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THE EFFECTS OF TWO VARIABLES ON THE PROBLEM-SOLVING ABILITIES OF FIRST-GRADE CHILDREN.

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NINETY FIRST GRADE CHILDREN WERE RANDOMLY SELECTED FROM 3 SCHOOLS (WHICH EACH USED A DIFFERENT ARITHMETIC PROGRAM) TO PARTICIPATE IN A STUDY TO INVESTIGATE THE EFFECTS OF 2 VARIABLES ON THE CHILDREN'S ABILITY TO SOLVE ADDITION PROBLEMS. THE VARIABLES WERE (1) THE PRESENCE OR ABSENCE OF AN EXISTENTIAL QUANTIFIER PRECEDING THE START OF THE PROBLEM, AND (2) EITHER THE PRESENCE OF 3 DIFFERENT NAMES FOR THE 3 SETS IN ANY PROBLEM OR THE PRESENCE OF COMMON NAMES FOR THE 3 SETS. THE CHILDREN WERE INDIVIDUALLY TESTED ON 20 PROBLEMS WHICH WERE READ TO THEM BY 1 EXPERIMENTER. FORTY-FIVE CHILDREN RECEIVED PROBLEMS WHICH INVOLVED AN EXISTENTIAL QUANTIFIER, AND 45 HAD PROBLEMS WHICH DID NOT. TEN OF THE PROBLEMS GIVEN TO EACH CHILD HAD DIFFERENT NAMES WITHIN THE PROBLEM SETS, AND 10 PROBLEMS HAD THE SAME SET NAMES. ANALYSIS OF VARIANCE OF THE DATA SHOWED THAT THE ONLY SIGNIFICANT DIFFERENCE OCCURRED BETWEEN THE PROBLEMS INVOLVING SET NAMES. ALTHOUGH THE STUDY SCOPE IS LIMITED, RESULTS SUGGEST THAT CHILDREN SHOULD BE GIVEN MORE CHANCES TO INTERPRET PROBLEMS PRESENTED VERBALLY AND THAT PICTORIAL REPRESENTATIONS OF SETS IN EXERCISE BOOKS SHOULD BE DESCRIBED BY DIFFERENT WORDS RATHER THAN BY 1 COMMON TERM. (MS)

THE EFFECTS OF TWO VARIABLES ON THE PROBLEM-SOLVING ABILITIES OF FIRST-GRADE CHILDREN

WISCONSIN RESEARCH AND DEVELOPMENT
CENTER FOR
COGNITIVE LEARNING

RESEARCH
COGNITIVE LEARNING
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Technical Report No. 21

THE EFFECTS OF TWO VARIABLES ON THE PROBLEM-SOLVING ABILITIES OF FIRST-GRADE CHILDREN

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PREFACE

The goal of the Center is to contribute to an understanding of, and the improvement of educational practices related to, cognitive learning by children and youth. Of primary concern are the learning of concepts, such as those which comprise the main body of organized knowledge in science or in mathematics, and the nurturing of related cognitive skills, such as those which are involved in problem solving, creative production, or in reading. Conditions within the learner and conditions within the learning situation are also relevant areas of research and development.

In the research reported herein, Dr. Steffe introduced two variations in problems presented to first-grade children who were using three different arithmetic programs. For the problem-solving skills investigated, the programs were of equal effectiveness. One of the problem variations, different names for the sets to be combined and for the total set, resulted in significantly different scores. The results illustrate that first-grade children have difficulty in solving addition problems presented to them verbally, with the difficulty more acute when the problems involve different names for the sets to be combined and for the total set than when all three sets have a common name. Dr. Steffe recommends that curriculum developers include additive situations in the curriculum in which the two sets to be combined may be described by different words and in which the total set may be described by still a different word. He also recommends that more work be given on interpretation, by the children, of situations presented to them verbally or symbolically.

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Co-Director for Research

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ABSTRACT

Ninety first-grade children were randomly selected from three school buildings, each of which used a different arithmetic program. These children were individually tested on twenty addition problems which were read to them by one experimenter. Forty-five of the children received problems which involved an existential quantifier and forty-five received problems which involved no quantifier. Ten of the twenty problems each child received were problems in which the names that described the three sets involved were all different; while in the remaining ten problems, these names were the same.

Analysis of variance showed no significant difference between the problems with an existential quantifier vs. those with no existential quantifier and no significant difference among the pupils from the three schools. A significant difference did occur, however, between the problems which involved common names vs. those which involved different names. A discussion of these results along with curricular and research implications is given.

I

INTRODUCTION

STATEMENT OF THE PROBLEM

An arithmetic problem has an additive structure if it is an instance of the union of two or more sets.[1] This definition partitions the set of arithmetic problems into two subsets: (1) those problems that possess an additive structure and (2) those problems that do not possess an additive structure. Problems that fall into category 1 may differ from each other in many different ways, one of which is given by the following examples.

(a) There are two robins and four sparrows in a tree. How many birds are in the tree?

(b) John has three pennies and Jack has four pennies. How many pennies do both boys have?

These two problems are strictly analogous to each other in that each possesses an additive structure. An obvious difference, however, is the way in which the three sets in each problem are described. In problem (a), the sets are described in part by the words "robins," "sparrows," and "birds." In problem (b), each set is described in part by the word "pennies."

A child, when confronted with the two problems, should go through quite similar procedures in order to arrive at a correct solution. Is it true, however, that having different names present makes the first problem more difficult than the second problem? There are preliminary indications that this may be true. Steffe,[2] in a study involving 132 first-grade children, administered, among others, the following three problems:

(1) John has three pennies in one hand and four pennies in his other hand. How many pennies does he have in his hands?

(2) There are some kittens in the kitchen. Two kittens are drinking milk and five kittens are sleeping. How many kittens are in the kitchen?

(3) In a zoo, there are three bears in one cage and five bears in another. How many bears are there in the cages?

Of the 132 children, 97 scored (1) correct, 60 scored (2) correct, and 93 scored (3) correct. The problems were read to the children by one experimenter in a random order. No visual aids were present during solution. As an explanation of the relative difficulty of item (2), one may consider that the two sets of kittens are <u>doing</u> different things and thereby it may be true that the children look upon the two sets of kittens as being labeled differently. Moreover, the containing set may not be construed by the children as having the same name as the two other sets. In problems (1) and (3), however, no such possibility apparently exists. There is one other dimension in which problem (2) differs from (1) and (3) in that an existential quantifier precedes the statement of the problem in (2) whereas the problems in (1) and (3) are stated immediately. It certainly may be true that the presence of the quantifier inhibits the solution of the problem because it focuses attention first on the total set (the kittens in the kitchen). The children then must refocus on the two subsets (the kittens drinking milk and the kittens sleeping) and then focus their attention again on the total set in question.

Piaget,[3] while studying set inclusion, that is, $A \subset C$, $B \subset C$ while $A \cup B = C$, asserts that many children have difficulty simultaneously thinking of the total set (set C) and the parts (the subsets A and B) in that they forget the whole set when thinking of the two subsets and vice versa, even though they understand the definition of the sets. Elkind,[4] in a study which replicated that of Piaget, found that 50 per cent of five-year-olds and 32 per cent of six-year-olds had trouble when responding to the question, "Are there more boys (or girls depending on the sex of the child) or more children in your class?" It must be noted this question involved three sets (1) boys, (2) girls, and (3) children, each with a different name.

The above results of Piaget, Elkind, or Steffe lead one to conjecture that: (1) arithmetic addition problems which possess different names for the sets involved, such as in (a) above, are more difficult for children to solve than problems which possess common names for the sets involved, as in (b) above; and (2) arithmetic problems in which an existential precedes the statement of the problem are more difficult for children to solve than problems in which an existential quantifier does not precede the statement of the problem.

LEVELS OF PROBLEM SOLVING

Various levels of abstraction may be identified in problem solving at the primary school level. For example, children may look at a visual aid, either physical or pictorial, and be trained to make a certain response to the visual by writing a number sentence to tell the "story" of the visual. Verbalization may or may not be presented by the teacher and/or child. While it is not the purpose of this study to fully discuss levels of abstraction and draw conclusions about the abstractness of a problem situation, it is construed by the investigator that problems which are read to children and which do not have a visual aid present are more difficult for them to solve than those problems which are read and which do have a visual aid present. This opinion is not without empirical support, as it has been found that problems which are read to children without visuals present are more difficult to solve than problems which are read to children with visuals present.[5]

With the above empirical support, then, one may provisionally scale problems according to a level of abstractness, from a lesser abstraction to a greater abstraction:

(a) The case where the problem is stated verbally to the child in the presence of a visual, and he makes a verbal response. Here, various categories may be defined among which are the following two:

(1) The visuals are parallel to the sets named in the problem.

(2) The visuals are in no way related to the sets named in the problem.

(b) The case where the problem is stated verbally to the child with no visual present, and he makes a verbal response.

There are no doubt levels that come before (a) or after (b), such as the case where children walk through a situation. Levels may also exist between those stated, such as in the case where children interpret a problem verbally stated with visuals provided them. That is, they act out the problem but do not necessarily verbalize it.

Problems that fall in category (b) were selected for the study for two reasons. The first is that data were available for comparison purposes, and the second is that the study was construed to be an exploratory study, so that no attempt at total generalizability would be made, that is, generalizability to all levels of abstraction in problem solving. For the same reason, subtraction problems were excluded.

PROBLEMS WITH A TRANSFORMATION

Arithmetic problems may be viewed as either involving a transformation or not involving a transformation.[6] At the various levels of abstraction given above, the transformation may take on different forms. For example, in (a), the transformation may be implied by the picture. That is, some motion of one of two sets is indicated. In the case of (b), the transformation may be only verbally described. For this study, problems that do not involve a transformation were used for the same two reasons given in the last section.

A DESCRIPTION OF THREE ARITHMETIC SERIES

The results of an exploratory study should be such that they can be considered valid regardless of the particular arithmetic series in which the children in the study participated. This type of generalizability usually is inhibited by practical considerations. However, an attempt was made to use children from a school system which used more than one arithmetic series. Such a school system, namely, Monona Grove, Wisconsin, was found in which three different arithmetic series are being used. They are (a) Greater Cleveland Mathematics Program,[7] (b) Patterns in Arithmetic,[8] and (c) Numbers We Need.[9]

A description of each program follows:

(a) Greater Cleveland Mathematics Program: The concepts of (1) one-to-one correspondence, (2) numbers from zero through ten, (3) addition including combinations which sum to four, (4) subtraction with a minuend of four, (5) order of the numbers zero through ten, and (6) ordinal numbers through "tenth" had been covered by all the children in the sample from this school. In almost all addition situations which involved pictures, a common name could be associated with each of the three sets the children had to work with. The school that used this series was designated as School 1.

(b) Patterns in Arithmetic: This is a televised arithmetic series which is at the present time in a field-testing stage of development. Twelve programs had been covered at the time the testing was done for this study. The concepts of (1) one-to-one correspondence, (2) transitivity of "more than," "fewer than," "as many as," (3) the numbers from one to ten and their order, (4) conservation of numerousness, and (5) addition combinations through five had been presented. In their exercise books, the children had to symbolize some situations in which the two original sets could be described by different names. For example, in one situation, five bees were flying to join two monkeys. The total set had no apparent name and was defined by the objects in the picture, as was the case in many of the exercises. The school which used this program will be designated as School 2.

(c) Numbers We Need: The concepts of (1) the numbers from one to one hundred, (2) counting by two's and ten's, (3) addition combinations with sums to five (some to seven), (4) telling time, (5) subtraction, minuend of six (some seven), (6) fractions (one-half), and (7) coins (cent, nickel, dime) had all been covered by each child in the sample from this school. In the addition situations which involved pictures, almost all of the sets involved could be described by a single name. In some cases, such things as "2 dogs + 1 dog = 3 dogs"[10] were encountered. The school that used this series was designated as School 3.

II
METHOD

SUBJECTS

Ninety first-grade children were randomly selected from 245 first-grade children at Monona Grove, Wisconsin. The 245 children were in three school buildings, with about 80 first graders in each building. The children in any one building were using the same arithmetic program, but children in any two buildings were using different programs. The order in which the children were randomly selected was recorded and the first thirty in the sample that were in any school building were considered as subjects. Nine alternates were selected from each building in a similar fashion. Only one alternate was used. The children were tested the weeks of December 5 and December 12, 1966. All the testing was done on Monday, Tuesday, and Wednesday of each week. The IQ scores were obtained from a previously administered Pintner IQ test. The mean IQ's of the children in the sample for each school were as follows. (The IQ's of only 20 of the 30 children in School 3 were available.)

Table 1

Mean IQ's By School

School	1	2	3
IQ	101.6	108.2	104.4

PROCEDURE

Each child was tested individually by a single experimenter who read each problem to each child. The experimenter was a substitute teacher for the Monona Grove School System as well as a certified elementary school teacher. In each school building, fifteen children were randomly assigned to the problems that involved a quantifier and fifteen children were randomly assigned to the problems that involved no quantifier. Each child was given twenty addition problems to solve, ten of which involved common names for the three sets described in the problem and ten of which involved three different names for the three sets involved. A different random order was assigned to the twenty problems for each child. No time limit was imposed. The time taken to do the twenty problems was about fifteen minutes per child. In order to make the problems challenging enough for all the children, addition combinations which summed to 5, 6, and 7—excluding any combination with one as an addend—were randomly assigned to the problems. It has been noted that the children using Numbers We Need had had experience with combinations through at least those which sum to six, while the children in the two other series had combinations which summed through at most five. If a training factor exists, it then would be a school effect or an interaction between schools and the factor of a common name vs. no common name.

MATERIALS

A total of forty problems was given to the children. These problems, which follow, are in four categories of ten problems each.

No Quantifier: Different Names

1. John has 5 jacks in one pocket and 2 marbles in another pocket. How many toys does he have in his pockets?
2. Mary has 4 kittens and 2 goldfish. How many pets does Mary have?
3. There are 2 monkeys in one cage and 3 bears in another cage. How many animals are in the cages?
4. Peter has 3 pennies and 3 nickels. How many coins does Peter have?

5. There are 3 robins in one tree and 2 pigeons in another tree. How many birds are in the trees?
6. There are 2 watermelons and 4 pumpkins on a table. How many vegetables are on the table?
7. In Bob's toybox, there are 4 toy pistols and 2 toy rifles. How many toy guns are in the toybox?
8. There are 2 elephants and 5 monkeys in a circus ring. How many animals are in the circus ring?
9. There are 3 cups and 2 plates on a table. How many dishes are on the table?
10. There are 4 boys and 2 girls swinging. How many children are swinging?

No Quantifier: Common Name

11. There are 5 cars in one parking lot and 2 cars in another parking lot. How many cars are in these parking lots?
12. There are 4 cookies on one plate and 2 cookies on another plate. How many cookies are on the plates?
13. There are 2 blocks in one pile and 3 blocks in another pile. How many blocks are there in the piles?
14. There are 3 houses on one side of a stream and 3 houses on the other side of the stream. How many houses are by the stream?
15. There are 3 balls in a pile and 2 balls in another pile. How many balls are there in the piles?
16. There are 2 candles on a table and 4 candles on another table. How many candles are there on the tables?
17. John has 4 pennies in one hand and 2 pennies in his other hand. How many pennies does he have in his hands?
18. In a zoo, there are 2 bears in one cage and 5 bears in another cage. How many bears are there in the cages?
19. There are 3 ducks swimming on a pond and 3 ducks swimming on another pond. How many ducks are swimming on both ponds?
20. There are 4 fish in a fishbowl and 2 fish in another fishbowl. How many fish are in both fishbowls?

Quantifier: Different Names

1. John has some toys in his pockets. He has 5 jacks in one pocket and 2 marbles in another pocket. How many toys does he have in his pockets?
2. Mary has some pets. She has 4 kittens and 2 goldfish. How many pets does Mary have?
3. There are some animals in two cages. There are 2 monkeys in one cage and 3 bears in the other cage. How many animals are in the cages?
4. Peter has some coins. He has 3 pennies and 3 nickels. How many coins does Peter have?
5. There are some birds in two trees. There are 3 robins in one tree and 2 pigeons in another tree. How many birds are in the trees?
6. There are some vegetables on the table. There are 2 watermelons and 4 pumpkins. How many vegetables are on the table?
7. Bob has some toy guns in his toybox. He has 4 toy pistols and 2 toy rifles. How many toy guns are in the toybox?
8. There are some animals in a circus ring. There are 2 elephants and 5 monkeys in the circus ring. How many animals are in the circus ring?
9. There are some dishes on a table. There are 3 cups and 2 plates on the table. How many dishes are there on the table?
10. Some children are swinging. There are 4 boys and 2 girls swinging. How many children are there swinging?

Quantifier: Common Name

11. There are some cars in two parking lots. There are 5 cars in one parking lot and 2 in the other parking lot. How many cars are in these parking lots?
12. There are some cookies on two plates. Thereare 4 cookies on one plate and 2 on the other plate. How many cookies are on the plates?
13. There are some blocks in two piles. There are 2 in one pile and 3 in the other pile. How many blocks are there in the piles?
14. There are some houses by a stream. There are 3 houses on one side and 3 on the other side. How many houses are by the stream?
15. There are some balls in two piles. There are 3 balls in one pile and 2 in the other. How many balls are there in the piles?
16. There are some candles on two tables. There are 2 candles on one table and 4 on the other. How many candles are there on the tables?

17. John has some pennies in his hands. He has 4 pennies in one hand and 2 pennies in his other hand. How many pennies does he have in his hands?
18. There are some bears in two cages in a zoo. There are 2 bears in one cage and 3 in the other. How many bears are there in the cages?
19. There are some ducks swimming in two ponds. There are 3 ducks swimming in one pond and 2 ducks swimming in the other. How many ducks are swimming in both ponds?
20. There are some fish in two fishbowls. There are 4 fish in one bowl and 2 fish in the other bowl. How many fish are in both fishbowls?

Each problem under the general category of "Quantifier" has an exact parallel under the general category of "No Quantifier," so that the two tests of twenty problems each were exact replicas with the exception of the presence of the quantifier in one test.

DESIGN

The basic design used in this study is a 3 x 2 x 2 factorial design with repeated measures on the last factor.[11] Schools were used as a blocking variable, of which three levels exist. Hereafter, it will be referred to as Factor S. The second factor is not repeated and is the quantifier vs. no quantifier factor, hereafter referred to as Factor Q. The repeated measures factor is the factor of common names vs. different names, hereafter referred to as Factor N. A diagram of the design follows.

Table 2

Diagram of Design

S	Q	Ind	Common Name	Diff. Names
1	Quantifier	1 2 . . . 15	x 1111 x 2111 . . . x 15 111	x 1112 x 2112 . . . x 15 112
	No Quantifier	1 2 . . . 15	x 1121 x 2121 . . . x 15 121	x 1122 x 2122 . . . x 15 122
. . .				
3	Quantifier	1 2 . . . 15	x 1311 x 2311 . . . x 15 311	x 1312 x 2312 . . . x 15 312
	No Quantifier	1 2 . . . 15	x 1321 x 2321 . . . x 15 321	x 1322 x 2322 . . . x 15 322

In the diagram X_{ijkl} represents the i^{th} individual, the j^{th} school, the k^{th} level of Factor Q, and the l^{th} level of Factor N. Substantively, it represents a score of the i^{th} individual on any one of the four tests and therefore can be any number from and including zero to and including ten. With this design, it is possible to detect any possible differences in the means of:

1. The three levels of Factor S. These means are calculated over the 60 observations in each school. In effect, they represent the means of the scores of the 30 children in each school on the total problem-solving test.

2. The two levels of Factor Q. These means are calculated over the 90 observations across schools. In effect, they represent the means of the scores of the 45 children who took tests at each level.

3. The two levels of Factor N. These means are calculated over the 90 observations across schools. In effect, they represent the means of the scores of the 90 children who took tests at each level.

It is also possible to test for the following interactions:

1. S X Q
2. S X N
3. Q X N
4. S X Q X N

The first three interactions are of considerable interest for the study, and will be discussed fully in the results section.

In addition to the basic design above, internal-consistency reliability[12] coefficients were calculated for each test on which an <u>F</u> ratio is reported. This was done by means of an available computer program.[13]

III
RESULTS AND DISCUSSION

THE RELIABILITY STUDIES

The internal-consistency reliability of the total test and each subtest is reported below in Table 3.

Table 3

Internal Consistency Reliabilities

Test		Reliability
Total	(20 Items)	.93
Common Name	(10 Items)	.87
Different Name	(10 Items)	.87
Quantifier	(20 Items)	.93
No Quantifier	(20 Items)	.94

The reliability coefficient for the Total Test was computed on 90 children, so that any item involved a quantifier for 45 of the children and no quantifier for 45 of the children. The reliability coefficients for the subtests, "Common Name" or "Different Names," also were computed on items which involved a quantifier for 45 of the children and no quantifier for the remaining 45 children. The reliability coefficients for the tests "Quantifier" or "No Quantifier" were computed on items taken by 45 children, where ten of the items involved a common name and ten of the items involved different names.

The distribution of the total score for each test is given in Table 4. Table 5 gives the difficulty levels of each item for each test.

An investigation of these two tables reveals that no practical difference exists in the means or standard deviations of the problems with a quantifier and the problems with no quantifier. An inspection of the difficulty levels of the items as given in Table 5 shows that the difficulty levels oscillate between the quantifier and no quantifier. That is, in some cases, an item with a quantifier is easier than that same item with no quantifier, while in other cases, the reverse is true. This suggests no consistent superiority of one level of Factor Q over another. The means of the two levels of Factor N are different as are the distributions which are both given in Table 4. The statistical tests for these means will be given in the next section.

THE STATISTICAL ANALYSIS

The analysis used in this study is an 3 x 2 x 2 ANOVA with repeated measures on the last factor. The first factor, schools (S), is a blocking variable of which three levels exist. In any level, there is one school building represented. The school buildings comprise the elementary schools of Monona Village, Wisconsin. The second factor, quantifier vs. no quantifier (Q), is not a repeated factor. Forty-five children were randomly assigned to each level of this factor. The third factor, common names vs. different names (N), is the repeated factor. The results of the analysis of variance outlined earlier are given below in Table 6.

FACTOR S

The effect due to schools, the blocking variable, is not significant. The means are given in Table 7. Even though no statistical significance is present, it is interesting to make some observations. First, the mean IQ of the children in School 2 was greater than that of the two other schools, with the mean IQ of the children in School 3 greater than those of School 1, as noted earlier. Incomplete data were present in the case of School 3, so valid comparisons can be made for only Schools 1 and 2. The arithmetic programs in which the children from these two schools participated were similar in that the children were basically through only combinations through four. It was true that the children

Table 4

Distribution of Frequencies of Total Scores by Tests

Total Score	Total Test	Quantifier	No Quantifier	Common Name	Different Name
0	2	0	2	3	4
1	3	2	1	7	7
2	2	0	2	2	10
3	2	2	0	2	4
4	2	2	0	6	7
5	6	2	4	7	5
6	2	1	1	6	6
7	3	1	2	7	8
8	3	3	0	9	10
9	3	1	2	15	12
10	2	1	1	26	17
11	4	2	2	--	--
12	2	1	1	--	--
13	4	3	1	--	--
14	4	1	3	--	--
15	3	2	1	--	--
16	4	2	2	--	--
17	6	3	3	--	--
18	12	5	7	--	--
19	8	4	4	--	--
20	13	7	6	--	--
Mean	13.00	13.07	12.98	7.98	6.04
Std. Dev.	6.23	6.07	6.38	3.11	3.29

Table 5

Difficulty Levels of Items by Tests[a]

Item	Total Test	Quantifier	No Quantifier
1	.59	.60	.58
2	.64	.67	.62
3	.61	.64	.58
4	.63	.60	.67
5	.68	.64	.71
6	.53	.56	.51
7	.59	.67	.51
8	.50	.49	.51
9	57	.60	.53
10	.68	.71	.64
11	.70	.67	.73
12	.69	.67	.71
13	.68	.67	.69
14	.77	.78	.76
15	.73	.71	.76
16	.61	.62	.60
17	.73	.75	.71
18	.59	.53	.64
19	.80	.80	.80
20	.68	.69	.67

[a](Items 1-10 Different Names)
(Items 11-20 Common Names)

Table 6

ANOVA Table

Source of Variation	df	MS	F
Between Subj.			
S	2	47.072	2.45
Q	1	.089	< 1
S X Q	2	23.439	1.22
Subj./gps.	84	19.177	
Within Subj.			
N	1	39.200	32.64**
S X N	2	.317	< 1
Q X N	1	2.222	1.85
S X Q X N	2	.039	< 1
N X Subj./gps.	84	1.201	

**p< .01

Table 7

Means of the Three Schools

Schools	1	2	3	Total
Mean	11.00	14.30	13.77	13.02

in School 2 were exposed to combinations which summed to five, but they were only beginning those combinations. The children in School 3 were accelerated insofar as addition combinations are concerned, with some of the children through combinations which summed to seven and all of the children through combinations which summed to five. If this additional training had any effect on the performance of the children on the test given to them, it certainly is not apparent from this data.

The overall mean was about 65 per cent. This compares quite favorably with the mean of 63 per cent obtained on the three verbal addition problems by 132 first-grade children cited from an earlier study.[14] It is not very comforting, however, that these means correspond so closely, because, in the earlier study, the test was given four months later in the school year. The children in the earlier study were participating in a curriculum different from those cited in this study, so an extrapolation is somewhat dangerous.

FACTOR Q

The effect of Factor Q is insignificant as can be seen by Table 6. The means are given in Table 8. It apparently makes no difference

Table 8

Means of Factor Q

Q	Quantifier	No Quantifier
Means	13.07	12.98

in the difficulty of a problem if first the child's attention is directed toward the total set in question, then to the two component sets, and then back to the total set by the way in which the problem is worded. Even though the interaction of Factors S and Q is insignificant, it is interesting to look at the means, which are given in Table 9. The children in School 1 did better on the problems with no quantifier than on problems with a quantifier, which is what was expected. However, the reverse was true for those children in School 2.

Table 9

Means by Factor S and Factor Q

Q \ S	1	2	3
Quantifier	9.80	15.60	13.80
No Quantifier	12.20	13.00	13.73

It may be of interest to note that the children in School 2 were introduced to addition in a slightly different way than were the children in the two other schools. Before a standard number name was given to two sets of objects, one of which had joined the other, the children had first to write only the sum for the total objects present. For example, if one man is joined by four men (situations such as this were actually shown on the TV screen), then the children had to write 1 + 4 to tell how many men were there. It was not until later that the standard name "5" was introduced. The added verbage of the quantifier may have thereby helped these children in that it first focused their attention on a total set.

From that point, their attention was focused on two component subsets and then back to the total set. It may be that, due to their training, this was the situation with which they were most familiar.

FACTOR N

The effect of Factor N is highly significant. The means of the two levels of Factor N, given in Table 10, represent between a 9 and 10 per cent difference.

Table 10

Means of the Two Levels of Factor N

N	Common Name	Different Names
Means	6.98	6.04

Apparently, then, at this time in the first grade, verbal addition problems with different names attached to (1) the total set and (2) the two subsets whose union is the total set are more difficult for first-grade children than are verbal addition problems with common names associated with all three sets encountered in a problem. Due to the lack of an interaction of Factor N with Factor S and with Factor Q, one can say this variable (Factor N) is operative across schools (arithmetic programs) and also across the presence of a quantifier vs. no quantifier. These interactions are given in Tables 11 and 12 below.

Table 11

Means by Factors S and N

N \ S	1	2	3
C. N.	6.00	7.53	7.40
D. N.	5.00	6.77	6.37

Table 12

Means by Factors Q and N

N \ Q	Quantifier	No Quantifier
C. N.	6.89	7.07
D. N.	6.18	5.91

It is quite appropriate to continue the discussion deferred in the section in which Factor S was discussed. There, it was noted that the grand mean of this study compared favorably with the grand mean of three problems reported from an earlier study which the author conducted. It is now possible to make a further comparison. The problems with different names involved had a mean of 60.4 per cent. The second problem cited earlier in this report from the previous study had a mean of 45.5 per cent, which is considerably lower than that of 60.4 per cent. However, an investigation of Table 3 shows problems with different names involved having as low as 50 per cent correct responses, so no hard and fast statement may be made relative to the comparison. The deviation is great enough, however, to make one wonder if there isn't something about the second problem which hasn't been fully explained by this study. It certainly may be a more complex problem than any used here. It is different in that it describes two sets of similar objects as doing different things, rather than having different names.

The two problems with common names involved quoted earlier from the previous study have a mean score of about 72 per cent. The ten problems in this study with common names involved have a mean score of 69.8 per cent which is quite comparable to that of the previous study. One may then conjecture that, if the experiences of the two different sets of children in the two studies are similar, those experiences are in no way improving the problem-solving abilities of first-grade children in the case of verbal addition problems over a four-month time interval. This conjecture is supported by the lack of a school difference reported earlier. If the above conjecture is true, then something is lacking in the arithmetic curricula of the first grade. Certainly the solution of verbal problems such as those used in this study should be a forerunner of a knowledge of addition facts. That is, they should be a necessary step toward the acquisition of the addition facts. The evidence given here can only recommend that research be conducted in which (1) verbal addition problems are interpreted by the children using visuals and (2) verbal addition problems are interpreted by the children symbolically. These two proposed phases of a curriculum should be in addition to that already given in the case of addition. That is, an experiment should be conducted in which children not only verbalize or symbolize usually presented stimuli, but also interpret situations presented symbolically and verbally using visuals. In the case of the symbolic presentation, the interpretation could be also a verbal one, and in the case of a verbal presentation, the interpretation could be a symbolic one or just a verbal response.

Table 13 gives the interaction S x Q x N. In the case of School 2, higher scores on the problems with a quantifier present than those with no quantifier present exist in the case of problems with a common name as well as problems with different names. This should support an interpretation given earlier as to why children in School 2 did better on the problems involving a quantifier than those which involved no quantifier.

Table 13

Means by Factor S, Factor Q, and Factor N

N	Q	S 1	S 2	S 3
CN	Quan	5.27	8.07	7.33
	No Quan	6.37	7.00	7.47
DN	Quan	4.53	7.53	6.47
	No Quan	5.48	6.00	6.27

IV
SUMMARY AND CONCLUSIONS

This study was designed to investigate the effects of two variables on the ability of first-grade children to solve arithmetic addition problems read to them by one experimenter. The variables were: (1) the presence of an existential quantifier preceding the statement of the problem vs. no existential quantifier and (2) the presence of three different names for the three sets in the addition problems vs. common names for the three sets. The sample of subjects was taken from a school district which used three different arithmetic programs in three different school buildings, which constituted the elementary schools of the city in which they were located. A total of 40 problems were administered, 20 per individual. Each individual was randomly assigned to either the problems with a quantifier or those with no quantifier, until a total of 45 individuals were in each category, 15 from each school. Ten of the 20 problems each individual was administered involved common names and ten involved different names. The design used to analyze the data was a 3 x 2 x 2 ANOVA with repeated measures on the last factor. Reliability coefficients were computed on each test on which an *F* ratio was reported. These reliabilities were all quite substantial, ranging from .87 to .94, and thereby support any conclusions which may be drawn about the variables in the study. No significant differences were present in the means of: (1) schools and (2) quantifier vs. no quantifier. Also, no significant interactions were present. The variable of common names vs. different names was significant favoring the problems with common names. The mean of these problems was about 9.4 per cent higher than those with a different name.

Limitations of the study certainly exist, among which are the following: (1) At the present time it is not known whether Factor N (common names vs. different names) is operative over all levels of abstraction that children go through when learning about addition. For example, is it more difficult for children to symbolize pictorial representations of additive situations where the pictures are representative of the type of problems with different names, than to symbolize pictorial representation of additive situations where the pictures are representative of the type of problems with common names? Similar questions may be asked at varying levels of abstraction. (2) Does Factor N operate in the same way with different groups of children, where these groups are defined by some external measure such as IQ or level of conservation of numerousness? (3) Questions (1) and (2) may be repeated with reference to Factor Q. (4) Can one observe the same phenomena in the case of subtraction?

It is questionable whether this study, standing alone, is of sufficient scope to give definitive guidelines for curriculum builders to follow. However, there are certainly indications and conjectures which cannot be overlooked. (1) It seems advisable to include, at least in the children's exercise books, pictorial representations of additive situations in which the two sets may be described by different words and in which the total set may be described by still a different word. The emphasis to be placed on this type of pictorial representation has yet to be determined. (2) It seems advisable that children be given more chances to interpret, first using visuals and then symbolically, addition problems which have been verbally stated to them by their teacher. This activity should be in addition to the usual verbalization and/or symbolization of pictorial situations. (3) It seems advisable that children be given more chances to interpret number sentences such as 2 + 3 = 5 using visuals or verbally. The amount of emphasis that should be placed on (2) and (3) has again not yet been ascertained. The best that can be said at the present time is that they should not be ignored.

APPENDIX

RAW DATA

School No.	Subject No.	Quantifier: Different Names 1	2	3	4	5	6	7	8	9	10	Quantifier: Common Names 11	12	13	14	15	16	17	18	19	20
2	1	0	0	1	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1	0
	2	1	1	1	1	1	0	1	0	1	1	1	1	1	1	0	0	1	0	1	1
	3	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	4	1	1	0	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	5	0	1	1	1	0	1	1	0	1	1	1	1	1	1	1	1	1	1	0	1
	6	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	0	1	1
	7	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	8	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	1	1	1
	9	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	0	1	1
	10	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1
	11	1	0	1	1	0	1	1	0	1	1	1	1	0	1	1	1	1	0	1	1
	12	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	13	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	0	1	1
	14	1	1	1	1	1	1	1	0	0	1	0	1	1	1	1	1	1	1	1	1
	15	0	1	0	1	0	0	1	1	1	0	1	1	1	1	1	0	0	1	1	1
1	16	0	0	0	1	0	0	0	1	0	0	0	0	0	0	0	0	0	1	0	0
	17	1	0	0	1	0	0	0	0	0	0	1	1	1	1	0	0	0	0	1	0
	18	0	0	1	1	0	1	1	0	0	0	0	1	1	1	0	1	1	1	1	0
	19	0	0	1	0	1	0	0	1	0	1	0	0	1	1	1	0	0	0	1	1
	20	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1
	21	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	22	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	23	1	1	1	0	1	1	1	1	0	1	1	0	1	1	0	1	1	1	1	1
	24	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	25	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
	26	0	0	0	0	1	0	0	0	0	1	0	0	0	0	0	1	1	0	0	0
	27	1	1	1	1	0	0	1	0	0	1	0	0	0	1	1	1	1	0	1	1
	28	0	1	0	0	0	1	0	1	0	1	0	0	0	1	0	0	0	0	1	0
	29	0	1	1	0	0	0	1	0	1	0	1	0	1	1	1	1	1	0	1	1
	30	0	0	0	0	0	0	1	0	0	0	0	1	0	1	1	0	1	0	0	0
3	31	0	1	1	0	1	1	0	0	0	1	1	0	0	0	1	1	1	0	1	0
	32	0	0	0	0	1	0	0	0	1	0	1	0	1	1	1	0	1	0	1	0
	33	0	1	0	0	1	1	1	0	1	0	1	1	0	1	1	0	1	0	1	0
	34	1	0	0	0	0	0	0	0	1	1	1	0	1	0	1	0	1	0	0	1
	35	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	36	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	37	0	1	1	1	1	0	0	0	0	1	0	1	0	1	1	1	1	1	1	1
	38	0	0	1	0	1	0	1	0	0	1	1	1	0	1	0	1	0	0	0	0
	39	1	1	0	0	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1
	40	1	1	1	1	1	1	0	1	1	1	1	0	1	1	1	1	1	1	1	1
	41	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	42	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	43	1	0	0	0	0	0	1	0	0	1	0	0	0	0	0	0	0	1	0	0
	44	1	1	1	1	1	1	1	0	1	1	1	1	1	0	1	1	1	0	1	1
	45	1	1	0	1	1	0	0	1	0	1	0	1	1	1	1	0	1	1	1	1

APPENDIX (continued)

School No.	Subject No.	No Quantifier																			
		Different Names										Common Names									
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	46	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	47	1	1	1	0	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1
	48	1	1	1	1	1	1	0	1	0	0	1	1	1	1	1	1	1	1	1	1
	49	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	0	1	1
	50	1	0	0	0	0	0	0	1	0	1	1	0	1	1	1	0	0	1	1	1
	51	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	52	0	1	0	1	0	0	0	1	0	1	0	1	0	0	0	1	1	1	0	1
	53	1	1	0	0	1	0	0	0	0	1	1	1	0	1	1	1	1	0	0	1
	54	1	0	0	1	0	0	0	0	0	0	1	1	1	0	0	0	0	1	1	0
	55	1	1	1	1	1	0	0	0	1	0	1	1	1	1	1	1	1	1	1	0
	56	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1
	57	0	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1
	58	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	59	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
	60	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	61	1	1	1	1	0	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1
	62	0	0	0	0	0	1	0	0	0	1	1	0	1	1	0	0	0	0	0	0
	63	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	64	0	1	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	1	1
	65	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	66	1	0	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	0	1
	67	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	68	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	69	1	1	1	1	1	0	1	0	1	1	1	1	1	1	1	1	1	1	1	1
	70	0	1	1	1	1	1	1	0	0	1	1	1	0	1	1	1	1	1	1	1
	71	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1	1	1	0	1
	72	0	0	0	1	1	1	1	1	1	1	1	1	0	1	1	0	1	0	1	1
	73	1	1	1	0	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	74	0	0	1	0	0	0	0	0	0	1	1	0	1	1	1	0	0	0	1	0
	75	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0
2	76	0	1	1	0	1	1	0	0	0	1	0	1	1	0	1	1	0	0	1	1
	77	0	1	0	1	1	0	0	0	0	0	0	1	1	0	1	0	1	0	1	1
	78	1	0	1	1	1	0	0	1	1	0	1	0	1	1	1	0	0	1	1	0
	79	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
	80	0	0	0	0	1	0	1	0	0	0	0	0	0	1	0	1	0	1	1	0
	81	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	82	1	1	1	1	1	1	1	0	1	1	0	1	1	1	1	0	1	1	1	1
	83	0	0	1	1	1	1	1	1	1	0	1	1	0	1	0	1	1	1	1	0
	84	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	85	1	0	1	1	1	0	0	1	1	0	1	0	1	1	1	0	1	1	1	0
	86	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1
	87	1	1	0	1	1	1	1	0	0	1	1	1	1	1	1	1	1	0	1	1
	88	1	0	0	0	0	0	0	1	0	0	1	1	0	0	0	0	0	1	0	0
	89	1	1	0	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1
	90	0	1	1	1	1	0	1	0	0	1	1	1	1	1	1	0	1	0	1	1

NOTES

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