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THE MATH GO-ROUND, A UNIT OF MATHASBORG.

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THIS BOOKLET, ONE OF A SERIES, HAS BEEN DEVELOPED FOR THE PROJECT, A PROGRAM FOR MATHEMATICALLY UNDERDEVELOPED PUPILS. A PROJECT TEAM, INCLUDING TEACHERS, IS BEING USED TO WRITE AND DEVELOP THE MATERIALS FOR THIS PROGRAM. THE MATERIALS DEVELOPED IN THIS BOOKLET INCLUDE (1) NUMERALS AND GEOMETRICAL PATTERNS, (2) ACTIVITIES FOR DISCOVERING PATTERNS IN MULTIPLICATION AND DIVISION, (3) TESTS FOR DIVISIBILITY, AND (4) ACTIVITIES INVOLVING PRIME AND COMPOSITE NUMBERS. ACCOMPANYING THESE BOOKLETS WILL BE A "TEACHING STRATEGY BOOKLET" WHICH WILL INCLUDE A DESCRIPTION OF TEACHER TECHNIQUES, METHODS, SUGGESTED SEQUENCES, ACADEMIC GAMES, AND SUGGESTED VISUAL MATERIALS. (RP)

THE MATH GO-ROUND

A UNIT OF
MATHASBORG

ESEA TITLE III

PROJECT MATHEMATICS

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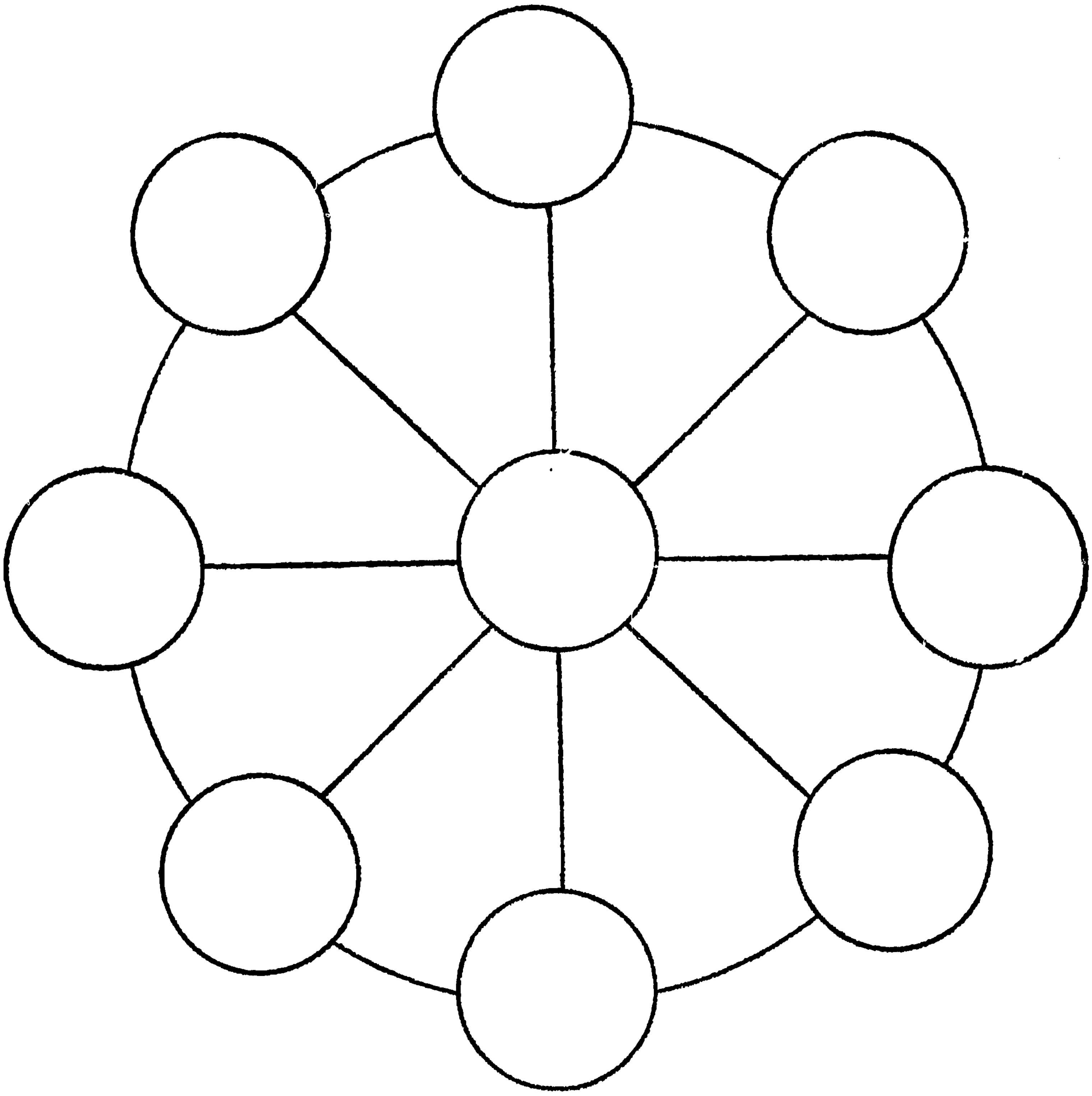
Math can be fun! How many times have you heard that statement and then wondered if the person saying it was really telling the truth?

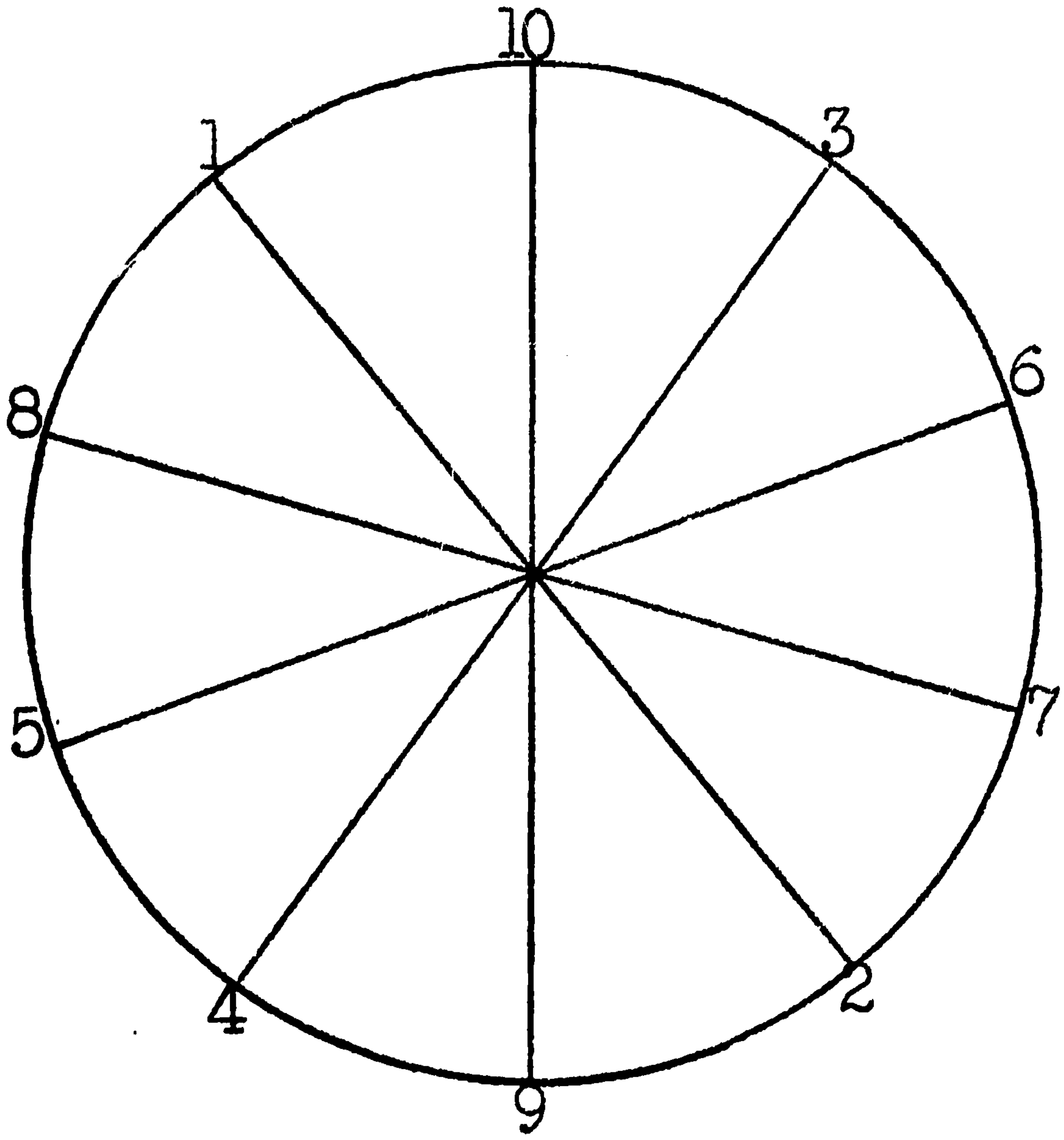
This unit can be fun! It is filled with a mixture of patterns and pieces, games and rules.
Go ahead! Enjoy it! Put a little math in your life and become another Euler!

Would you believe Gauss?

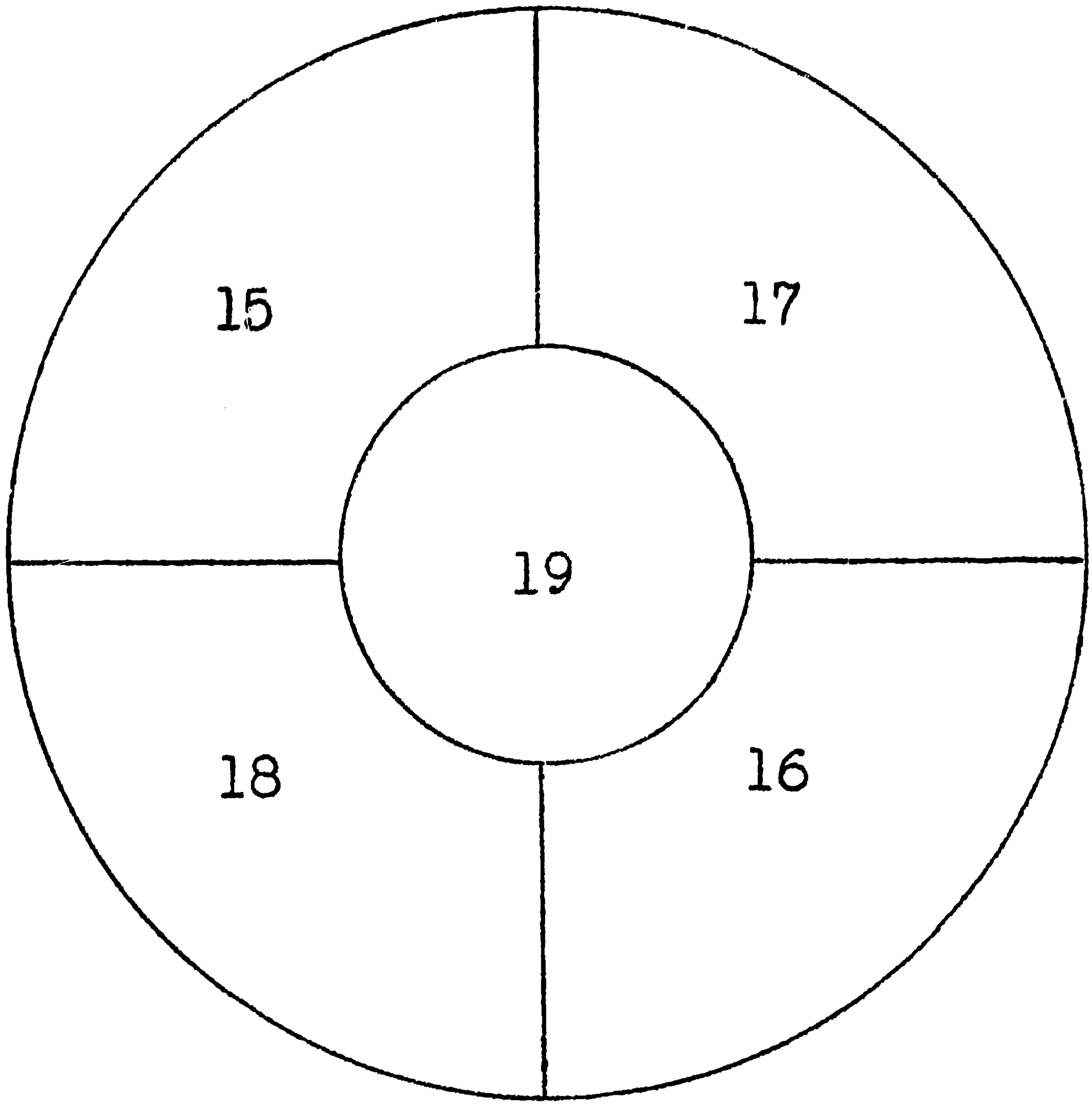
How about a happy student?



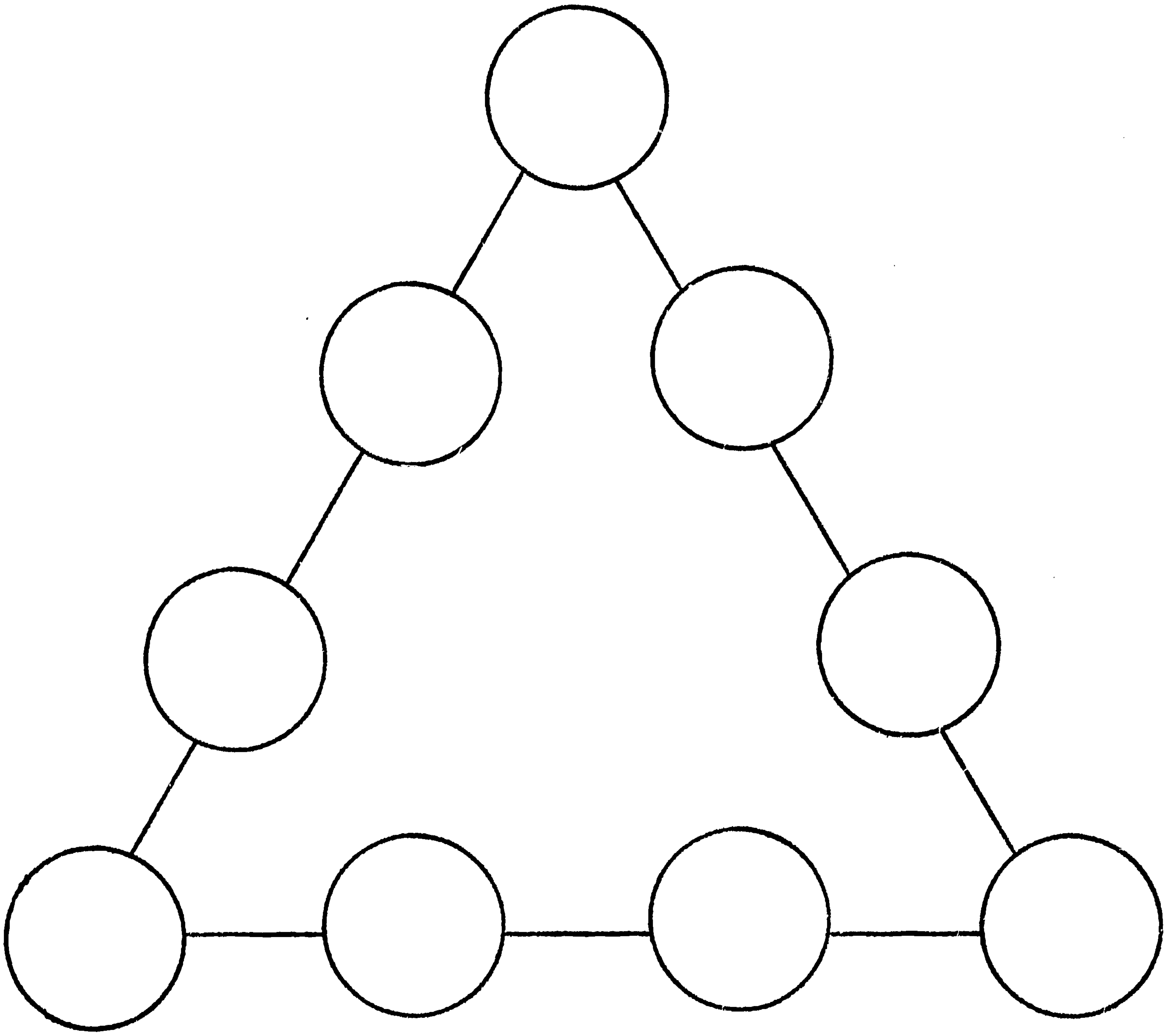
Can you arrange the nine digits (1, 2, 3, 4, 5, 6, 7, 8, 9,) in the circles above in such a way as to have the sum of any line equal to 15?



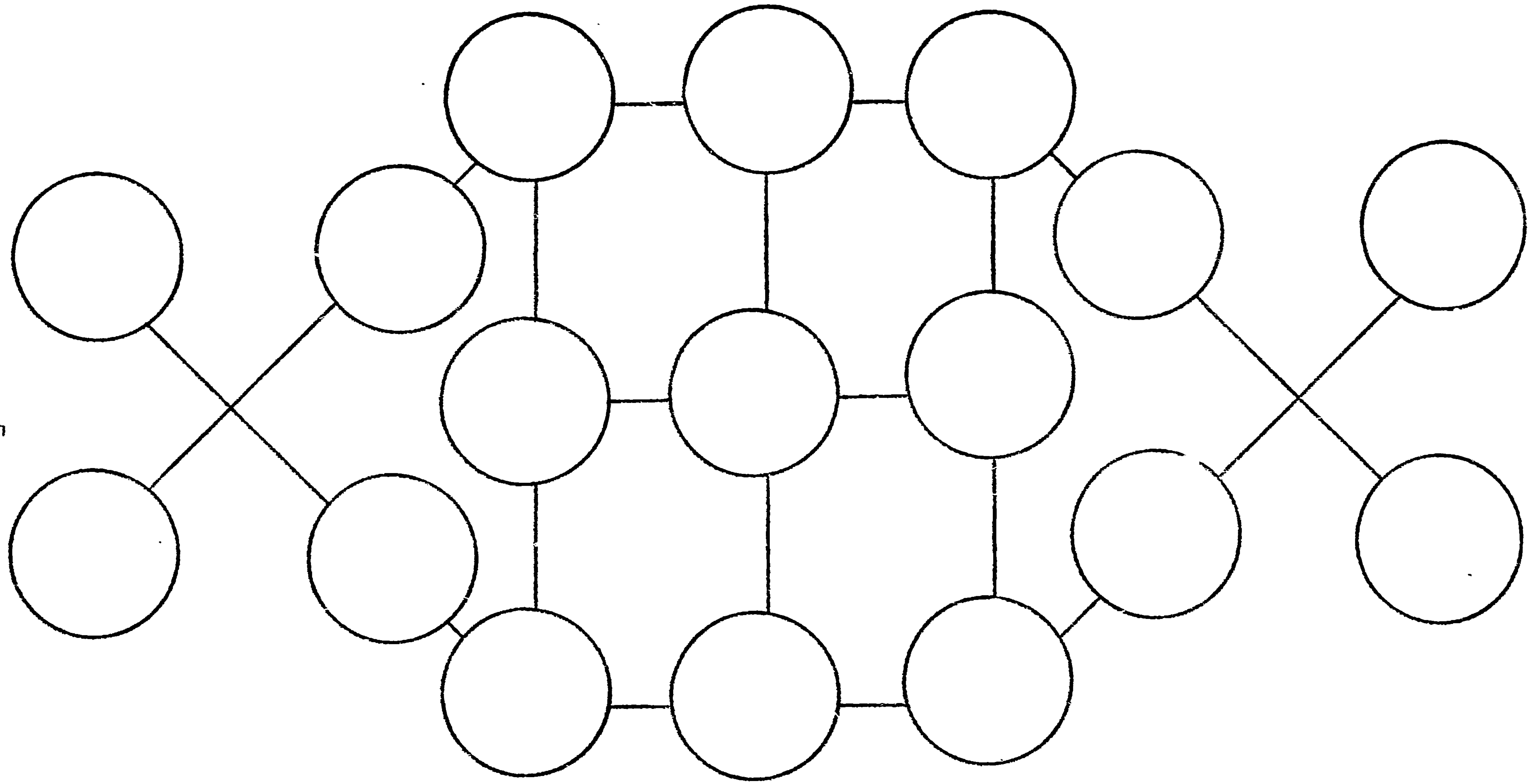
Can you discover what makes this wheel almost magic?
Look carefully!



Using the above target, I was able to score exactly 100 while practicing with my rifle. How many shots were necessary and where did they hit the target? Can you find more than one combination that will solve this problem? Will you always use the same number of bullets?



Can you write each of the nine digits (1, 2, 3, 4, 5, 6, 7, 8, 9,) in the circles of the triangle in such a way as to have a total of 20 on each side?



Can you place the numbers 1 through 17 into the above circles so they total the same magic number in all the directions the connecting lines show? This is a hard one!

PATTERNS

Can you complete these patterns? Look and think carefully?

5	7	0	8	6	1
10	14	0			

5	6	1	8	0	2	9
6	7	2				

2	4	7	9	0
6	12	21		

2	4	7	9	0	1
7	13	22			

14	22	45	67	23
13	21			

86	75	23	45	60
82	71			

7	12	8	9	13
5	7		7	
2	5	3		5

7	5	4	8	2
5	2		3	
35	10	24		18

3	6	5	8	9	2	1
0	0	0		0		
0	0	0	0		0	0

4	6	2	9	5	3
12	18				

Pascal, a French philosopher and mathematician of the 17th century, was very interested in roulette and other games of chance. This keen interest led him to discover some important facts about probability. In the EVENTS AND CHANCE unit the idea of tossing a penny was used to discover the chances of heads or tails appearing as the penny came to rest.

Pascal did this also and recorded his findings in the form of a triangular chart. (See chart diagramed on next page.)

Can you see where the 4 comes from in the tossing of two coins? What is 1 + 2 + 1 = ? How about the 8 in the three coins' toss? Check: 1 + 3 + 3 + 1 = ?

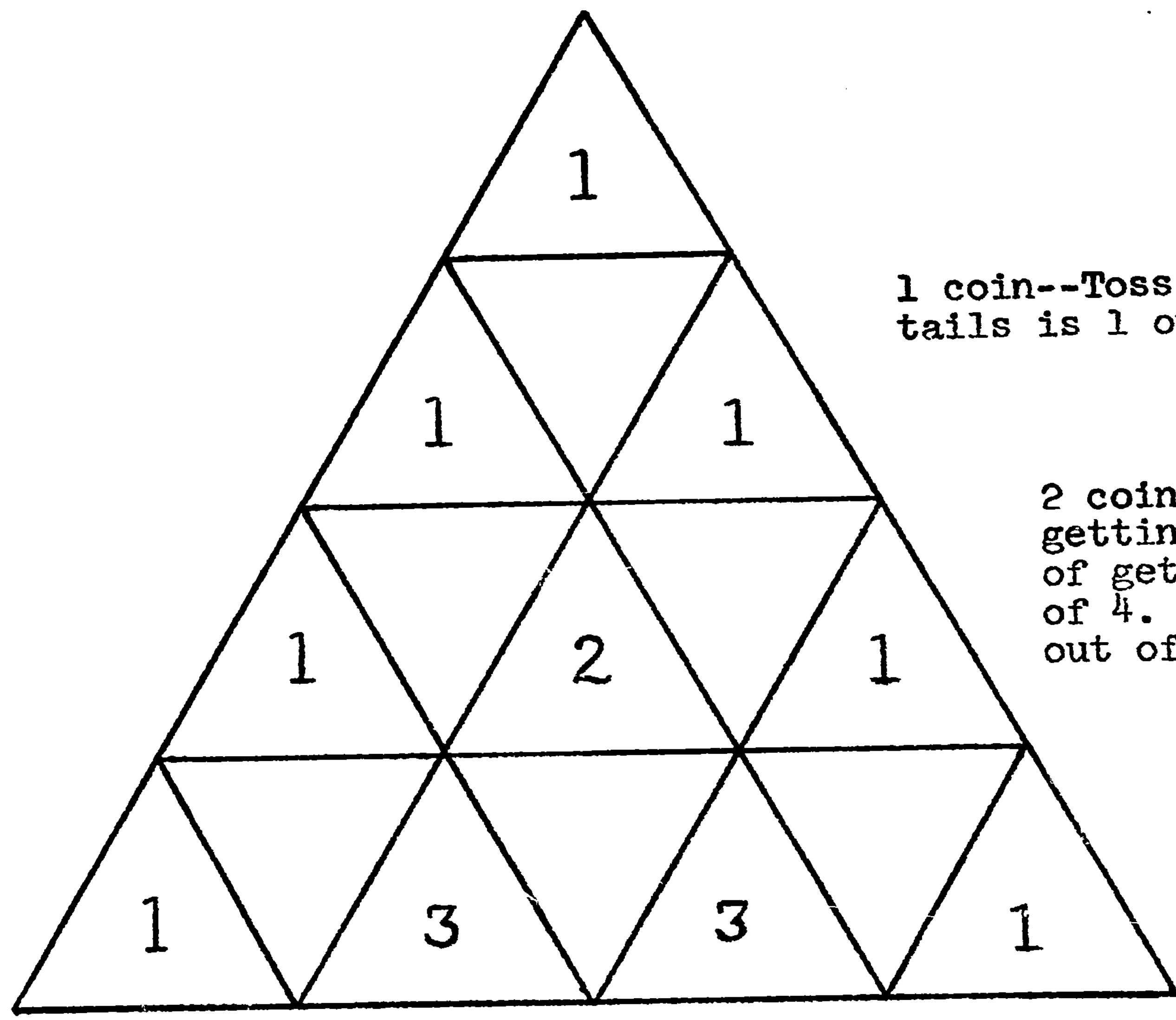
Can you now decide what the next row of Pascal's Triangle will tell us?

1 4 6 4 1

Did you notice that, starting at the top of the triangle, the outside left row and outside right row are completely filled with ones?

What can you see in the second row from left? (diagonal row) Do you see the same in the second row from the right?

What other patterns can you find in this unusual triangle?



1 coin--Toss one coin. The chance of getting tails is 1 out of 2.

2 coins--Toss two coins. The chance of getting 2 tails is 1 out of 4. Chances of getting 1 head and 1 tail are 2 out of 4. Chances of getting 2 heads is 1 out of 4.

Three coins--Toss three coins. Chance of getting 3 tails is 1 out of 8. Chances of getting 2 heads and 1 tail are 3 out of 8. Chances of getting 2 tails and 1 head are 3 out of 8. Chances of getting all heads is 1 out of 8.

Discovering Patterns

Have a person write a number on the board. It doesn't matter how many digits it has: but the larger the number, the greater the effect.

Tell the person to write another number having the same number of digits. Then tell the person that you would like to write a number in his column. Don't write just any number. Your number placed under his must make with his the sum of nine. (See the example below.)

Have the person write as many numbers as he wishes, but each time you must also write a number ("a nine sum") under his number. Then, when you wish, say you are ready to write the sum of all these numbers.

You can write the sum quickly. Count the number of times you wrote a number. This is the first digit in the sum. The remaining digits of the answer are the digits of the first number the student wrote less the number of times you wrote a number.

Example:

Other person	34678	
Other person	19632	9 sums
You	80367	
Other person	26580	9 sums
You	73419	
Other person	35428	9 sums
You	64571	
You wrote 3 times →	334675	8 - 3 = 5

Have a person write any number he wishes on board. (Four or more digits are more effective.) Instruct the person to write another number under the first, using all the digits of the first but in a different order. Then have the person subtract the two numbers. Have the class check his results. Then tell him to give you all but one of the digits of the answer, and you will give him the missing digit. (This is very effective if you hide your head, keep back turned to board, etc.)

Example:

Person	97645
Person	- 45679
	5[redacted]966

Person covers 1 and tells you remaining digits of answer

5966

You (mentally)

$$5 + 9 + 6 + 6 = 26$$

$$2 + 6 = 8$$

$$9 - 8 = 1$$

Missing digit is 1.

PATTERNS OF NINE

Look for a pattern and complete the exercies. Discuss each pattern.

1. 123456789 x 9 =

   123456789 x 18 =

   123456789 x 27 =

   123456789 x 36 =

   123456789 x 45 =

   123456789 x 54 =

   123456789 x 63 =

   123456789 x 72 =

   123456789 x 81 =

2. 999999 x 2 =

   999999 x 3 =

   999999 x 4 =

   999999 x 5 =

   999999 x 6 =

   999999 x 7 =

   999999 x 8 =

   999999 x 9 =

3. 9 x 9 =

   99 x 99 =

   999 x 999 =

   9999 x 9999 =

   99999 x 99999 =

   999999 x 999999 =

   9999999 x 9999999 =

   99999999 x 99999999 =

   999999999 x 999999999 =

4. 222222222 x 9 =

   333333333 x 9 =

   444444444 x 9 =

   555555555 x 9 =

   666666666 x 9 =

   777777777 x 9 =

   888888888 x 9 =

   999999999 x 9 =

5. 1 x 9 =

   2 x 9 =

   3 x 9 =

   4 x 9 =

   5 x 9 =

   6 x 9 =

   7 x 9 =

   8 x 9 =

   9 x 9 =

(Examine the products for discussing patterns.)

6. Write down a three digit number. Then sum the digits and subtract the sum from the original number. The result is always a multiple of (divisible by) 9.

```
 421 --> (4 + 2 + 1 = 7)
-  7
 414 --> (4 + 1 + 4 = 9)
```

If you ask a person to do this and he gives you two digits of the answer, could you always (without looking) tell him the missing digit?

7. Select a three digit number in which the digit in the ones' position is less than the digit in the hundreds' position. Now, reverse the ones' and hundreds' digits, and subtract from the original number.

$$\begin{array}{r} 421 \\ -124 \\ \hline 297 \end{array}$$

Will the tens' digit of the answer always be a nine? Will the hundreds' digit plus the ones' digit of the answer always add to nine?

If so, if a person will tell you only the ones' or hundreds' digit of the answer, could you tell him the missing digits?

8. Examine the pattern below and complete the missing answers.

$1 \div 9 = \frac{1}{9} = .11111...$

$2 \div 9 = \frac{2}{9} = .2222...$

$3 \div 9 = \frac{3}{9} =$

$6 \div 9 = \frac{6}{9} =$

$12 \div 99 = .121212...$

$16 \div 99 = .161616...$

$23 \div 99 =$

$88 \div 99 =$

$91 \div 99 =$

$142 \div 999 = .142142142...$

$387 \div 999 =$

$854 \div 999 =$

$1284 \div 9999 = .12841284...$

$6531 \div 9999 =$

$87641 \div 99999$

What pattern is being followed above? Make up some other problems that fit this pattern and check your answers.

9. An interesting pattern based on 9 is:

987654321	x 9	=	8888888889
	x 18	=	17777777778
	x 27	=	26666666667
	x 36	=	35555555556
	x 45	=	44444444445
	x 54	=	53333333334
	x 63	=	62222222223
	x 72	=	71111111112
	x 81	=	80000000001

METHODS OF INTEREST--MULTIPLICATION

1. Patterns in Multiplication

Form A	Form B
95 x 95 = 9025	99 x 91 = 9009
85 x 85 = 7225	92 x 98 = 9016
75 x 75 = 5625	87 x 83 = 7221
65 x 65 = ____	72 x 78 = 5616
55 x 55 = ____	68 x 62 = ____

a) Do patterns exist in the above illustration? Compare Form A to Form B, what similarities exist? Do you see how to multiply two digit numbers together of either form?

2. Complement Method

This procedure is the best for two numbers that are near 100. For example: 98 x 97, 99 x 94, etc.

1) First write the two numbers in vertical form.

2) To the right side of each number, indicate the number which, if added, would make 100.

```
    98 → 2 }
x   94 → 6 } x
  9212
```

3) Multiply the two numbers obtained in (2) for the last two digits of your product. (If a one digit product, a zero must be placed in the tens position--99 x 97.) Subtract the 6 from 98 for the other two digits.

Try the problems below using the complement method.

```
  99        95
x 97      x 95


  96        91
x 93      x 91
```

3. Doubling and Halving Method

Select one side to double and halve the other. For each time one side is double the other must be halved. Discard all fractional remainders on the side you halve. Stop when the side you are halving gets to (1). Select the even numbers on the side you halve and cross out the corresponding double. Then sum up the numbers left on the side you have doubled. This sum is also the product of the two original numbers.

Example:

(Double)		(Halve)	
~~27~~		~~26~~	even
54	———	13	
~~128~~		~~6~~	even
256	———	3	
512	———	1	stop

(27 x 26) = 822

Try these problems using the halving and doubling method.

33 x 34   13 x 15   15 x 84

4. Another interesting method of multiplying is called the lattice method. Check your library and see if you can find this method.

TESTS FOR DIVISIBILITY

Below is a summary of divisibility for certain numbers. Use these ideas for the next few exercises.

1. Every natural number is divisible by unity.

2. If the last digit of a number is divisible by 2, then the number is divisible by 2.

3. If the sum of the digits of a number is divisible by 3, then the number is divisible by 3.

4. If the last 2 digits of a number are divisible by 4, then the numbers are divisible by 4.

5. If the last digit of a number is 5 or 0, then the number is divisible by 5.

6. If the number is divisible by 2 or 3, then the number is divisible by 6.

7. NOTE: Certain tests exist for divisibility by 7, but it is usually easier and faster to perform the division.

8. If the last 3 digits of a number are divisible by 8, then the number is divisible by 8.

9. If the sum of the digits of a number is divisible by 9, then the number is divisible by 9.

10. If the last digit of a number is zero then the number is divisible by 10.

11. Add alternate digits of your number. Subtract the two numbers (obtained by adding alternating digits). If the difference is divisible by 11, the original number is divisible by 11.

Draw a circle (○) around the numbers that are divisible by two.

1. 465
2. 121
3. 543
4. 169
5. 145
6. 456
7. 691
8. 101
9. 16940
10. 2242
11. 6971
12. 48320
13. 7891
14. 593
15. 1516
16. 1967
17. 1769
18. 760
19. 1812
20. 1851
21. 1492
22. 1324
23. 3615
24. 1952
25. 1945

Draw a rectangle (▭) around the numbers that are divisible by three.

1. 999
2. 399
3. 464
4. 5100
5. 4350
6. 89
7. 1935
8. 1908
9. 666
10. 1515
11. 6969
12. 24
13. 2444
14. 9493
15. 3624
16. 464
17. 123
18 321
19. 412
20. 231
21. 111
22. 910
23. 4444
24. 984
25. 3939

Draw two circles (◎) around the numbers that are divisible by four.

1. 3623
2. 1111
3. 8
4. 1228
5. 0008
6. 144
7. 7172
8. 2222
9. 1221
10. 1237
11. 585
12. 732
13. 6648
14. 1694
15. 5824
16. 5761
17. 64
18. 1022
19. 8801
20. 3668
21. 4761
22. 007
23. 1452
24. 5824
25. 3360

Check (✓) the numbers that are divisible by five.

1. 254
2. 500
3. 6980
4. 4807
5. 950
6. 4005
7. 362636
8. 9055
9. 121212
10. 13
11. 225
12. 705
13. 954
14. 1894
15. 2141
16. 284
17. 420
18. 1211
19. 1213
20. 900
21. 500
22. 483200
23. 1250
24. 585
25. 8801

Draw a circle and a rectangle (⧇) around the numbers that are divisible by six.

1. 2436
2. 36
3. 4928
4. 42612
5. 5
6. 39
7. 36240
8. 72
9. 21
10. 7172
11. 2159
12. 1022
13. 585
14. 8640
15. 642
16. 69
17. 582
18. 100
19. 689
20. 12
21. 4350
22. 3826
23. 360
24. 1420
25. 101

Draw a double circle and a triangle (◬) around the numbers that are divisible by eight.

1. 166
2. 8997
3. 3320
4. 4645
5. 6169
6. 3196
7. 64
8. 72
9. 435
10. 40
11. 24
12. 2456
13. 808
14. 6424
15. 36
16. 1452
17. 83
18. 5214
19. 91
20. 26
21. 18
22. 4718
23. 108
24. 96
25. 104

Draw two rectangles () around the numbers that are divisible by nine.

1. 639
2. 900
3. 333
4. 29
5. 36
6. 108
7. 109
8. 105
9. 3456
10. 15363
11. 810
12. 843
13. 9991
14. 4824
15. 6126
16. 300
17. 345
18. 6381
19. 4554
20. 6756
21. 3663
22. 909
23. 7227
24. 1881
25. 3336

Draw an X beside the numbers that are divisible by eleven.

1. 2077
2. 14161
3. 3249
4. 3643
5. 4455
6. 121
7. 1320
8. 9999
9. 87641
10. 7664
11. 8930
12. 57463
13. 1234
14. 45556
15. 19836
16. 5847
17. 476
18. 132
19. 220
20. 39485
21. 4837
22. 473648
23. 4657
24. 190890
25. 78901

FACTORS

A number is a factor of another if it will divide evenly (leave a zero remainder). Or, another way to say this is that a number is a factor of another number if it will multiply by a whole number and give the other number as a product.

Write true or false for these statements:
(Use divisibility tests when you can.)

Example True 1. 6 is a factor of 42 since 6x7 = 42 or $6\overline{)42}$ = 7, 42, 0

_____2. 4 is a factor of 16

_____3. 5 is a factor of 75

_____4. 7 is a factor of 27

_____5. 2 is a factor of 103

_____6. 9 is a factor of 63

_____7. 8 is a factor of 64

_____8. 7 is a factor of 107

_____9. 3 is a factor of 39

_____10. 6 is a factor of 416

_____11. 4 is a factor of 364

_____12. 11 is a factor of 66, 55

_____13. 5 is a factor of 20, 34

_____14. 3 is a factor of 24, 336

_____15. 8 is a factor of 56, 148

_____16. 2 is a factor of 402, 100

_____17. 7 is a factor of 77, 245

_____18. 5 is a factor of 200, 345

_____19. 6 is a factor of 90, 216

_____20. 11 is a factor of 1999, 8881

LARGE PRIMES

For about 75 years prior to 1951, the largest known prime number was:

170,141,183,460,469,231,687,303,715,884,105,727

Two Englishmen, J.C.P. Miller and D.J. Wheeler, using the EDSAC at Cambridge, England, found more than ten primes greater than this number. In 1951 they announced this tremendous prime number:

$$180 \times (2^{127} - 1)^{2} + 1$$

Until 1960 this was the largest known prime number. Then a prime number was found which has 969 digits.

2 591 17086 013 20262 777 62467 679 22441 530 94181 888 75531 254 27303
974 92316 187 40192 665 86362 086 20120 951 68004 834 06550 695 24173 319 41774
416 89509 238 80701 741 03777 095 97512 042 31306 662 40829 163 53517 952 31118
615 48622 656 04547 691 12759 584 87756 105 68757 931 19101 771 14088 262 52153
849 03583 040 11850 721 16424 747 46182 303 14713 983 40229 288 07454 567 79079
410 37288 235 82070 589 23510 684 33882 986 88861 665 86502 809 27692 080 33960
586 93087 905 00409 503 70987 590 21190 183 71991 620 99400 256 89351 131 36548
829 73911 265 67973 032 41986 517 25011 641 27035 097 05427 773 47797 234 98216
764 43446 668 38311 932 25400 996 48994 051 79024 162 40565 190 54483 690 80961
606 16257 430 42361 721 86333 941 58524 264 31208 737 26659 196 20617 535 35748
892 89459 962 91951 830 82621 860 85340 093 79328 394 20261 866 58614 250 32514
507 73096 274 23537 682 29386 494 07127 700 84607 712 42118 230 80804 139 29808
705 75047 138 25264 571 44837 937 11250 320 81826 126 56664 908 42516 994 53951
887 78961 365 02484 057 39378 594 59944 433 52311 882 80123 660 40626 246 86092
121 50349 937 58478 229 22371 443 39628 858 48593 821 57388 212 32393 687 04616
067 73629 093 15071

Computer Discovers New Prime Number

A computer has discovered a new prime number. It is the biggest prime number ever proved -- 2,917 digits long. Proof that the number could be divided evenly by only itself took the Illiac II high-speed electrical computer a mere 85 minutes. In this time it did three-quarter billion multiplications and additions. By hand, the calculations would take 80,600 man years.

Prof. Donald B. Gillies of the University of Illinois Computer Laboratory, Urbana, Ill., developed the program for the computer. (SCIENCE NEWS LETTER, 83:291 May 11, 1963)

PRIME AND COMPOSITE NUMBERS

Remember that a number is prime if it has exactly two different divisors (factors). Composite numbers are numbers (whole) greater than one that are not prime.

Tell whether each of these numerals are prime or composite. (List all the divisors of each.)

1. 122
2. 15,220
3. 121
4. 217
5. 113
6. 143
7. 871
8. 314
9. 101
10. 5035
11. 1919
12. 931
13. 289
14. 6851
15. 382
16. 24
17. 58
18. 69
19. 483
20. 728
21. 695
22. 584
23. 47
24. 384
25. 596

PATTERNS OF SQUARES

Examine the pattern, then supply the missing numbers.

1. $(2 \times 4) + 1 = 9 = 3^2$
2. $(3 \times 5) + 1 = 16 = 4^2$
3. $(4 \times 6) + 1 = 25 = 5^2$
4. $(\square \times 7) + \square = \square = 6^2$
5. $(\square \times 8) + 1 = \square = \square^2$
6. $(12 \times \square) + 1 = \square = 13^2$ (careful)
7. $(10 \times \square) + 1 = \square = 11^2$

This unusual triangle of numbers is called Pascal's triangle. Study the pattern and supply the numbers in three missing rows. (The triangle can continue on indefinitely.)

Row

0 — 1

1 — 1 1

2 — 1 2 1

3 — 1 3 3 1

4 — 1 4 6 4 1

5 — □ □ □ □ □ □

6 — □ □ □ □ □ □ □

7 — □ □ □ □ □ □ □ □

A NETWORK--OR TWO

Can you begin at a given point, trace the network, return to your starting point, and not go over the same line two times?