REPORT RESUMES

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IDEAS FROM NUMBER THEORY.

BY- FOLEY, JACK L.

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THIS BOOKLET, ONE OF A SERIES, HAS BEEN DEVELOPED FOR THE PROJECT, A PROGRAM FOR MATHEMATICALLY UNDERDEVELOPED PUPILS. A PROJECT TEAM, INCLUDING INSERVICE TEACHERS, IS BEING USED TO WRITE AND DEVELOP THE MATERIALS FOR THIS PROGRAM. THE MATERIALS DEVELOPED IN THIS BOOKLET INCLUDE ELEMENTARY IDEAS CONCERNING (1) WHOLE NUMBERS, (2) OPERATIONS WITH SETS, (3) DIVISORS AND MULTIPLES OF A NUMBER, AND (4) ACTIVITIES INVOLVING DIVISORS AND MULTIPLES. ACCOMPANYING THESE BOOKLETS WILL BE A "TEACHING STRATEGY BOOKLET" WHICH WILL INCLUDE A DESCRIPTION OF TEACHER TECHNIQUES, METHODS, SUGGESTED SEQUENCES, ACADEMIC GAMES, AND SUGGESTED VISUAL MATERIALS. (RP)

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# IDEAS FROM NUMBER THEORY

ESEA Title III

# PROJECT MATHEMATICS

## Project Team

Dr. Jack L. Foley, Director
Elizabeth Basten, Administrative Assistant
Ruth Bower, Assistant Coordinator
Wayne Jacobs, Assistant Coordinator
Gerald Burke, Assistant Coordinator
Leroy B. Smith, Mathematics Coordinator for Palm Beach County

## Graduate and Student Assistants

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Kathleen Whittier
Jeanne Hullihan
Barbara Miller
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Mr. Kelly Williams
Mr. Lloyd Williams

August, 1967

For information write: Dr. Jack L. Foley, Director
Bldg. S-503, School Annex
6th Street North
West Palm Beach, Florida

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# WHOLE NUMBERS

## The Set of Whole Numbers

Do you ever talk about the Johnsons, Smiths, or perhaps the Campbells? We have named only three families of people. What if you had to name all the families in the world? What a large list you would have. The three families would be only a "part of" the list of all families.

We can use this same idea to show the numbers we are going to study. They are called the whole numbers, and they are a subset of "all" numbers. For the set of whole numbers, we begin with 0. It is the first whole number. To get to the "next" whole number we simply add 1. We repeat this to get to the next whole number. This addition of 1 goes on and on. We can show this in an easy way. We use dots to show that the numbers continue on.

$$\{0,\ 0 + 1,\ 0 + 1 + 1,\ 0 + 1 + 1 + 1,\ \ldots\}$$
$$\{0,\ 1,\ 2,\ 3,\ \ldots\}$$

Could you continue these numbers in this way? Would there be a last one? You're right. You would never reach a "last one" since you could always add 1 and get to another.

Remember that we will only study the whole numbers in this unit. This means that we will exclude common fractions, decimals, negative numbers, and all other numbers that are not whole numbers.

## Subsets of the Set of Whole Numbers - Even Numbers and Odd Numbers

We can break the whole numbers down into subsets. For example, every whole number is either odd or even. The two subsets are shown below.

$$\text{Whole Numbers} \left\{ \begin{array}{l} \text{Even} -\{0, 2, 4, 6, \ldots\} \\ \text{Odd} \ \ -\{1, 3, 5, 7, \ldots\} \end{array} \right)$$

A number is even if 2 divides the number and leaves a 0 remainder. If you divide by 2 and have a remainder of 1, then it is odd.

The Greeks thought of an even number as representing a collection of objects that could be arranged in two rows with the same number of objects in each row. An odd number represented a collection that could not be arranged in this way. One row would have one more object than the other row. This is to say that an even number can be written as a product of two factors, with 2 as one of these factors.

For example:

6 = 2 X 3 (2 rows - 3 objects in each row)
14 = 2 X 7 (2 rows - 7 objects in each row)
18 = 2 X 9 (2 rows - 9 objects in each row)

In general terms, an even number can be written as 2 X w (w is any whole number). How could an odd number be shown in general terms?

Of course, (2 X w) + 1 or (2 X w) - 1.

## Activities

### Even and Odd Whole Numbers

1. TRUE OR FALSE

a) __________ If two even numbers are added, the sum will always be even.

b) __________ If two odd numbers are added, the sum will be even.

c) __________ If two even numbers are multiplied, the product is odd.

d) __________ If two odd numbers are multiplied, the product is even.

2. For adding consecutive odd numbers, starting with 1, can you see an easy way to arrive at the sum?
Continue this pattern until you have ten addends. Check each of your sums by actually adding.

| Indicated Sum | Number of Addends (terms) | Sum |
|---|---|---|
| 1 | 1 | 1 X 1 = 1 |
| 1 + 3 | 2 | 2 X 2 = 4 |
| 1 + 3 + 5 | 3 | 3 X 3 = 9 |
| 1 + 3 + 5 + 7 | 4 | 4 X 4 = 16 |
| 1 + 3 + 5 + 7 + 9 | 5 | 5 X 5 = 25 |
| ☐ | ☐ | ☐ |
| ☐ | ☐ | ☐ |
| ☐ | ☐ | ☐ |
| ☐ | ☐ | ☐ |
| ☐ | ☐ | ☐ |

3. For adding consecutive even numbers, starting with 2, can you see an easy way to arrive at the sum? Extend this pattern and check by adding. Extend until there are eight addends in your indicated sum.

| Indicated Sum | Pattern | Sum |
|---|---|---|
| 2 | (1X1) + 1 or $1^2 + 1$ | 2 |
| 2 + 4 | (2X2) + 2 or $2^2 + 2$ | 6 |
| 2 + 4 + 6 | (3X3) + 3 or $3^2 + 3$ | 12 |
| 2 + 4 + 6 + 8 | (4X4) + 4 or $4^2 + 4$ | 20 |
| | | |
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4. For adding consecutive whole numbers, beginning with 1, can you see an easy way to arrive at the sum? Extend this pattern until you have eight addends in the indicated product. Do your checking again by adding.

| Indicated Sum | Pattern | Sum |
|---|---|---|
| 1 | $\frac{1 X 2}{2}$ | 1 |
| 1 + 2 | $\frac{2 X 3}{2}$ | 3 |
| 1 + 2 + 3 | $\frac{3 X 4}{2}$ | 6 |
| 1 + 2 + 3 + 4 | $\frac{4 X 5}{2}$ | 10 |
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5. For multiplying three consecutive whole numbers, can you see an easy way to arrive at the product? Extend this pattern four more times and check your answers.

| Indicated Product | Pattern | Product |
|---|---|---|
| 1 X 2 X 3 | (2X2X2) - 2 or $2^3$ - 2 | 6 |
| 2 X 3 X 4 | (3X3X3) - 3 or $3^3$ - 3 | 24 |
| 3 X 4 X 5 | (4X4X4) - 4 or $4^3$ - 4 | 60 |
| 4 X 5 X 6 | (5X5X5) - 5 or $5^3$ - 5 | 120 |
| | | |
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6. For multiplying two whole numbers whose difference is 2, can you extend this pattern?

| Indicated Product | Pattern | Product |
|---|---|---|
| 3 X 5 | (4X4) - 1 or $4^2$ - 1 | 15 |
| 5 X 7 | (6X6) - 1 or $6^2$ - 1 | 35 |
| 7 X 9 | (8X8) - 1 or $8^2$ - 1 | 63 |
| 8 X 10 | (9X9) - 1 or $9^2$ - 1 | 80 |
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7. Extend the following pattern up to (19 X 19) - 1.

| **Odd Numbers (start with 3)** | **Pattern (even numbers)** | **Product** |
|---|---|---|
| (3 X 3) - 1 | 2 X 4 | 8 |
| (5 X 5) - 1 | 4 X 6 | 24 |
| (7 X 7) - 1 | 6 X 8 | 48 |
| (9 X 9) - 1 | 8 X 10 | 80 |
| | | |
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## Subsets Formed Using Divisors

We can also break down the whole numbers into subsets using other ideas. Suppose we use the idea of division. We will call a number a <u>divisor</u> of another number if you get a 0 remainder when dividing with this number.

**For example:**

6 is a divisor of 12

3 is a divisor of 9

2 is a divisor of 14

We can make use of these divisors to put numbers in certain subsets. Let's start by listing divisors for each whole number. We can begin with 0 since it is our first whole number. We will write the divisors opposite the number and enclose them with braces. The braces enclose members of a set.

| Number | | Divisors of Number |
|---|---|---|
| 0 | - | {1, 2, 3, 4, 5, 6, 7, 8, ...} |
| 1 | - | {1} |
| 2 | - | {1, 2} |
| 3 | - | {1, 3} |
| 4 | - | {1, 2, 4} |
| 5 | - | {1, 5} |
| 6 | - | {1, 2, 3, 6} |
| 7 | - | {1, 7} |
| 8 | - | {1, 2, 4, 8} |
| 9 | - | {1, 3, 9} |
| 10 | - | {1, 2, 5, 10} |
| 11 | - | {1, 11} |
| 12 | - | {1, 2, 3, 4, 6, 12} |
| 13 | - | {1, 13} |

Would you like to look over this divisor list as far as we have gone? Do you notice anything unusual or different about 0 and 1? Maybe we can show some ideas from our list.

Idea I - Every number (whole number) other than 0 is a "divisor of" 0.

Idea II - The number 1 has only one divisor -- itself.

Idea III - The number 1 is a divisor of every number.

Idea IV - Every number is a divisor of itself, except 0.

Idea V - Every number "greater than" 1 has at least two different divisors, one and itself.

Idea VI - Zero is not a divisor of any number.

We can use our ideas now to show our subsets. Let 0 and 1 be two different subsets. Let those numbers greater than 1 with exactly two different divisors be a third subset and those numbers greater than 1 with more than two different divisors be our fourth subset. Dots show that numbers go on and on.

$$\text{Whole numbers}\left\{\begin{array}{l}\{0\}\\ \{1\}\\ \{2,\ 3,\ 5,\ 7,\ 11,\ 13,\ 17,\ 19,\ldots\ \text{- prime}\}\\ \{4,\ 6,\ 8,\ 9,\ 10,\ 12,\ 14,\ 15,\ldots\ \text{- composite}\}\end{array}\right\}$$

The numbers with exactly two different divisors are called prime numbers. Notice that 2 is the only even prime. The numbers "greater than 1" with more than two different divisors are called composite numbers.

## Factors

A composite number can always be represented by a rectangular array. This means that every composite number can be written as the product of at least two smaller numbers. These smaller numbers are called factors. A factor is each number listed in an indicated product.

For example: 3 X 2 X 5 = 30

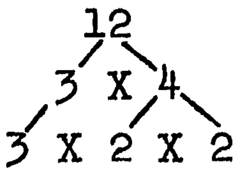
The 3, 2, and 5 are called factors. Every non-zero factor is also called a divisor.

The factors of a composite number can be shown if we represent the composite number as a rectangular array of dots or other objects. One factor represents the number of rows and the second factor represents the number of objects in each row. (A prime number cannot be written as a rectangular array since it will never have the same number of objects in each of its rows.)

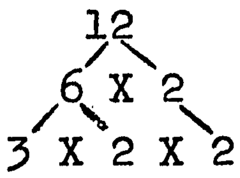
| Composite Number | Array | Product of Two Smaller Numbers (Factors) |
|---|---|---|
| 10 | 5<br>2 xxxxx<br>xxxxx | 2 X 5 (2 rows with 5 objects in each row) |
| 12 | 4<br>3 xxxx<br>xxxx<br>xxxx | 3 X 4 (3 rows with 4 in each row) |
| 14 | 7<br>2 xxxxxxx<br>xxxxxxx | 2 X 7 (2 rows with 7 in each row) |
| 9 | 3<br>3 xxx<br>xxx<br>xxx | 3 X 3 (3 rows with 3 in each row) |

Do you notice that 10, 14, and 9 cannot be shown in a different rectangular array? Do you also see that 12 can? Our factors, each number in the indicated product, could give us a clue as to why this is so. The factors for 10 are 2 and 5. For 14 we have the factors 2 and 7, and for 9 each factor is a 3. All of these factors are prime numbers.

Are both factors shown for 12 prime? You are right, 4 is not a prime. Could 4 be shown as an array? Suppose we take a composite number and continue factoring until each factor is a prime. We can use 12 and follow the pattern below.

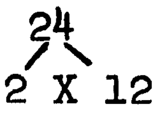


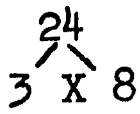
This figure is called a factor tree. With many composite numbers we can start out with different factorizations. Our 12 factor tree could have looked like this.

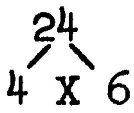


Notice again that we can continue factoring until only prime numbers are found at the base of the factor tree.

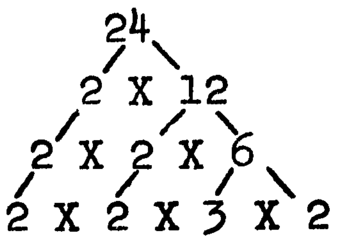
Think of the ways we could start out writing 24 as a product of 2 factors.

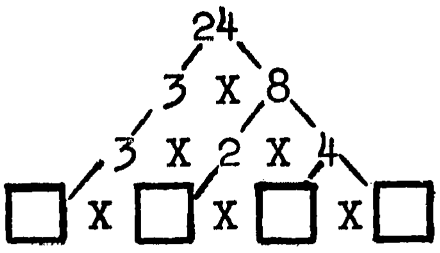


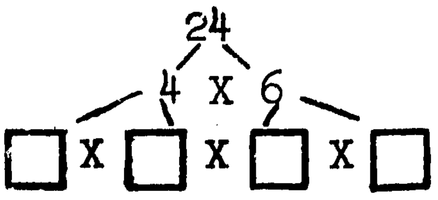




We can continue writing each factor as a product of two smaller factors until all factors are prime numbers. Please complete the last two factor trees.







In each example we ended up with the same set of prime factors except for the way they are arranged (order). You notice that the 3 appears in different positions in the order of the factors. Will it change a product if you rearrange factors? You are right. It does not change the product.

The fact that every composite number can be expressed as a product of primes in one and only one way (except for order) is called The Fundamental Theorem of Arithmetic.

## Activities

### Prime and Composite Numbers

1. TRUE or FALSE

    a) ________ Some whole numbers do not have a divisor.

    b) ________ All odd numbers are prime.

    c) ________ Three is a divisor of 18.

    d) ________ Every number, other than 0, is a divisor of 0.

    e) ________ The number 16 has five divisors.

    f) ________ Fifteen objects cannot be represented in a rectangular array.

2. Make a factor tree for each of these numbers: 15, 20 and 18.

3. Show 20 objects in 2 different rectangular arrays. Different means that each array has a different number of rows and columns (objects in each row).

4. Complete each factor tree that is started for the number 60.

5. Circle the numbers in the set below which represent a collection of objects that can be arranged into a rectangular array.

    { 6, 5, 17, 18, 19, 23, 26 }

6. Circle the numbers in the set below that are prime numbers.

$$\{ 2, \ 4, \ 6, \ 8, \ 9, \ 11, \ 13, \ 15, \ 17 \}$$

7. If a number is written in expanded notation, can you easily see that each position is even, except possible the <u>ones</u> position?

Example: $$\underline{3533} = \underset{\text{even}}{\underset{\downarrow}{\underline{3000}}} + \underset{\text{even}}{\underset{\downarrow}{\underline{500}}} + \underset{\text{even}}{\underset{\downarrow}{\underline{30}}} + \underset{\text{odd}}{\underset{\downarrow}{\underline{3}}}$$

Can you figure out why?

8. Using only the prime number 3, see how many consecutive whole numbers you can write. Any operations you know may be used. This idea is started below.

$$0 = 3 - 3$$

$$1 = \frac{3}{3}$$

$$2 = \frac{3 + 3}{3}$$

$$3 = 3$$

$$4 = \frac{3 + 3 + 3 + 3}{3}$$

$5 =$

$6 =$

$7 =$

.

.

.

9. The <u>only</u> <u>even</u> prime number is 2. Can you give a reason why any $\overline{\text{even}}$ $\overline{\text{number}}$ greater than 2 will have more than two different divisors?

10. The pattern below will give a result that is a prime number up to n = 41. Check five other numbers less than n = 41.

(Pattern is $n^2 - n + 41$.)

| n | n X n | (n X n) - n | (n X n) - n + 41 | Result |
|---|---|---|---|---|
| 1 | 1 X 1 = 1 | 1 - 1 = 0 | 0 + 41 | 41 |
| 2 | 2 X 2 = 4 | 4 - 2 = 2 | 2 + 41 | 43 |
| 3 | 3 X 3 = 9 | 9 - 3 = 6 | 6 + 41 | 47 |
| 4 | 4 X 4 = 16 | 16 - 4 = 12 | 12 + 41 | 53 |
| 5 | 5 X 5 = 25 | 25 - 5 = 20 | 20 + 41 | 61 |
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11. With the use of the high-speed computer, some very large prime numbers have been discovered. Use your library to see if you can find the largest discovered up to this date. Don't think that this will be <u>the</u> largest prime; it is merely the largest that man <u>knows</u> at the moment. There is no largest prime, for the prime numbers go on and on.

## SETS

Two basic operations with sets are union and intersection. You have probably studied both of these. The intersection of sets will be used in looking at common divisors. In case you've forgotten, we will take a look at the meanings of sets and at the union and intersection of sets.

The word "set" implies a collection of objects. The objects are called elements or members of the set. We use a capital letter to denote a set. Lower case letters are used for the elements. The members of a set are usually enclosed in braces. The set of members must be described in some way. Some examples will clear up this idea. Let's describe some sets and then put them in set form.

Description - The set of vowels. We can choose any capital letter to represent our set. For this set let's choose A.
A = {a, e, i, o, u}

Description - The first eight even numbers.
B = {0, 2, 4, 6, 8, 10, 12, 14}

Description - The first four odd numbers.
C = {1, 3, 5, 7}

Description - The set of even prime numbers.
D = {2}

### Union

When we think of the word "union," we may think of uniting or bringing together. First, a description of union will be given and then some examples.

Union - For any two sets A and B, form another set by listing all elements that are in A or B. Each element is listed only once. A capital "U" means Union.

Example: Let A = {a, b, c, d, e, f}

Let B = {a, e, g}

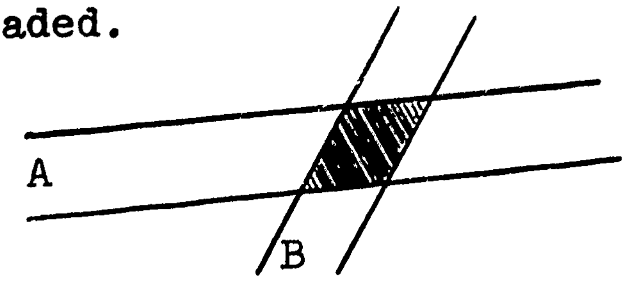
Then A Union B = {a, b, c, d, e, f, g}

or

A U B = {a, b, c, d, e, f, g}

## Intersection

When we think of the word "intersection," we think of two highways crossing. Call one highway A and the other B. The intersection is shaded.



Which highway would claim the shaded portion? Of course, it belongs to both.

Intersection - For any two sets A and B, form another set by listing all elements that are in both A and B. The symbol for intersection is ∩.

Example: Let A = {a, b, c, d}

Let B = {a, e, f}

Then A intersection B = {a}

or

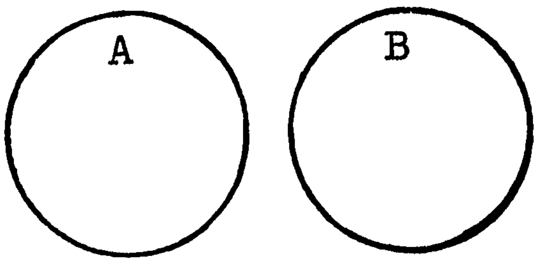
$A \cap B = \{a\}$

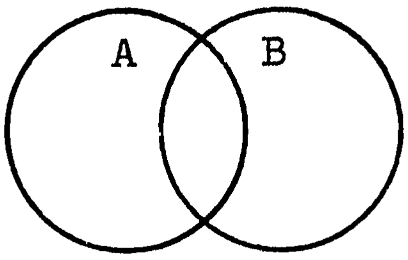
## Activities

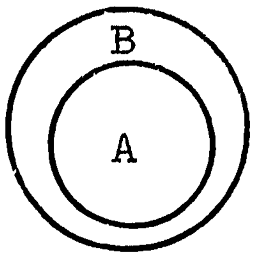
### Sets

1. If A = {1, 3, 5, 7, 9} and B = {3, 4, 5, 6}, show:
   a) $A \cup B =$
   b) $A \cap B =$

2. If A = {a, b, c} and B = {a, b, c, d, e, f, g}, which drawing below would best show how A and B are related?





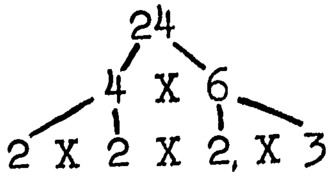


3. Use set notation to illustrate each of the following:
   Example: The set of months beginning with the letter J.
   A = {January, June, July}
   a) The set of odd numbers less than 10.
   b) The set of even numbers between 15 and 31.
   c) The set of composite numbers between 10 and 30.
   d) The set of prime numbers between 5 and 23.

## DIVISORS

### Number of Divisors

When we write a number as a product of primes, we can use the prime factors to find out how many divisors the number has. You can accept this fact now, see how it works, and later in your study of numbers you can prove it. Let's take another look at the factorization of 24.



Here is how you find how many divisors 24 has:

1. Count how many times each prime appears as a factor: 2 X 2 X 2 X 3

   (3 times) (1 time)

2. Add one to each of these numbers of times: 3 + 1 and 1 + 1 (which is 4 and 2)

3. Multiply these together: 4 X 2 = 8 (the 8 is the number of exact divisors)

Then 24 has 8 exact divisors. How do we find the divisors? The divisors of any whole number are:

1. 1

2. The number's different prime factors chosen one at a time. (The different prime factors of 24 are 2 and 3.)

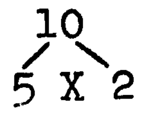
3. The possible products when 2 of the number's primes are selected. (For 24 this would be 2 X 2 = 4 and 2 X 3 = 6.)

4. The possible products when 3 of the number's primes are selected. (For 24 this would be 2 X 2 X 2 = 8 and 2 X 2 X 3 = 12.)

5. The possible products when 4 of the number's primes are selected and so on until all the primes are selected at the same time. This last product will be the number itself. (For 24 the product of 4 primes is 2 X 2 X 2 X 3 = 24. For 24, selecting 4 primes at a time is the same as selecting all the primes, which we said would result in the number itself.)

Now the 8 divisors of 24 that we have just found are the underlined numbers above. They are the set: {1, 2, 3, 4, 6, 8, 12, 24}

Example: How many divisors does 10 have and what are they?



1. Count - Two appears 1 time and 5 appears 1 time.

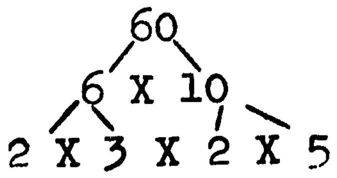
2. Add - Now add 1 to the number of times each prime appears:
1 + 1 and 1 + 1

3. Multiply - 2 X 2 = 4

Then 10 has 4 exact divisors. These are:
1. 1
2. 5, 2
3. 5 X 2 = 10

1, 2, 5, 10

Example: How many divisors does 60 have and what are they?



1. Count - 2, 1, 1

2. Add - 3, 2, 2

3. Multiply - 3 X 2 X 2 = 12

Then 60 has 12 exact divisors. These are:

1. 1
2. 2, 3, 5 (numbers of different factors chosen one at a time)
3. 2 X 3 = 6, 2 X 2 = 4, 3 X 5 = 15 (factors chosen two at a time)
4. 2 X 3 X 2 = 12, 2 X 3 X 5 = 30, 2 X 2 X 5 = 20 (factors chosen three at a time)
5. 2 X 3 X 2 X 5 = 60 (all factors chosen)

We have found the 12 divisors of 60 to be:
{1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60}

It is sometimes easier to use the idea of an exponent to find how many exact divisors a number has. We merely write down each different factor one time. Then we place a number above and to the right of this factor which tells how many times it appears as a factor. The number which tells how many times the factor appears is called an exponent.

For example, examine the product:

2 X 2 X 3 X 2 X 3 X 5

How many times does the 2 appear as a factor? That is right, it appears 3 times. The 3 appears 2 times and the 5 appears 1 time. We now write:

2 X 2 X 3 X 2 X 3 X 5 = $2^3$ X $3^2$ X $5^1$

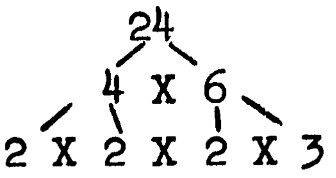
The small numbers placed above and to the right are exponents. Now let's use this idea to find how many divisors 24 has.

Step I: Find the prime factorization of 24.

Step II: Put the factorization into exponent form.

Step III: Add 1 to each exponent and express the result as a product. The product is the number of exact divisors of 24.

Step I:



Step II: $2^3$ X $3^1$

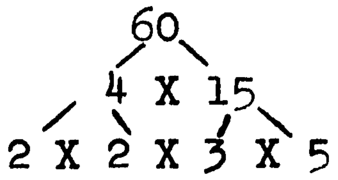
Step III: Have exponents 3 and 1.

Then, (3 + 1) X (1 + 1)
4 X 2 = 8

Then 24 has 8 exact divisors.

For 60, we have:

Step I:



Step II: $2^2$ X $3^1$ X $5^1$

Step III: Have exponents 2, 1 and 1.

Then,

(2 + 1) X (1 + 1) X (1 + 1) =
3 X 2 X 2 = 12

Then 60 has 12 exact divisors.

This is really what we did at first, but we did not put our factorization into exponent form.

## Activities

Use the idea of an exponent to find the number of exact divisors of the following numbers:

Example: 34

I. 34
17 X 2

II. $17^1$ X $2^1$

III. (1 + 1) X (1 + 1) =
2 X 2 = 4

1. 54

I.

II.

III.

2. 28

I.

II.

III.

3. 86

I.

II.

III.

## Sum of Divisors

List the divisors of some particular numbers and find the sum of these divisors. For example:

| Number | Divisors | Sum of Divisors |
|---|---|---|
| 10 | {1, 2, 5, 10} | 1 + 2 + 5 + 10 = 18 |
| 15 | {1, 3, 5, 15} | 1 + 3 + 5 + 15 = 23 |
| 20 | {1, 2, 4, 5, 10, 20} | 1 + 2 + 4 + 5 + 10 + 20 = 42 |

Each divisor of a number that is less than the number itself is called a proper divisor. The proper divisors of 10 are: 1, 2, 5.

If the sum of the proper divisors is less than the number itself, the number is called a deficient number. The number 10 is deficient as 1 + 2 + 5 = 8, and 8 "is less than" 10.

Show some other deficient number. ____________________

If the sum of the proper divisors is equal to the number itself, it is called a perfect number. The number 6 is perfect as 1 + 2 + 3 = 6.

Can you find another perfect number? ____________________

If the sum of the proper divisors "is greater than" the number itself, it is called an abundant number. The number 20 is an abundant number as:

$$1 + 2 + 4 + 5 + 10 = 22$$

and 22 "is greater than" 20.

Can you find other abundant numbers? ____________________

## Common Divisors

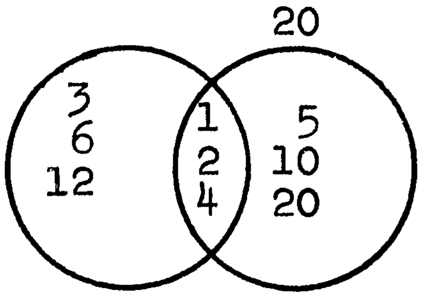
A common divisor is a divisor that appears in each set of divisors for two or more numbers. Suppose we find the common divisors of 12 and 20. The divisors for each number can be listed and the common divisors selected.

| Number | Set of Divisors |
|---|---|
| 12 | {1, 2, 3, 4, 6, 12} |
| 20 | {1, 2, 4, 5, 10, 20} |

The intersection of these two sets gives the set of common divisors. These common divisors are: {1, 2, 4}

## Greatest Common Divisor

Pick the greatest common divisor of 12 and 20. It is 4. This is the largest number that will divide both 12 and 20. We could draw two circles, called Venn diagrams, and place the divisors of 12 in one circle and the divisors of 20 in the other circle.



This drawing shows the intersection of the two sets.

As another example, find the greatest common divisor of 16, 24, and 30.

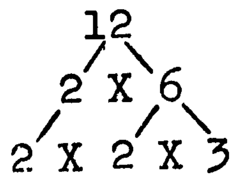
| Number | Set of Divisors |
|---|---|
| 16 | {1, 2, 4, 8, 16} |
| 24 | {1, 2, 3, 4, 6, 8, 12, 24} |
| 30 | {1, 2, 3, 5, 6, 10, 15, 30} |

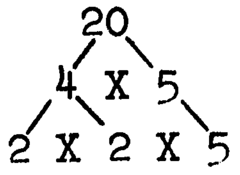
Common divisors are: {1, 2}

Greatest common divisor is: 2

The greatest common divisor (g. c. d) can also be found by making factor trees, using the <u>prime</u> <u>factor</u> <u>form</u>. First, express each of the numbers as a product of primes. This is called <u>prime</u> <u>factorization</u>.

Below we will find the g.c.d. of 12 and 20 by using this form.

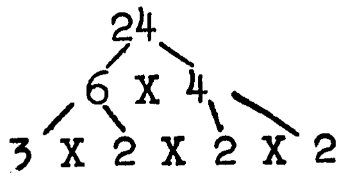


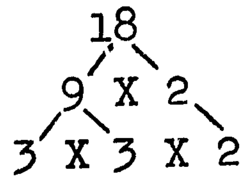


1. List each common prime. If a common prime appears more than once in <u>each</u> factor tree, we list it as many times as it appears <u>common</u>. Since <u>2</u> appears as a common factor two times in the above example, we put it down twice: 2, 2.

2. Express these common factors as a product, 2 X 2 = <u>4</u> 4 is the g.c.d. (greatest common divisor.) This may also be referred to as h.c.f. or highest common factor.

As another example, try <u>24</u> and <u>18</u>.





3 X 2 = <u>6</u>, greatest common divisor

$$\begin{array}{rl} \text{or:} \quad 24 = & 3 \text{ X } 2 \text{ X } 2 \text{ X } 2 \\ & \downarrow \quad\; \downarrow \\ 18 = & 3 \text{ X } 2 \text{ X } 3 \\ & \downarrow \quad\; \downarrow \\ \text{g.c.d.} = & 3 \text{ X } 2 = \underline{6} \end{array}$$

One of the older ways to find the g.c.d. of two numbers is called <u>Euclid's Algorithm</u>. This process can be proved, but it will only be illustrated here. First, let's state the steps to follow.

Step I - Divide the smaller of the two numbers into the larger.

Step II - Disregard the quotient and dividend and divide the <u>remainder</u> into the <u>divisor</u>.

Step III - Continue dividing each remainder into the divisor until it goes evenly (0 remainder). When this happens, the divisor of the last step is the g.c.d. of the two numbers you started with.

Example: The g.c.d. of 12 and 20

Step I:

$$\begin{array}{r} 1 \\ 12\overline{)\,20} \\ \underline{12} \\ 8 \end{array}$$

Step II:

$$\begin{array}{r} 1 \\ 8\overline{)\,12} \\ \underline{8} \\ 4 \end{array}$$

Step III:

$$\begin{array}{r} 2 \\ 4\overline{)\,8} \\ \underline{8} \\ 0 \end{array}$$

Since this division gives a 0 remainder, the divisor 4 is the g.c.d. of 12 and 20.

Example: The g.c.d. of 18 and 24

$$\begin{array}{r} 1 \\ 18\overline{)\,24} \\ \underline{18} \\ 6 \end{array}$$

$$\begin{array}{r} 3 \\ 6\overline{)\,18} \\ \underline{18} \\ 0 \end{array}$$

The g.c.d. = $\underline{6}$

Example: The g.c.d. of 23 and 14.

$$\begin{array}{r} 1 \\ 14\overline{)\,23} \\ \underline{14} \\ 9 \end{array} \qquad \begin{array}{r} 1 \\ 9\overline{)\,14} \\ \underline{9} \\ 5 \end{array} \qquad \begin{array}{r} 1 \\ 5\overline{)\,9} \\ \underline{5} \\ 4 \end{array} \qquad \begin{array}{r} 1 \\ 4\overline{)\,5} \\ \underline{4} \\ 1 \end{array} \qquad \begin{array}{r} 1 \\ 1\overline{)\,4} \\ \underline{4} \\ 0 \end{array}$$

The g.c.d. = $\underline{1}$

You can use the g.c.d. of two or more numbers to find all other common divisors of the numbers. All common divisors of two or more numbers are divisors of the g.c.d.

For example, the g.c.d. of 18 and 24 is 6. Now all divisors of 6, including 6 itself, are the common divisors of 18 and 24.

| g.c.d. | Divisors of 6 |
|---|---|
| 6 | {1, 2, 3, 6} |

Then the common divisors of 18 and 24 are: {1, 2, 3, 6}

Check:

| Number | Divisors |
|---|---|
| 18 | {1, 2, 3, 6, 9, 18} |
| 24 | {1, 2, 3, 4, 6, 8, 12, 24} |
| Common Divisors | {1, 2, 3, 6} |

Try an example using the g.c.d. of three numbers.

Some ideas for discussion are stated below regarding the g.c.d. of two or more numbers.

Idea I - The g.c.d. is never larger than the smallest of these numbers.

Idea II - More than one way can be used to find the g.c.d.

Idea III - All common divisors will evenly divide the g.c.d.

Idea IV - Two or more numbers will always have a g.c.d.

Idea V - The number 1 is always a common divisor and is sometimes the g.c.d.

From Idea V we can show two or more numbers whose g.c.d. is 1.

| Number | Divisors |
|---|---|
| 5 | {1, 5} |
| 16 | {1, 2, 4, 8, 16} |

Common Divisors: {1}

g.c.d. = 1

If two or more numbers have 1 as a g.c.d., then they are called relatively prime.

14 and 23 are relatively prime. This does not mean that either is a prime number. It merely means that the largest number that will divide both numbers is 1.

# MULTIPLES

When we count by 2, we begin with 2 and count all multiples of 2 . This can be shown in two ways. The dots are used to show there is no last multiple of 2.

| Number | Multiples of Number |
|---|---|
| 2 | { 2, 4, 6, 8, 10, 12, 14, ...} |
| or | ↓ ↓ ↓ ↓ ↓ ↓ ↓ |
| 2 | { 1X2, 2X2, 3X2, 4X2, 5X2, 6X2, 7X2, ...} |

We can show multiples of 5 in the same way.

| Number | Multiples of Number |
|---|---|
| 5 | { 5, 10, 15, 20, 25, ...} |
| or | ↓ ↓ ↓ ↓ ↓ |
| 5 | { 1X5, 2X5, 3X5, 4X5, 5X5, ...} |

From the description above of multiples, how would you describe a multiple in general terms?

In general terms, if n is any whole number, the multiples of n are: n, 1 X n, 2 X n, 3 X n, ...

Or we could say that for any whole number n, a <u>multiple</u> of n is any whole number, other than <u>zero</u>, that n will evenly divide.

## Common Multiples

A common multiple of two or more numbers is a multiple that each of the numbers will divide.

Suppose we want to know the common multiples of <u>12</u> and <u>18</u>. We could list some multiples of each until we arrive at some common multiples.

| Number | Multiples of Number |
|---|---|
| 12 | { 12, 24, 36, 48, 60, 72, ...} |
| 18 | { 18, 36, 54, 72, 90, ...} |

Common Multiples: { 36, 72, 108, 144, ...}

or { 1 X 36, 2 X 36, 3 X 36, 4 X 36, ...}

Do you notice that there is no <u>last</u> common multiple of 12 and 18? This is true for two or more numbers.

## Least Common Multiple

What is the least common multiple of 12 and 18? It is 36, the first common multiple we arrived at.

Is there a largest common multiple of 12 and 18? The answer is no, because we know that there is no last common multiple.

The set of common multiples for two or more numbers is the intersection of the set of multiples for each number. It might take a long time to find the least common multiple (l.c.m.) for two or more numbers if we used only the process of writing the multiples of each number and then selecting the smallest common one. Several ways to find the l.c.m. are known which are faster. However, these ways do not show the meaning of multiples as well as this procedure.

Now let's look at the other ways to find the l.c.m. We can start by finding the l.c.m. for only two numbers. The following steps are used.

Step I: Find a number, greater than 1, that will divide both numbers. If no such number exists, the l.c.m. is the product of the two numbers.

Step II: If you find a number that will divide both numbers, then divide both by that number. Now look at your quotients and find another number that will divide your two quotients. Continue finding numbers until the only number that will divide your newest quotient is 1. (g.c.d. = 1)

Step III: Collect all your divisors and the two quotients for which you could not find a g.c.d. greater than 1 and multiply these together. Your product is the l.c.m.

Example: Find the l.c.m. of 12 and 18.

Step I: 2|12 18 (2 is greater than 1 and will divide both 12 and 18.)

Step II: 3| 6 9 (the quotients 6 and 9 can be divided by 3)
2 3 (the quotients 2 and 3 have 1 as a g.c.d.)

Step III: 2 X 3 X 2 X 3 = 36 = l.c.m.

**Example:** Find the l.c.m. of 24 and 36.

$$\begin{array}{r|rr} 6 & 24 & 36 \\ \cline{2-3} 2 & 4 & 6 \\ \cline{2-3} & 2 & 3 \end{array}$$

$$\text{l.c.m.} = 6 \times 2 \times 2 \times 3 = \underline{72}$$

We can modify this process and find the l.c.m. for three numbers. The steps for finding the l.c.m. for three numbers are:

**Step I:** First divide out any divisors common to all three numbers.

**Step II:** Divide out common divisors for <u>any two</u> of the numbers and simply bring the other number down. Repeat until the g.c.d. of any pair of the numbers is <u>1</u>.

**Step III:** Find the product of divisors and final quotients.

**Example:** Find the l.c.m. of 14, 21, and 42.

**Step I:** $\begin{array}{r|rrr} 7 & 14 & 21 & 42 \\ \cline{2-4} \end{array}$ (7 will divide all three of our numbers.)

**Step II:** $\begin{array}{r|rrr} 2 & 2 & 3 & 6 \\ \cline{2-4} \end{array}$ (2 is a common divisor of 2 and 6, bring 3 down.)

$\begin{array}{r|rrr} 3 & 1 & 3 & 3 \\ \cline{2-4} \end{array}$ (3 is a common divisor of 3 and 3, and bring 1 down.)

$\begin{array}{rrrr} \phantom{3} & 1 & 1 & 1 \end{array}$ (g.c.d. of any pair is 1.)

**Step III:** $7 \times 2 \times 3 \times 1 \times 1 \times 1 = \underline{42} = \text{l.c.m.}$

**Example:** Find the l.c.m. of 12, 18, and 30.

$$\begin{array}{r|rrr} 2 & 12 & 18 & 30 \\ \cline{2-4} 3 & 6 & 9 & 15 \\ \cline{2-4} & 2 & 3 & 5 \end{array} \quad \text{(g.c.d. of any pair is 1)}$$

$$2 \times 3 \times 2 \times 3 \times 5 = \underline{180} = \text{l.c.m.}$$

**Example:** Find the l.c.m. of 12, 15, and 25.

3 | 12 15 24

4 | 4 5 8

1 5 2

3 X 4 X 1 X 5 X 2 = 120 = l.c.m.

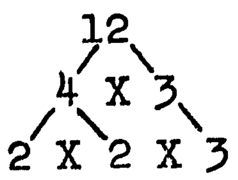
Another way to find the l.c.m. of two or more numbers is to use the prime factor form of the numbers. The l.c.m. will contain the factors, without repeating factors, of the numbers. We can say this in three steps.

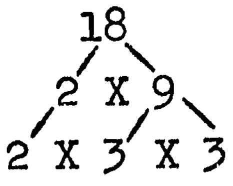
**Step I:** List the factorization of any one of the numbers involved. (The l.c.m. would have to contain these factors if the number is to divide it.)

**Step II:** Put in the list any factor of the other number or numbers that is not already included in the factorization list for the first number.

**Step III:** Find the product of these factors.

**Example:** Find the l.c.m. of 12 and 18.





12 = 2 X 2 X 3

18 = 2 X 3 X 3

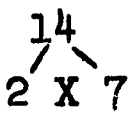
l.c.m. = 2 X 2 X 3 X 3 = 36

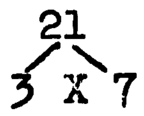
I. Factorization of 12 = 2 X 2 X 3

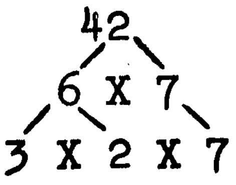
II. All we need to add as a factor is a 3 since all other factors of 18 are already listed. (In set language this is the union of prime factors for the two numbers.)

III. l.c.m. = 2 X 2 X 3 X 3 = 36 (Do you see both the factors of 18 and 12?)

**Example:** Find the l.c.m. of 14, 21, and 42.







14 = 7 X 2

21 = 7 X 3

42 = 7 X 3 X 2

l.c.m. = 7 X 3 X 2 = 42

Could you list some things you have learned about multiples? Would you agree with the following statements?

Idea I: Each number has an infinite set of multiples.

Idea II: Two or more numbers have an infinite set of common multiples.

Idea III: The least common multiple of two or more numbers can never be smaller than the larger of the numbers.

Idea IV: For two or more numbers, the least common multiple is a divisor of all the common multiples. (The l.c.m. is the product of the union of the numbers' prime factors.)

For any two numbers, such as 12 and 18, find the g.c.d. and l.c.m.

For 12 and 18: the g.c.d. = 6, and the l.c.m. = 36

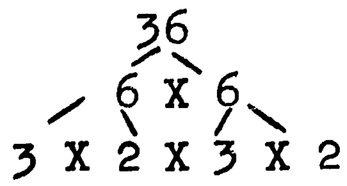
Multiply the two numbers: 12 X 18 = 216

Multiply the g.c.d. times the l.c.m.: 6 X 36 = 216

Comparing these two products, we find that they are equal. This is always true for <u>two</u> numbers.

## SUMMARY

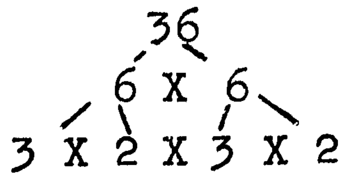
### Number of Divisors



I. three-2 times, two-2 times

II. 2 + 1 and 2 + 1
(add 1 to each number)

III. 3 X 3 = 9 = number of divisors



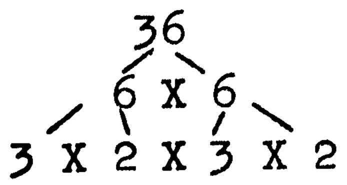
I. $3^2$ X $2^2$ (exponent form)

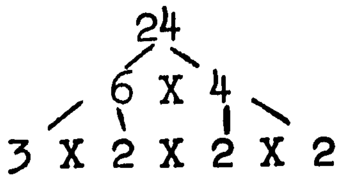
II. 2 + 1 and 2 + 1 (add 1 to each . exponent)

III. 3 X 3 = 9 = number of divisors

### Greatest Common Divisor (g.c.d.)

Factor Trees





36 = ③ X 3 X ② X ②
24 = ③ X ② X ② X 2
g.c.d. = ③ X ② X ②

I. ③ ② ②

II. 3 X 2 X 2 = 12 = g.c.d.

Euclid's Algorithm

I.

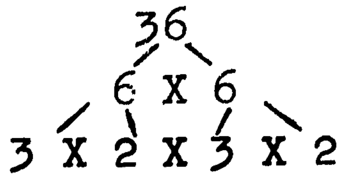
$$\begin{array}{r} 1 \\ 24\overline{)36} \\ \underline{24} \\ 12 \end{array}$$

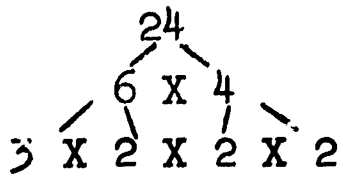
II. and III.

$$\begin{array}{r} 2 \\ \textcircled{12}\overline{)24} \\ \underline{24} \\ 0 \end{array}$$

g.c.d. = 12

## Least Common Multiple (l.c.m.) - Two Numbers





I. 3 X 2 X 3 X 2

II. 3 X 2 X 3 X 2 X 2

III. 3 X 2 X 3 X 2 X 2 = 72 = l.c.m.

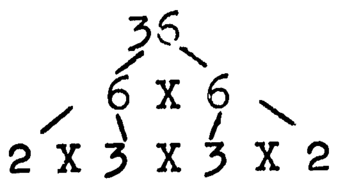
I. 6| 36 24

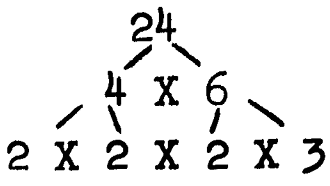
II. 2| 6 4

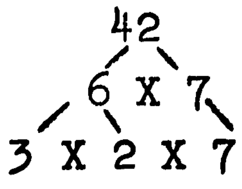
3 2

III. 6 X 2 X 3 X 2 = 72 = l.c.m.

## Least Common Multiple (l.c.m.) - Three or More Numbers







I. 2 X 3 X 3 X 2

II. 2 X 3 X 3 X 2 X 2 X 7

III. 2 X 3 X 2 X 3 X 2 X 7 = 504 = l.c.m.

I. 6| 36 24 42

II. 2| 6 4 7

3 2 7

III. 6 X 2 X 3 X 2 X 7 = 504 = l.c.m.

## Activities

### Divisors and Multiples

1. List the complete set of exact divisors (factors) for each of the following numbers.

   a) 24 -

   b) 60 -

   c) 32 -

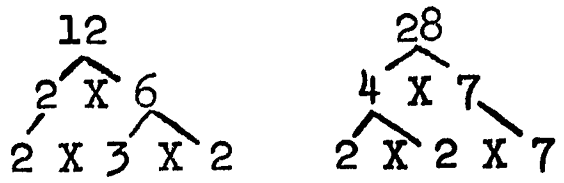
   d) 182 -

2. List the set of primes up to 40.

3. List the set of even primes.

4. Express each of the numbers as a product of primes. (Give the prime factorization.) Give the g.c.d. and l.c.m. by using these factorizations.

   Example: (12, 28)



   g.c.d. = 2 X 2 = 4

   l.c.m. = 2 X 3 X 2 X 7 = 84

   a) (26, 14)

   b) (32, 26)

c) (142, 64)

d) (15, 25, 21)

5. Goldbach's Conjecture - Any even number greater than 4 can be expressed as the sum of two odd primes. Give 5 illustrations of this. (Christian Goldbach, 1690-1764. This conjecture has never been proven.)

6. Twin Primes - This is a name given to a pair of primes which differ by 2. The first pair of twin primes is (3, 5). Note that 5 - 3 = 2. Give 3 more pairs of twin primes.

7. Find the g.c.d. of the following by the use of Euclid's Algorithm.

a) 15 and 40

b) 120 and 180

8. Use the fastest procedure you know to find the l.c.m. of:

   a) 20 and 70

   b) 18, 36, and 42

   c) 9, 15, and 60

9. TRUE AND FALSE

   a) _______ The prime factorization of 30 is 2 X 15.

   b) _______ The number 40 has eight exact divisors.

   c) _______ The l.c.m. of 15, 20 and 30 is 5.

   d) _______ The g.c.d. of 2 numbers is no smaller than the larger of the two numbers.

   e) _______ The g.c.d. of 18 and 36 is 9.

   f) _______ If two prime numbers are multiplied, the product will also be a prime number.

   g) _______ The following set contains all exact divisors of 36. {1, 36, 2, 3, 4, 6, 8, 9, 12}

10. MULTIPLE CHOICE - Circle correct answer.

    a) How many multiples of 4 are there between 25 and 50?

       1. 5
       2. 7
       3. 9
       4. 6

    b) The g.c.d. (greatest common divisor) and l.c.m. (least common multiple) of 18 and 42 are:

       1. 6 and 232
       2. 9 and 84
       3. 6 and 126
       4. 9 and 232
       5. 18 and 42

c) The l.c.m. of two numbers must be:

1. at least as large as their product.
2. no larger than the smaller of the two numbers.
3. no smaller than the larger of the two numbers.
4. larger than either of the two numbers.

d) Which of the following statements best describes a prime number?

1. a number which has no exact divisors
2. a number which does not have 2 as a factor
3. a whole number which has exactly 2 different divisors
4. every whole number which is not composite

e) The greatest common divisor (g.c.d.) of 18 and 24 is:

1. 3
2. 72
3. 6
4. 2
5. 144

f) This set {8, 9, 16, 20, 27, 72} contains all

1. odd numbers
2. even numbers
3. prime numbers
4. composite numbers

## NUMBER THEORY

### Illustration of Terms

**abundant number**, if the sum of the proper divisors "is greater than" the number.

1, 2, 3, 4, 6, 12 - are the divisors of 12
1, 2, 3, 4, 6 - are the proper divisors of 12
1 + 2 + 3 + 4 + 6 = 16
16 is greater than 12; so, 12 *is* an abundant number.

**addends**, the numbers involved in taking a sum.

5 + 12 + 7 + 3 = 27
5, 12, 7, 3 - are addends
3 + 4 = 7
3, 4 - are addends

**algorithm**, a way of computing using special order.

The division process is an algorithm. Euclid invented an algorithm for finding the highest common factor of two numbers. (See Euclid's algorithm.)

**array**, an arrangement of objects.

o o o o
o o o o
o o o o

A rectangular array

o
o o
o o o
o o o o

A triangular array

**{ } (braces)**, a device mathematicians use to enclose the elements of a set.

| | |
|---|---|
| A = {1, 2, 3} | Since 1, 2, and 3 are between the braces, they belong to the set A. |
| B = {pink elephants} | Since "pink elephants" are between the braces, the set B is made up of all "pink elephants." |

common divisors, numbers that divide two or more different numbers such that the remainder in each case is 0.

4 divides 12
4 divides 16
4 is a common divisor of 12 and 16.

3 divides 15
3 divides 27
3 is a common divisor of 15 and 27.

common multiple, a common multiple of two or more numbers is a multiple that each of the numbers will divide.

48 is a multiple of 16
48 is a multiple of 8
A common multiple of 16 and 8 is 48.

commutative principal, when a process can be performed in any order with the same result.

addition is commutative:
$5 + 3 = 3 + 5$
$4 + 8 = 8 + 4$
$a + b = b + a$

multiplication is commutative:
$2 X 3 = 3 X 2$
$7 X 8 = 8 X 7$

subtraction does not commute:
$2 - 5 = -3$
$5 - 2 = +3$

$6 - 4 = +2$
$4 - 6 = -2$

composite number, a number which has factors besides itself and 1; all counting numbers greater than one which are not prime.

computer, a machine which uses electronic circuits, storage units, and memory devices for the high speed performance of logical operations.

conjecture, a guess; a conclusion drawn from evidence which is not complete.

Often mathematicians have predicted certain relationships which are later proved true. Some of these conjectures are still not proved or disproved. Goldbach's conjecture, for example, remains unproven.

**consecutive**, following after in an ordered sequence.

1, 2, 3, 4, 5 are consecutive whole numbers

2, 4, 6, 8, 10 are consecutive even whole numbers

12 and 13 are consecutive numbers

**decimal**, a fraction whose denominator is a power of ten.

$$.13 = \frac{13}{100}$$

$$.125 = \frac{125}{1000}$$

**deficient number**, if the sum of the proper divisors "is less than" the number itself.

1, 2, 4, 8 are the divisors of 8
2, 4 are the proper divisors of 8
$2 + 4 = 6$
6 is less than 8, so 8 is a deficient number.

**difference**, the amount remaining after subtraction

$12 - 8 = 4$; 4 is the difference

**dividend**, the number being divided.

$$\begin{array}{r} 4 \\ 3\overline{)12} \end{array} \qquad \text{3 is the divisor}$$

$$\begin{array}{r} \text{quotient} \\ \text{divisor}\overline{)\text{dividend}} \end{array}$$

**. . . (three dots)**, continues in the same way.

The dots indicate some elements have been omitted, but we can tell what the elements omitted were by continuing in the same way.

{1, 2, 3, . . . 15} = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15}

{2, 4, 6, . . . 12} = {2, 4, 6, 8, 10, 12}

The dots are often used to save space; in other situations it is impossible to list the whole set, and the use of dots is a necessity.

{1, 2, 3, 4, . . .} This set never, never has a last element.

elements, the members of a set; the symbols, numbers, or letters which make up a set.

$A = \{1, 2, 3\}$

A is the set whose elements are 1, 2, and 3

Capital letters are used to identify sets while lower case (small letters) are used for elements.

Euclid's algorithm, a process for finding the highest common factor of two numbers.

$\underline{12}, \underline{40}$

$$\begin{array}{r} 3 \\ 12\overline{)40} \\ \underline{36} \\ 4 \end{array}$$

1. Divide smaller number into larger.

$$\begin{array}{r} 3 \\ 4\overline{)12} \\ \underline{12} \\ 0 \end{array}$$

2. Divide remainder into divisor until a remainder of 0 occurs.
3. Last divisor is the highest common factor.

$\underline{4}$ is the highest common factor of 12, 40.

even number, a number which has 2 as one of its factors; a number, which when divided by 2, leaves a remainder of 0.

$\{\text{even numbers}\} = \{2, 4, 6, 8, 10, 12, . . .\}$

expanded notation, writing a number as a sum whose summands (the numbers we are adding) are products of powers of ten and the digits of our number.

$12 = 10 + 2 = 1(10)^1 + 2$

$157 = 100 + 50 + 7 = 1(10)^2 + 5(10)^1 + 7$

$2503 = 2000 + 500 + 0 + 3 = 2(10)^3 + 5(10)^2 + 0(10)^1 + 3$

exponent, a shorthand device used by mathematicians to indicate the number of times a number is to be used as a factor.

$5^3 = 5 \text{ X } 5 \text{ X } 5$

exponent = 3 : so 5 is used as a factor 3 times

$5 \text{ X } 5 \text{ X } 5 = 125$

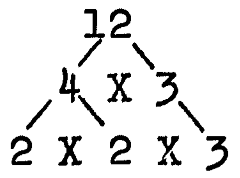
factorization, breaking a number into a product of factors.

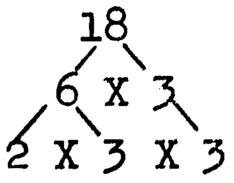
One factorization of 12 is: 4 X 3 (4 and 3 are factors)

Another factorization of 12 is: 6 X 2 (6 and 2 are factors)

Another factorization of 12 is: 2 X 2 X 3 (2 and 3 are factors. 2 is used as a factor twice.)

factor tree, a device used to show the prime factors of a number.





12 = 2 X 2 X 3

18 = 2 X 3 X 3

fractions, numbers expressible as a ratio of two integers (denominator not zero).

decimal fractions -- .572, .2, .33

common fractions -- $\frac{4}{7}$, $\frac{3}{8}$, $\frac{1}{4}$

Fundamental Theorem of Arithmetic, every composite number can be factored into a product of prime numbers: this factorization is different for every number. (see composite number)

12 = 4 X 3 = (2 X 2) X 3 = 2 X 2 X 3

If you multiply 2, 2, and 3 in any order, you always get the same result: 12.
No other number is the product of 2, 2, and 3.

(3 X 2 X 2 = 12, and 2 X 3 X 2 = 12)

Goldbach's Conjecture, the unproved guess that any even number greater than 2 can be represented as a sum of two primes.

4 = 2 + 2
6 = 3 + 3
8 = 4 + 4
10 = 5 + 5
12 = 6 + 6
14 = 7 + 7
16 = 11 + 5

greatest common divisor, the largest number which is a factor of all the numbers of a set. {10, 6}

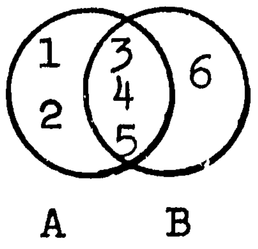
| factors of 10: | factors of 12: |
|---|---|
| 1 | 1 |
| 2 | 2 |
| 5 | 3 |
| 10 | 6 |

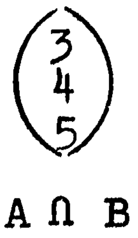
2 is the largest number which appears on both factor lists.

highest common factor, same as greatest common divisor intersection of two sets, the common members; the set of members which are in both sets.

A = {1, 2, 3, 4, 5}
↕ ↕ ↕
B = {3, 4, 5, 6}

A    B = {3, 4, 5}





least common multiple of two numbers, the smallest number which is a multiple of both numbers.

What is the least common multiple of 4 and 6?

Multiplication Tables

4 X 1 = 4
4 X 2 = 8
4 X 3 = 12
4 X 4 = 16.

4 X 5 = 20
4 X 6 = 24
4 X 7 = 28
4 X 8 = 32

6 X 1 = 6
6 X 2 = 12
6 X 3 = 18
6 X 4 = 24

12 is the smallest number which appears on both tables. Notice that 24 is a common multiple of 12 and 24, but it is not the smallest common multiple.

members of a set, the elements or objects which make up a set.

A = {1, 2, 3}

A is the set whose members are 1, 2, and 3.

multiples, a number is a multiple of another if it is the product of the given number and some other number.

20 = 4 X 5
20 is a multiple of 5
20 is a multiple of 4

12 = 3 X 4
12 is a multiple of 4
12 is a multiple of 3

negative numbers, a number on the number line to the left of zero (less than zero).

notation, a system of abbreviation, signs, or figures used for convenience.

**odd**, a number which is not divisible by 2; a number which when divided by two leaves a remainder of one; a number whose units digit is 1, 3, 5, 7, or 9.

**perfect number**, a number whose proper divisors add up to the number.

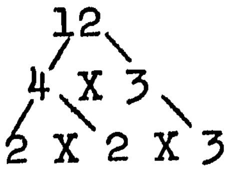
6 is a perfect number:
proper divisors 1, 2, 3
1 + 2 + 3 = 6

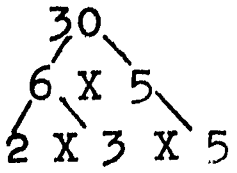
8 is not a perfect number:
proper divisors 1, 2, 4
1 + 2 + 4 = 7
$7 \neq 8$

**prime**, a number with exactly two different divisors. These divisors are 1 and the number itself.

2 is a prime--only divisors: 1, 2
5 is a prime--only divisors: 1, 5
7 is a prime--only divisors: 1, 7

**prime factorization**, to break a number into a product of prime factors.





2 and 3 are the prime factors of 12. 2 X 3 X 3 is the prime factorization of 12.

2, 3, and 5 are the prime factors of 30. 2 X 3 X 5 is the prime factorization of 30.

**product**, the number which results when two or more numbers are multiplied.

| | |
|---|---|
| 12 X 5 = 60 | 60 is the product |
| 8 X 3 = 24 | 24 is the product |

**proper divisors**, all divisors of a given number except the given number.

divisors of 12: 1, 2, 3, 4, 6, 12
proper divisors of 12: 1, 2, 3, 4, 6

**quotient**, the answer when one number divides another.

$$\begin{array}{r} 3 \\ 6\overline{)18} \\ \underline{18} \\ 0 \end{array}$$

3 is the quotient

rectangular array, an arrangement of objects so they are uniformly spaced and are shaped in a rectangle.

| | |
|---|---|
| o o o o o | Δ Δ Δ Δ Δ Δ |
| o o o o o | Δ Δ Δ Δ Δ Δ |
| o o o o o | Δ Δ Δ Δ Δ Δ |
| | Δ Δ Δ Δ Δ Δ |

remainder, the number which is left after a subtraction in the division process.

$$\begin{array}{r} 3 \\ 4\overline{)12} \\ \underline{12} \\ 0 \end{array}$$ 0 is the remainder.

$$\begin{array}{r} 4 \\ 4\overline{)17} \\ \underline{16} \\ 1 \end{array}$$ 1 is the remainder.

set, a collection of objects.

$A = \{1, 2, 3\}$

A is the set of elements 1, 2, and 3. A possible rule by which this set may have arisen is: the first three natural numbers.

subset, a set whose members are also members of another set.

$$\begin{array}{lcl} A & = & \{1,\ 2,\ 3,\ 4\} \\ & & \ \ \uparrow \qquad \uparrow \\ B & = & \{1, \qquad 3\} \end{array}$$

B is a subset of A.

sum, the answer to an addition problem.

$$\begin{array}{r} 4 \\ \underline{+3} \\ 7 \end{array}$$ 7 is called the sum.

theorem, some idea which has been shown true, or can be shown true, on the basis of certain other conclusions or facts.

twin primes, numbers which are prime and separated by only one whole number.

3 and 5 are prime
4 is the only number between 3 and 5
3 and 5 are twin primes

5 and 7 are primes
6 is the only number between 5 and 7
5 and 7 are twin primes

**union**, the elements which two or more sets are composed of; the elements present.

$A = \{1, 2, 3\}$

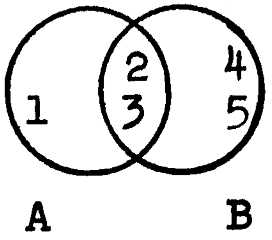
$B = \{2, 3, 4, 5\}$

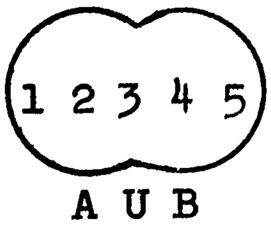
$A \cup B = \{1, 2, 3, 4, 5\}$

A is the set 1, 2, 3

B is the set 2, 3, 4, 5

A union B is the set of elements which are in A or in B.

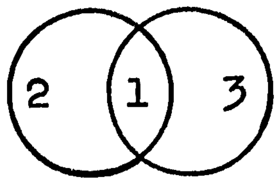




**Venn diagrams**, a diagram used to illustrate sets, their union and intersection.

$A = \{1, 2\}$

$B = \{1, 3\}$



$A \cup B = \{1, 2, 3\}$

$A \cap B = \{1\}$

The Venn diagram illustrates how the elements in each set are arranged in relation to the sets.

**whole numbers**, numbers by which we count things beginning with zero. We simply add 1 to get to the next whole number.