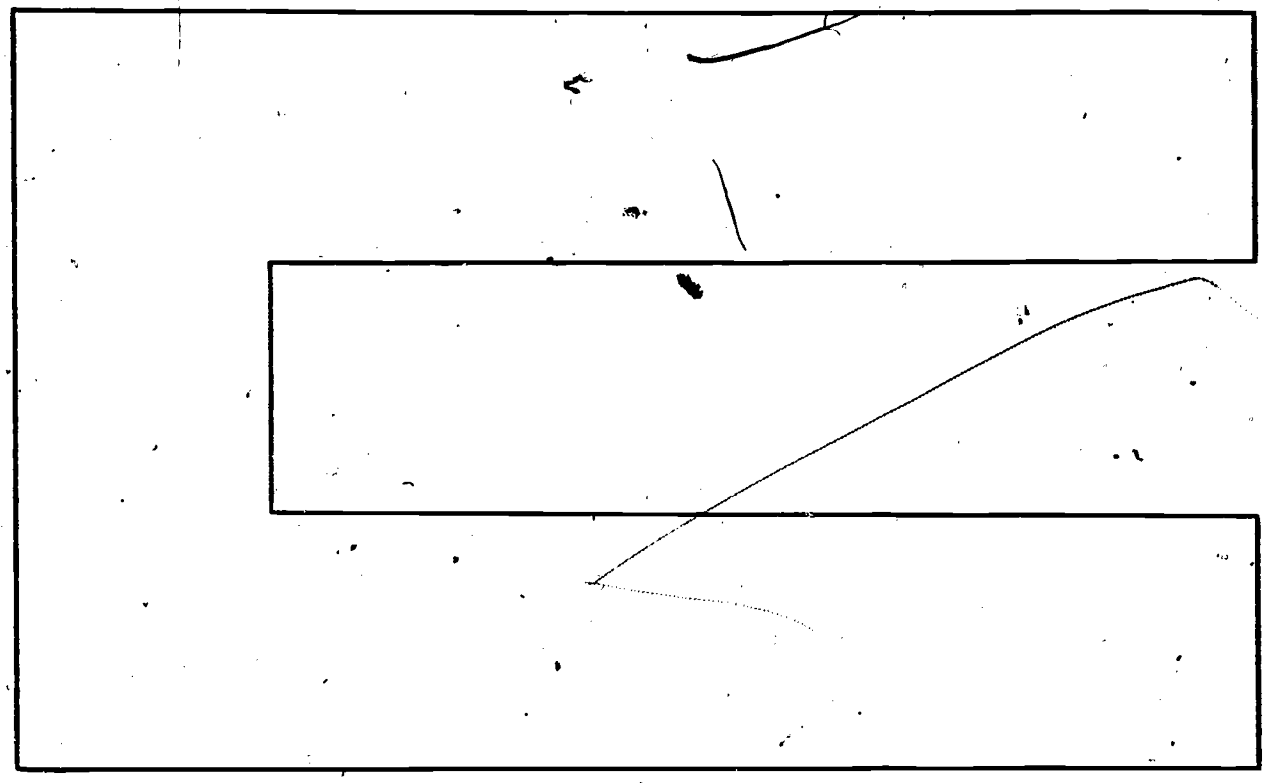
1.



2.



3.



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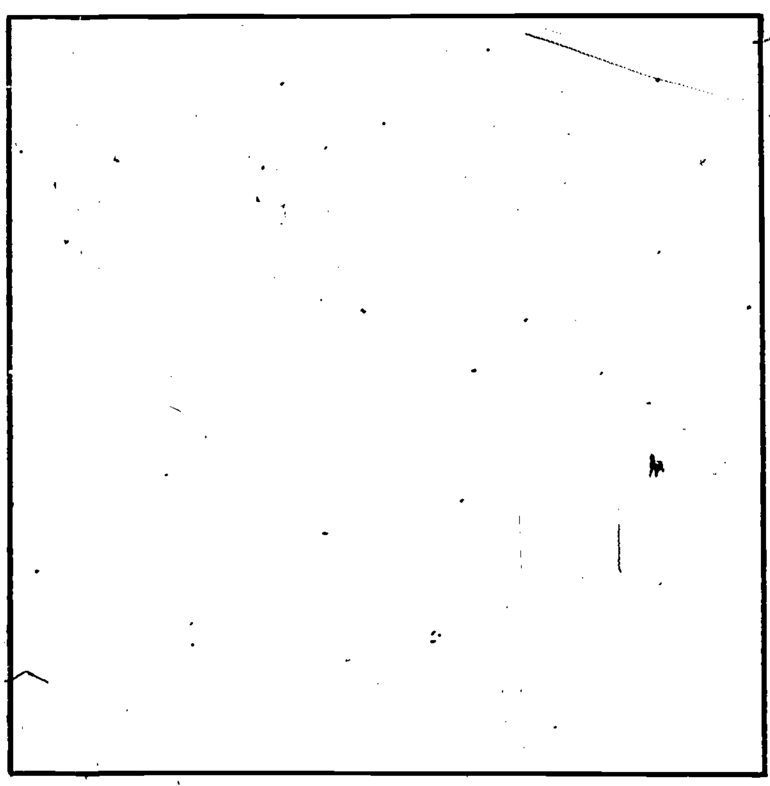

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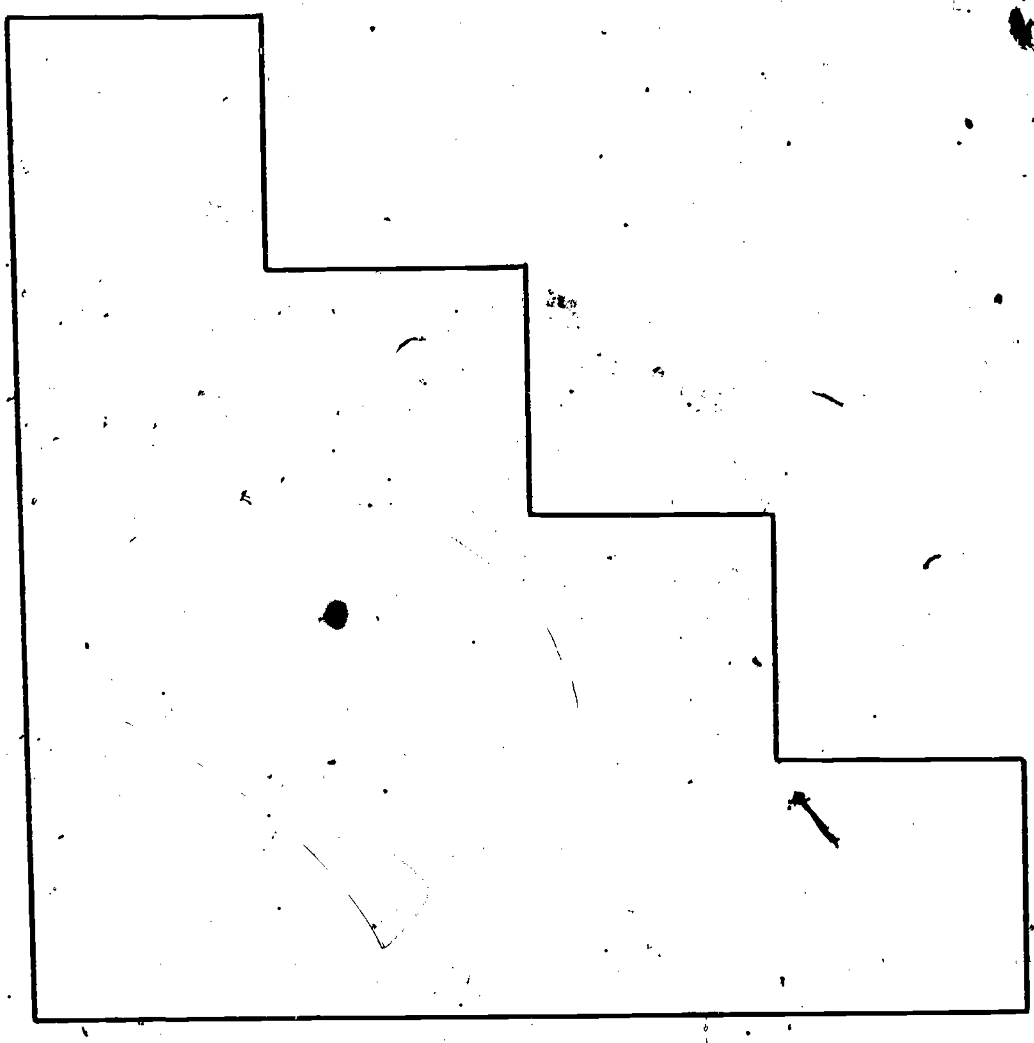

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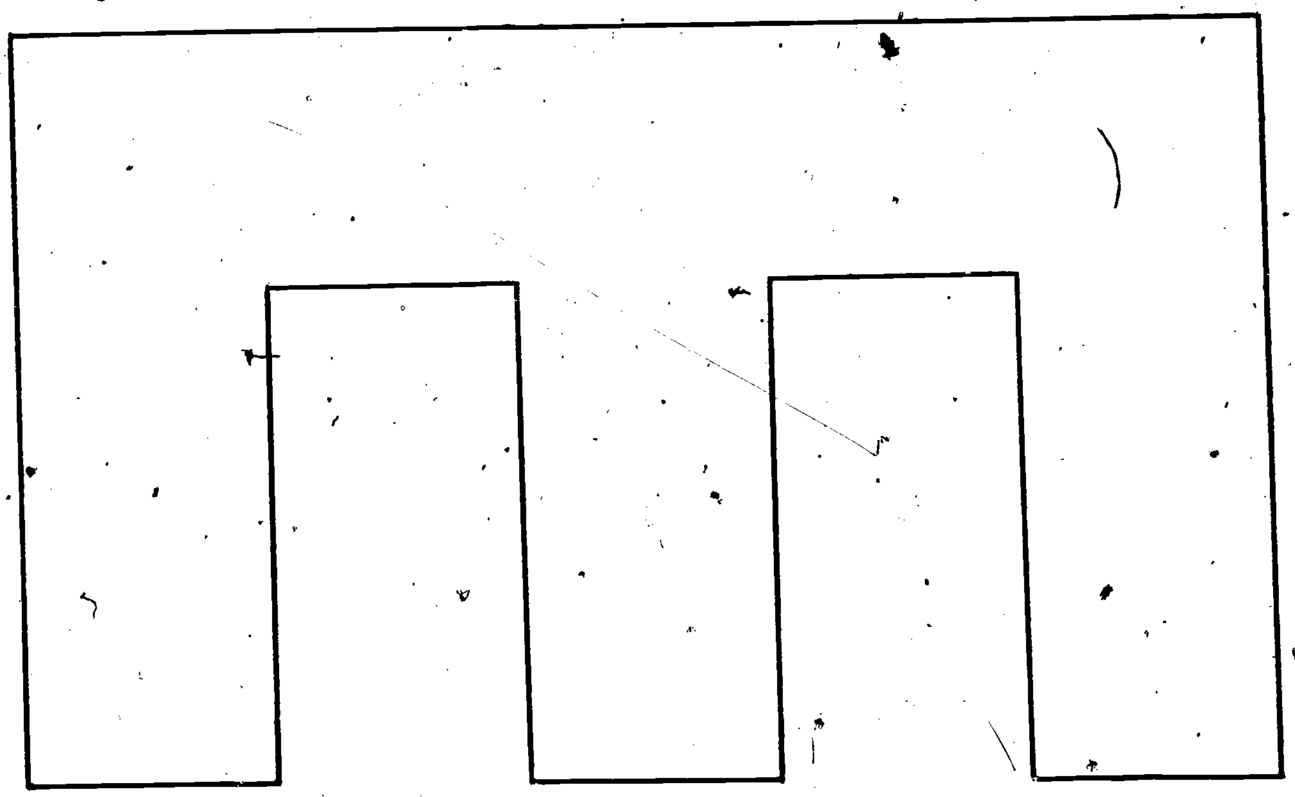

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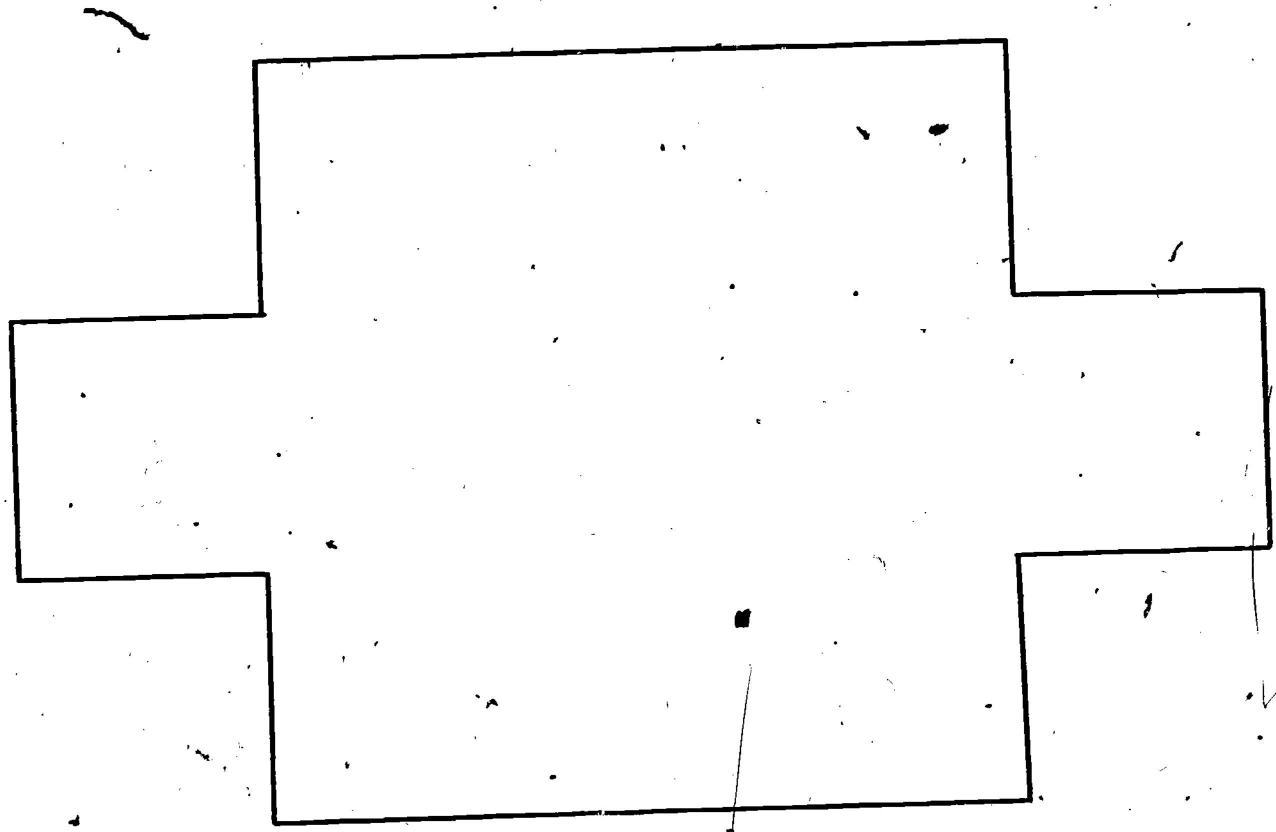

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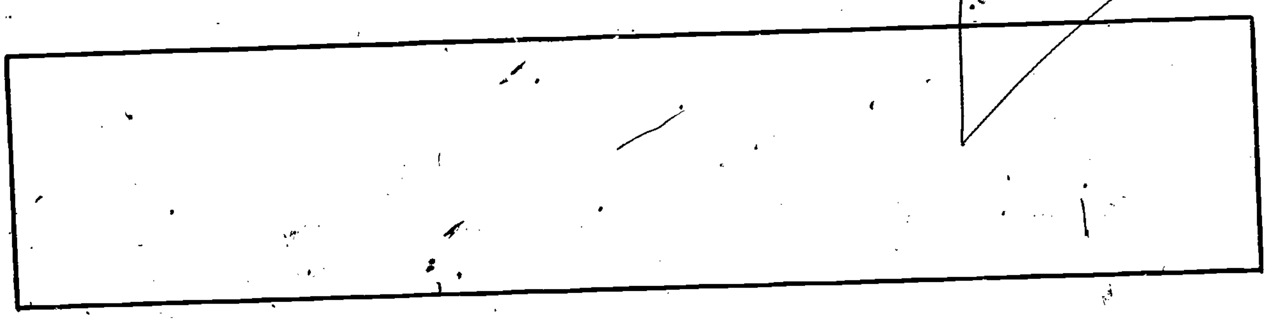

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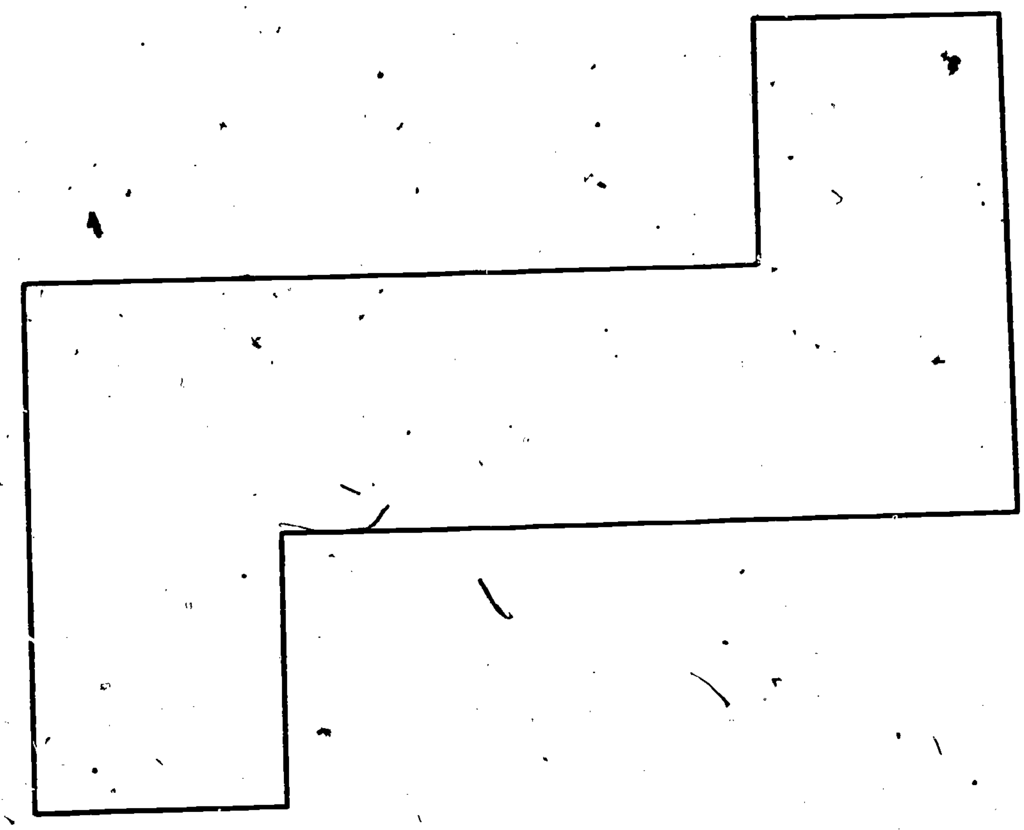

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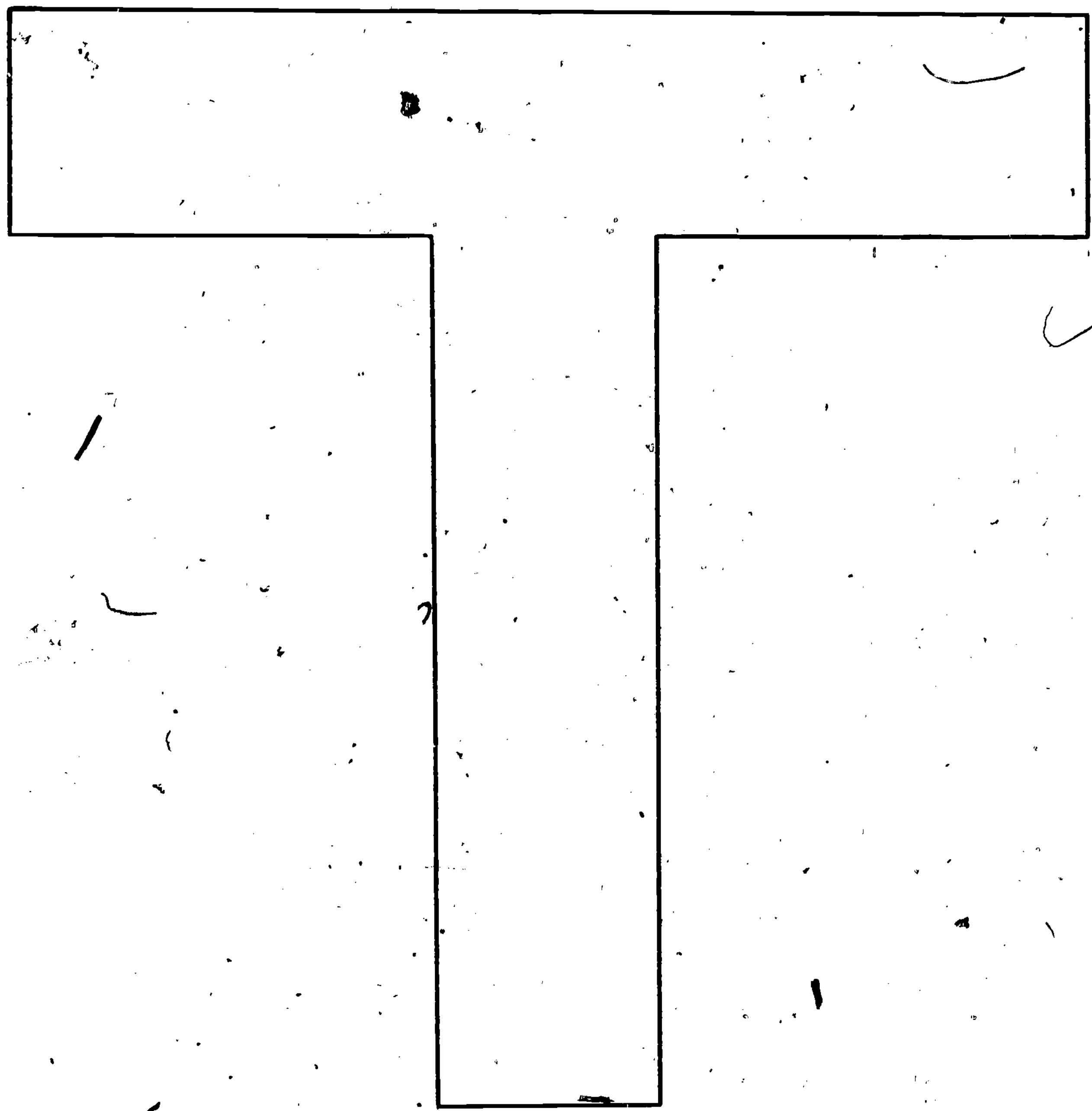
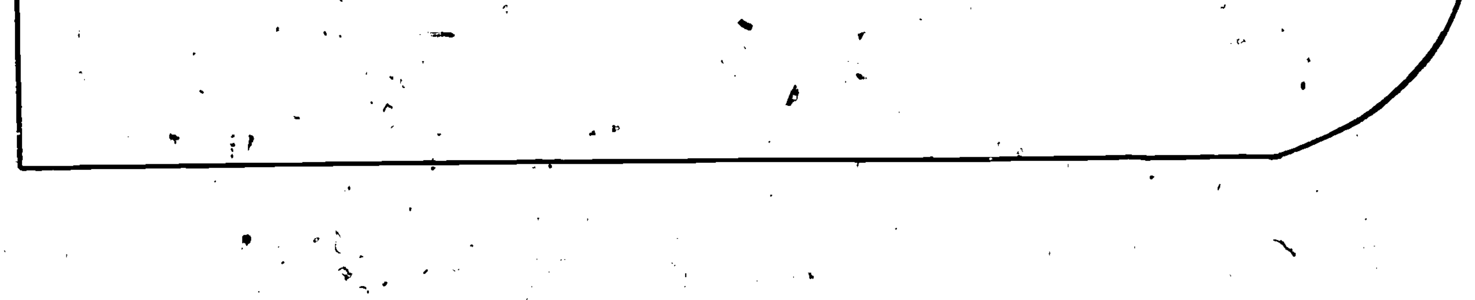
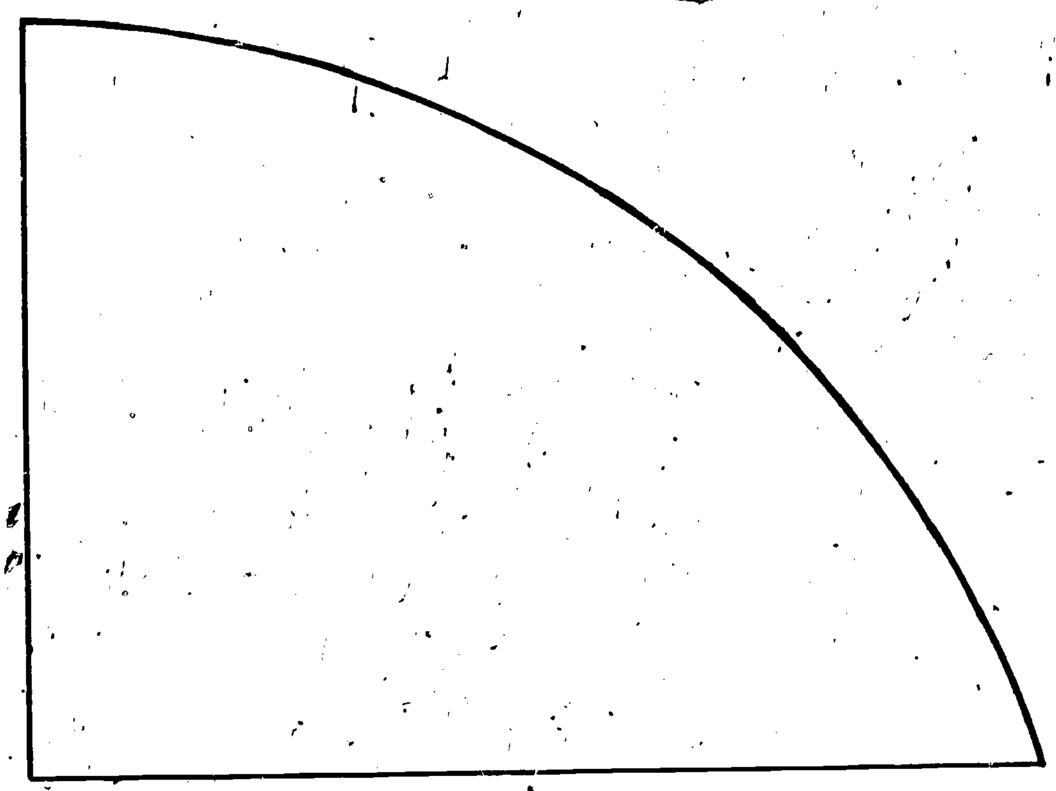


Find the area within each of the following plane geometric figures using your square inch and 1/4 square inch grids.

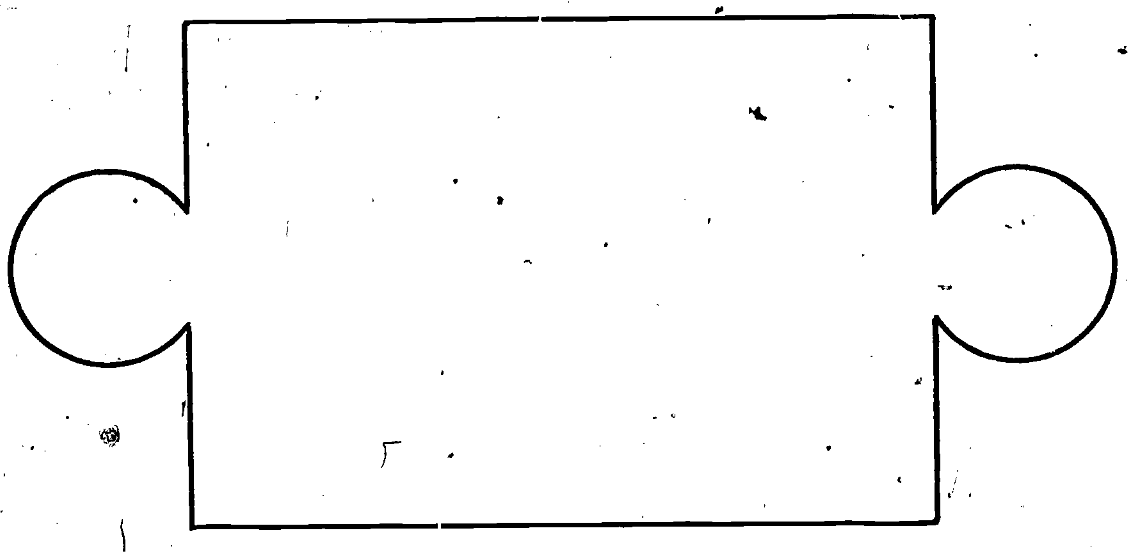
i.



2.



3.



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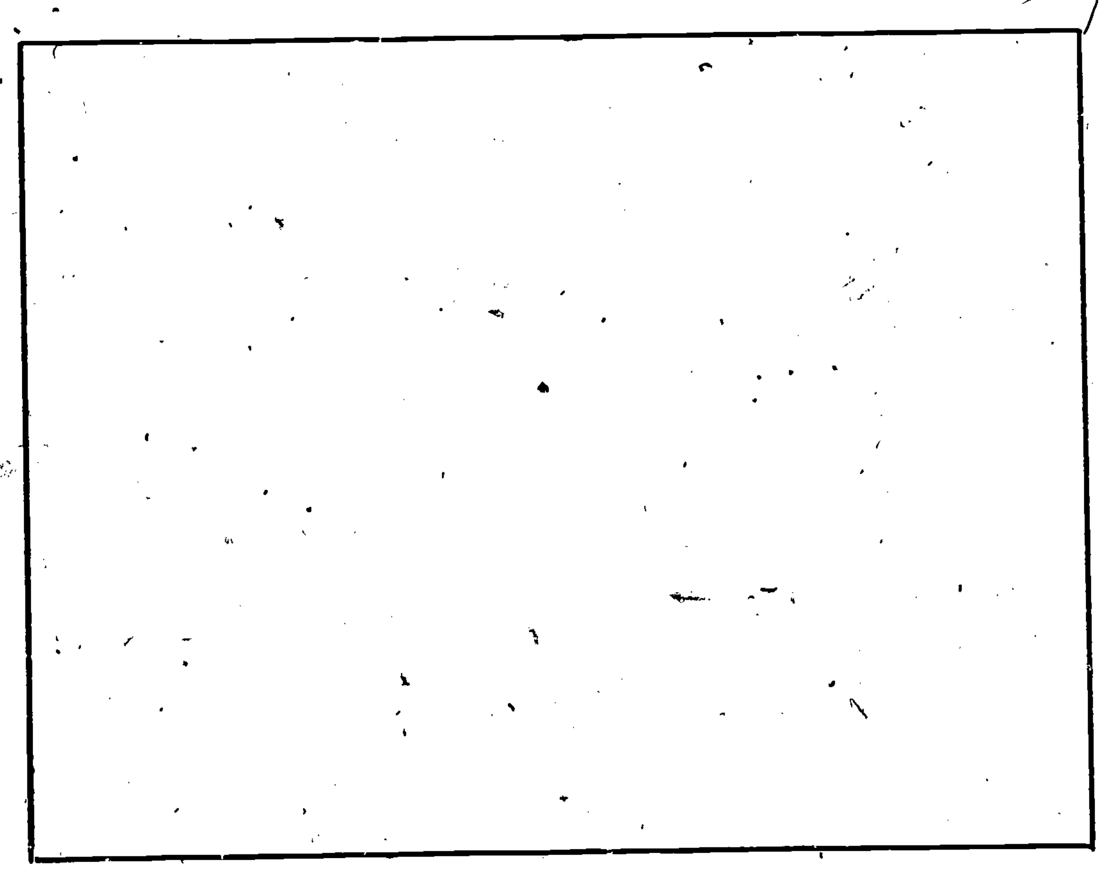

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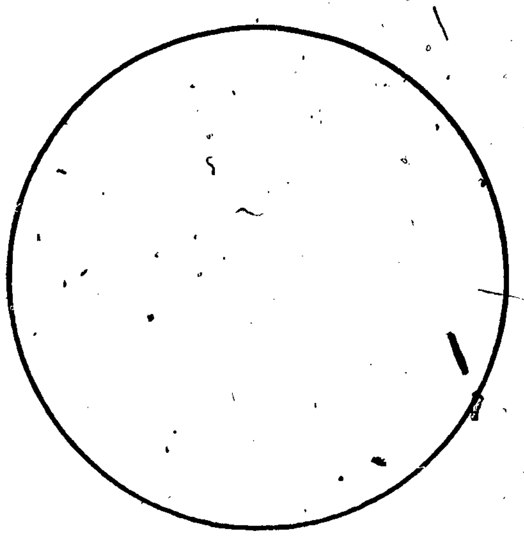
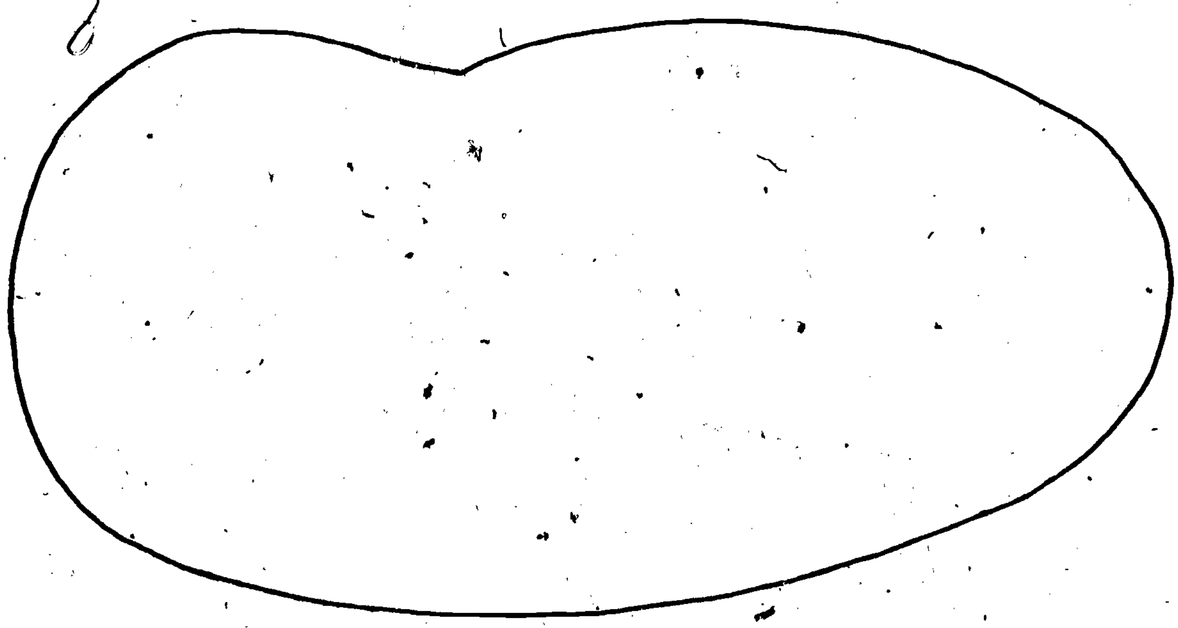


6.

7.



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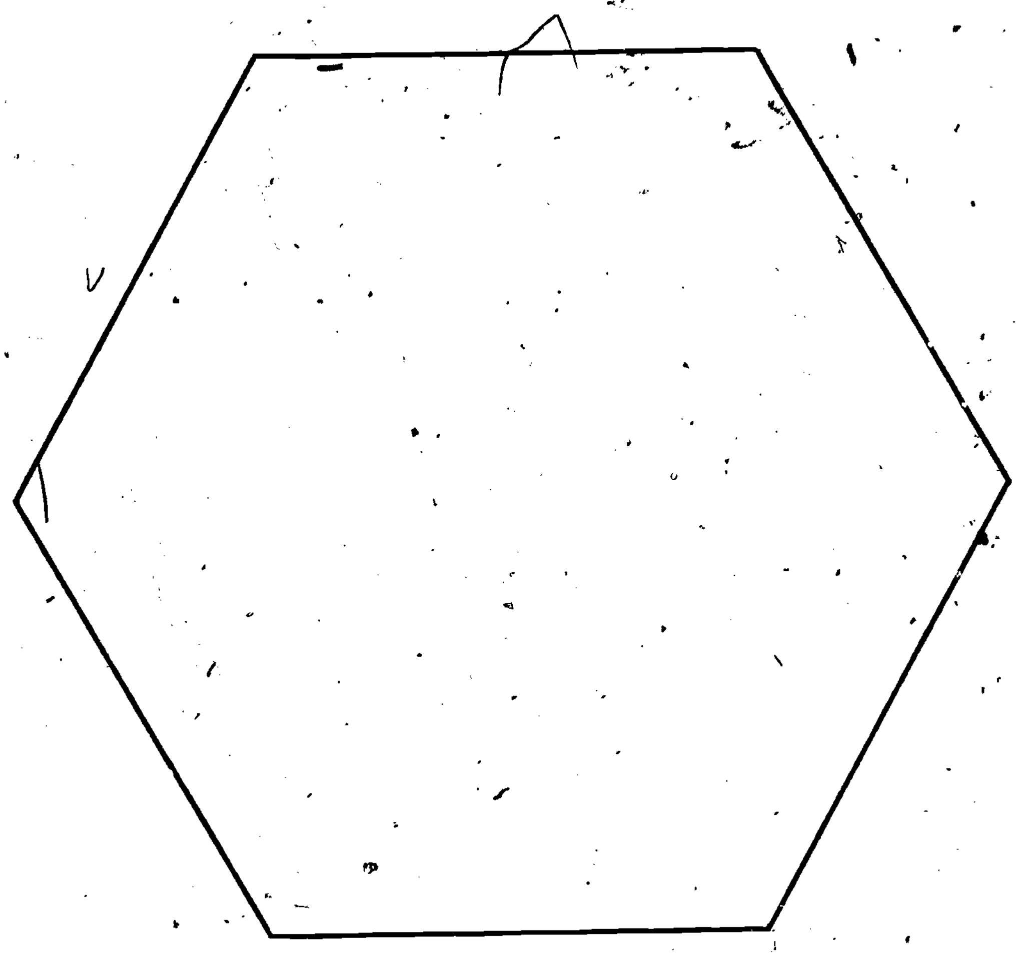

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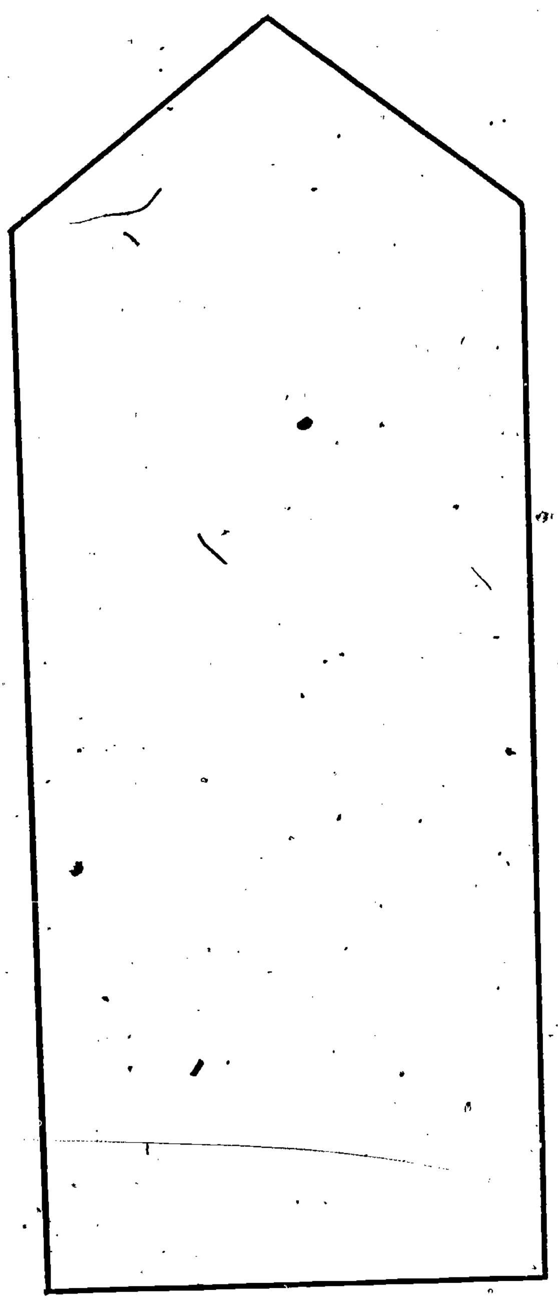

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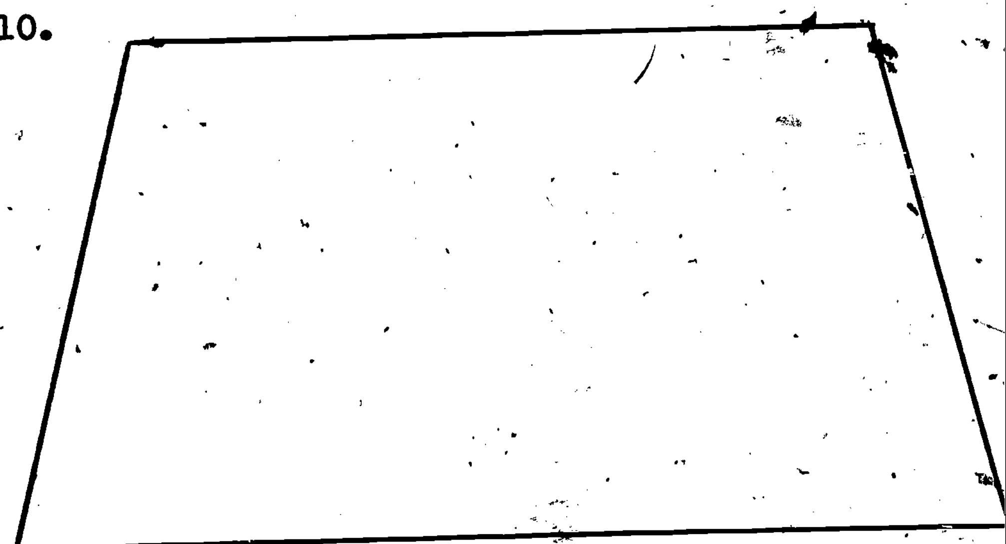
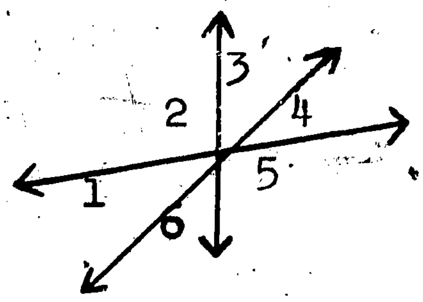


METRIC GEOMETRY
AREA MEASURE

Illustration of Terms

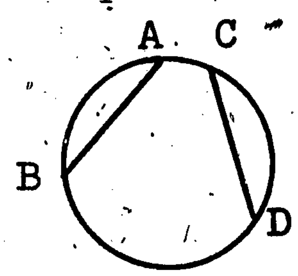


adjacent, next to; side by side.

Angles 1 and 2 are adjacent.
Angles 1 and 3 are not adjacent.

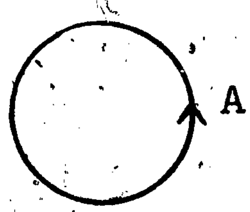
centimeter, unit of linear measure, $\frac{1}{100}$ of a meter, (1 cm.).

chords, line segments whose endpoints are points of the circle; a line segment from one point on the circle to another point on the circle.



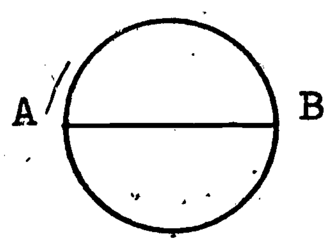
$\overline{AB}$ and $\overline{CD}$ are chords.

circumference, the distance around a circle.



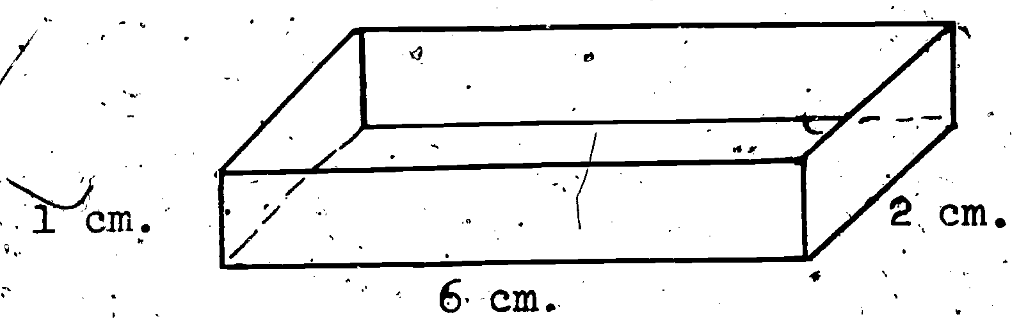
If you start at A and travel in only one direction on the circle, you will come back to A. This distance you moved is the circumference.

diameter, a line segment from a point on the circle through the center of the circle to a point on the circle.



$\overline{AB}$ is a diameter.

dimensions, the measurements in various directions which characterize a particular form.



The dimensions of this solid are 6 cm x 2 cm x 1 cm.

__equivalent__, different names for the same quantity.

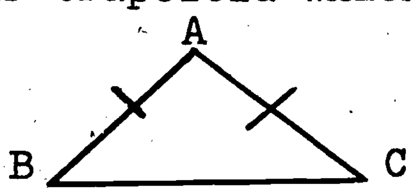
A quarter is equivalent to .25.
$\frac{4}{5}$ is equivalent to .8.

__estimate__, a guess about the answer to a problem.

__interior__, the part inside, located in the inside.

__intersection__, the points in common.

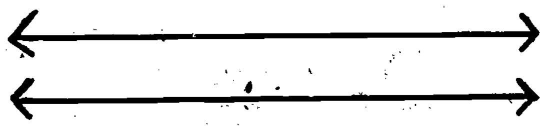
__isosceles__, a triangle or trapezoid which has two sides the same length.



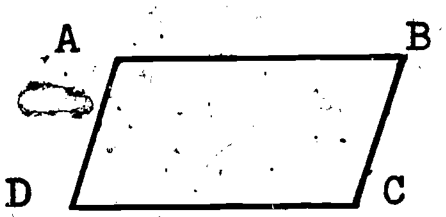
__linear measure__, measurement along lines.

__meter__, a unit of length about the same as a yard (actually 39.37 inches); 100 centimeters.

__parallel lines__, lines which stay the same distance apart. They neither cross nor meet.

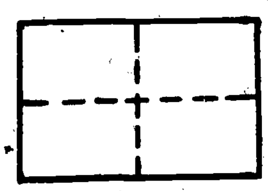


__parallelogram__, a four-sided plane figure with exactly two pairs of parallel sides.

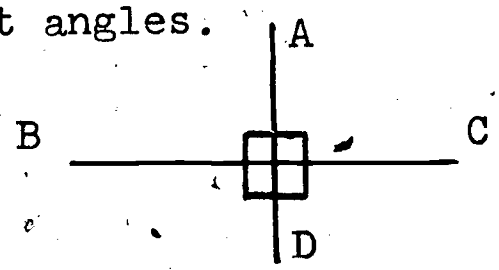


$\overline{AB}$ is parallel to $\overline{CD}$
$\overline{BC}$ is parallel to $\overline{AD}$
ABCD is a parallelogram

__partition__, to divide into parts.



<u>perpendicular</u>, meeting in such a way that the angles formed are right angles.



<u>π (pi)</u>, ratio of the circumference of a circle to its diameter; approximately $\frac{22}{7}$ or 3.14.

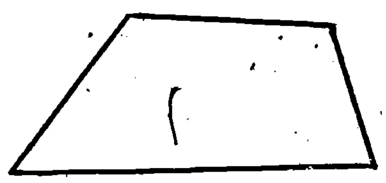
$$\frac{c}{d} = \pi$$

<u>plane</u>, a flat surface extending indefinitely in two of the three possible directions.

<u>product</u>, the number which results when two or more numbers are multiplied.

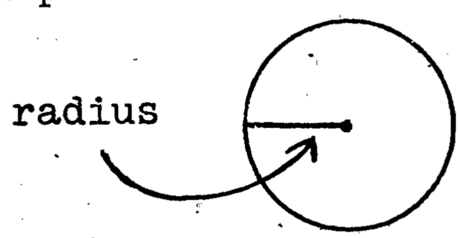
$5 \times 4 = 20$ -- 20 is the product

<u>quadrilateral</u>, a four-sided plane figure.



<u>radii</u>, plural of radius.

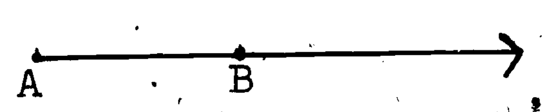
<u>radius</u>, the line segment whose endpoints are the center of the circle and a point on the circle.



<u>ratio</u>, the quotient of two numbers.

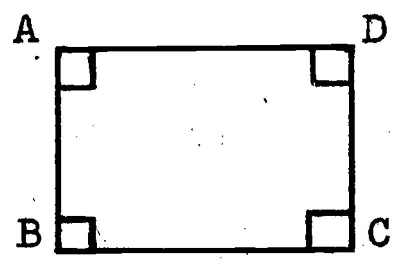
A ratio of 3 to 5 is expressed $\frac{3}{5}$.

<u>ray</u>, a line segment starting at a point and continuing unlimited in one direction.

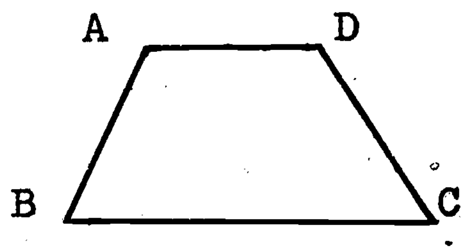


Ray AB starts at A, goes through B, and continues on forever.

<u>rectangle</u>, a four-sided figure whose opposite sides are parallel and whose angles are right.



<u>trapezoid</u>, a four-sided figure with exactly one pair of parallel sides.



$\overline{AD}$ is parallel to $\overline{BC}$
ABCD is a trapezoid

<u>vertex</u>, the point of intersection of the sides of an angle.