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DESIGN OF AN EARLY LEARNING CURRICULUM.

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THIS PAPER PRESENTS A RATIONALE FOR CURRICULUM DEVELOPMENT IN THE PRE-SCHOOL YEARS AND OUTLINES A METHODOLOGY FOR CURRICULUM DESIGN IN GENERAL. AN EXTENSIVE APPLICATION OF THIS METHOD TO THE DESIGN OF AN EARLY LEARNING CURRICULUM WILL BE UNDERTAKEN OVER THE NEXT FIVE YEARS BY THE PRIMARY EDUCATION PROJECT. THE PROJECT AIMS TO DESIGN AND IMPLEMENT AN INDIVIDUALIZED CURRICULUM AND SCHOOL ENVIRONMENT FOR A HETEROGENEOUS URBAN POPULATION. THE PROGRAM WILL BEGIN AT THE PRE-SCHOOL LEVEL, WITH THREE-YEAR-OLD CHILDREN, AND WILL EVENTUALLY BE DEVELOPED THROUGH THE THIRD GRADE. THE EXPERIMENTAL AND DEVELOPMENTAL WORK FOR THIS PROGRAM WILL TAKE PLACE IN A SINGLE PITTSBURGH ELEMENTARY SCHOOL, CHOSEN FOR ITS UNUSUAL DEGREE OF ECONOMIC AND RACIAL HETEROGENEITY. ULTIMATELY, THE PROJECT'S GOAL IS TO DEVELOP A PROGRAM THAT CAN BE IMPLEMENTED ELSEWHERE AT A REASONABLE COST AND WITHOUT THE CONTINUED SUPPORT OF AN OUTSIDE RESEARCH TEAM. DEVELOPED IN THE MAIN BODY OF THIS PAPER ARE (1) NEED FOR AN EARLY LEARNING PROGRAM, (2) CONTENT OF AN EARLY LEARNING PROGRAM, (3) ANALYSIS AND SEQUENCING OF OBJECTIVES, (4) CURRICULUM SEQUENCES AND THEIR USE IN THE CLASSROOM, AND (5) VALIDATION OF CURRICULUM SEQUENCES. THE PAPER CONCLUDES WITH APPENDED REPORTS ON (1) SKILLS FOR INCLUSION IN AN EARLY LEARNING CURRICULUM, (2) CURRICULUM SEQUENCE FOR TEACHING CLASSIFICATION IN A MATRIX FORMAT, AND (3) CURRICULUM SEQUENCE FOR NUMBER CONSERVATION. (RP)

# DESIGN OF AN EARLY LEARNING CURRICULUM

by

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With all the current discussion of the need for beginning education early, there has been astonishingly little systematic attention to the question of what the content of that education ought to be. There is plenty of debate over whether young children should be offered conceptually oriented formal instruction of any kind, and whether the classroom organization should be free and expressive or structured and directed. Yet with few exceptions, even those committed to early conceptual instruction offer no clear specification of curricula or rationale for curricular choices.

One possibility for early learning programs is that they simply begin instruction in the standard curriculum at a young age and thus make it likely that pupils will enter the first grade "advanced" in reading, arithmetic and related subjects. There can be little doubt that such an early start is possible, at least with some pupils, when the educational methods used are appropriate to the learning requirements of young children. This has been demonstrated in a number of settings, perhaps the best-known of which are O. K. Moore's Responsive Environments Project (Moore and Anderson, in press) and the many Montessori schools in this country and abroad. However, closer examination of most of these programs reveals that, in addition to the traditional "subjects," there is a whole series of preparatory skills and concepts which are also taught, if only implicitly. These preparatory behaviors, which are essential to subsequent learning, can well command primary attention in a systematic attempt to develop an early learning program.

## Need for an Early Learning Program

The child from an educated family who enters school at the age of six,

# Preface

This paper presents a rationale for curriculum development in the pre-school years and outlines a rigorous methodology for curriculum design in general. An extensive application of this method to the design of an early learning curriculum will be undertaken over the next several years by the research staff of the Primary Education Project.

The Primary Education Project is a cooperative undertaking of the University of Pittsburgh, the Pittsburgh Public Schools and the General Learning Corporation. The project aims, over a five year period, to design and implement an individualized curriculum and school environment for a heterogeneous urban population. The program will begin in the pre-school, with three-year-old children, and will eventually run through the third grade. The experimental work and developmental work for this program will take place in a single Pittsburgh elementary school, chosen for its unusual degree of economic and racial heterogenity. Ultimately, the project's goal is to develop a program that can be implemented elsewhere at a reasonable cost and without the continued support of an outside research team.

The project represents a new departure in collaboration between educational and business institutions. Within the total structure, the Pittsburgh Public Schools remain legally responsible for children in the experimental school and also provide the project's general director. The Learning Research and Development Center of the University of Pittsburgh is responsible for research and development work, including curriculum design and preparation of teaching materials. The University's School of Education will be responsible for developing in-service and pre-service training programs for teachers in the program. The General Learning Corporation will design a computer-based information management system to aid in individualization, in addition to providing the initial funding for the project.

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even if his parents have made no conscious effort to teach him, has behind him a learning history of considerable richness and complexity. As a result of his interaction with his environment, he has a perceptual discrimination repertoire that allows him to efficiently scan and categorize new visual stimuli. He has reasonably well-organized spatial and temporal concepts which permit him to locate objects and events in time and space and to orient himself with respect to them. He is fairly good at recognizing relevant aspects of a learning situation and focusing his attention on them. He can talk comfortably with adults and has a linguistic competence that allows him to learn from verbal exposure. He can use examples as a way of learning new concepts and can ask questions to gather new information. And so forth. The list of skills and concepts which constitute a well-endowed six year old's "readiness" for school would read like a catalogue of aptitude and intelligence measures. Yet the child was born with none of these specific abilities. In interaction with his environment he has learned them all.

Can an environment be created that will maximize the learning abilities of children from all kinds of home backgrounds? Herein lies the great challenge for early education. A few children learn from their families, or other interested adults, a whole range of specific abilities and learning sets that facilitate subsequent learning. These are the children whose scores on tests suggest that they are "intelligent," and whose performance in school normally confirms the appropriateness of the label. Yet what these children learn incidentally, almost by accident, the majority of children will learn only if they are systematically taught. Where the unprogrammed environment of the home and nighborhood does a weak or incomplete job, a "programmed" environment, designed for the pre-school, offers the best hope of providing the necessary learning experiences. This planned environment must be highly efficient in promoting learning, for the child will spend only a tiny fraction of his time during his pre-school years in classes of any kind. With so little time available, it is highly unlikely that a school environment which merely reproduces the unprogrammed qualities of the educated home will succeed in establishing the necessary skills.

The need, then, is to design an early learning program that is explicitly intended to teach children how to learn and how to think — in other words, to teach them the skills and concepts that underlie intelligent behavior. Such a program would represent a logical application of current theories which view intelligence as learned and responsive to experience rather than inherited and fixed (Hunt, 1961). This certainly does not mean, however, that children should be artificially "coached" to do well on standard intelligence tests. Such coaching usually has the effect of raising test scores, but not of generalizing to new learning situations. If an early learning program is ultimately to be counted successful, then instruction in the skills identified must not only lead to improved performance on intelligence tests; the increased intelligence scores must continue to be predictive of increased learning power, in school and elsewhere.

## Content of an Early Learning Program

Extensive study, both analytic and empirical, will be required to determine exactly which skills and concepts, systematically taught in the pre-school, will produce the kind of generalized learning power under consideration here. Initially, however, hypotheses concerning the probable range and character of the curriculum can be derived from a variety of sources. The literature in developmental cognitive psychology, while not pedagogically oriented, is rich in suggestions for curriculum development. Thus, several of the skill sequences discussed later in this paper draw heavily on the work of Piaget, Bruner and their followers. Further suggestions come from educators and pyschologists working with retarded and otherwise handicapped children. Perhaps because the children they work with do not learn easily from casual and unplanned exposure, investigators in the field of "special education" have had to pay closer than ordinary attention to the details of curriculum and instructional strategy, particularly in basic areas. Still other skills under consideration derive from our own observation and analysis of children's learning, both in the laboratory and in school settings.

From these sources, three general classes of skills can be identified for inclusion in an early learning curriculum. These can be termed 1) orienting and attending skills, 2) perceptual and motor skills, and 3) conceptual and linguistic skills. Each of these categories is discussed in the paragraphs below, and illustrative lists of skills for each class appear in Appendix A.

(1) Orienting and attending skills include the ability to concentrate on a task and resist distractions; the ability to attend to appropriate details; to follow directions; to accept rewards which are delayed rather than immediate, and verbal rather than concrete. Other examples of this class of skills appear in Section 1 of Appendix A (p. 20 ). These are the kinds

of skills that permit a child to function well in a classroom and to learn from the environment. In that sense, they are prerequisite to all of the other skills in an early learning curriculum. For some children a relatively long period of attention to these orienting and attending skills alone may be required (e.g. Berman, 1967). In any case, a pre-school program which ignores the need to develop and nurture these adaptive behaviors will almost certainly fail in its attempt to develop higher-order skills and concepts for all children.

(2) Perceptual-motor skills, which similarly underlie higher-order conceptual functioning, include the ability to use one's body efficiently, with awareness of its position in space, and the ability to make a wide range of sensory discriminations. These skills are listed in sections 2 through 5 of Appendix A (pp. 21-27). Both gross motor skills, such as are needed for controlling one's movement through space, and fine motor skills, such as fitting, placing and using writing instruments are included. In addition, there is a set of "positioning" skills, including the left-right discrimination and movement in specified directions, all cf which are prerequisite to the development of more complex spatial concepts. Finally, sensory skills comprise a range of visual, auditory, and haptic perception and discrimination behaviors. These are virtually synonymous with the child's earliest concepts. As he learns to discriminate, sort, and eventually label shapes, colors, sizes, and sounds, the child is learning elementary concepts that are later elaborated and combined in complex ways. In addition, particularly when systematic instruction highlights the processes, he learns methods for organizing and making sense out of the often confusing sensory stimulation his environment offers.

(3) The conceptual-linguistic category includes behaviors such as classification, reasoning, spatial relations, plan following, and memory, together with the language facility that supports and gives expression to competence in these areas. Early mathematical concepts and processes are also included in this category. Lists of these skills appear in Sections 6 through 16 of Appendix A (pp. 28 -- 42). Compared with motor and perceptual skills, generally exercised in pre-school tasks, conceptual and linguistic skills are still relatively undeveloped as targets of early instruction. Psychologists interested in cognitive and linguistic development, who have studied some of these concepts and skills in great detail, have been mainly concerned with describing the behaviors involved and identifying stages of cognitive development through which children pass "naturally." There have been relatively few investigations of the effects of direct intervention on this development, particularly of sustained and systematic instruction. On the other hand, most pre-school projects, which <u>are</u> concerned with intervention -- often long-term -- have addressed themselves to isolated pieces of conceptual and language behavior, without analyzing the entire range of such skills and preparing a carefully sequenced curriculum based on that analysis. Thus, the task of analyzing and sequencing a broad range of conceptual and language objectives for young children constitutes a research undertaking of major dimensions and importance.

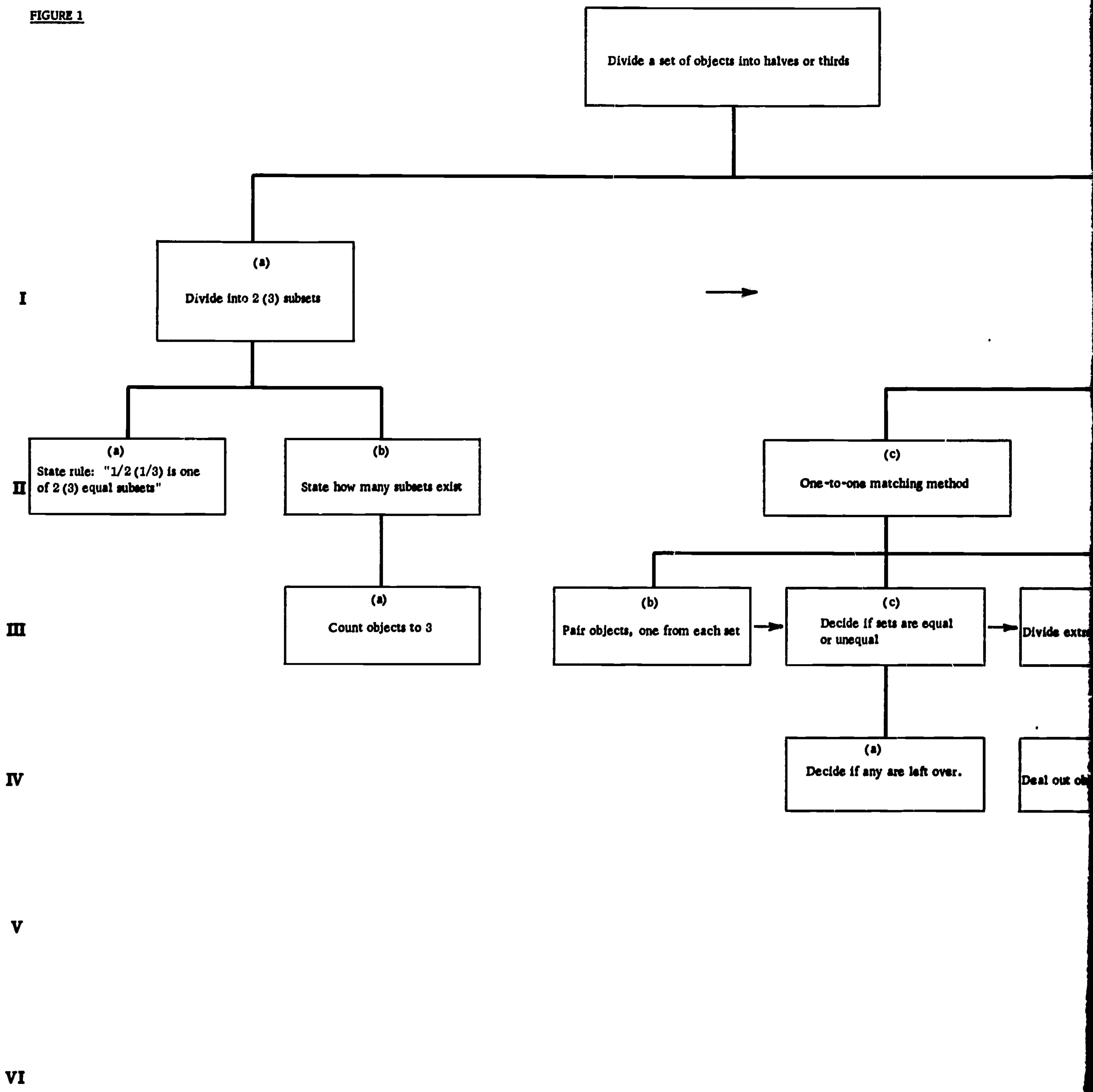
Analysis and Sequencing of Objectives

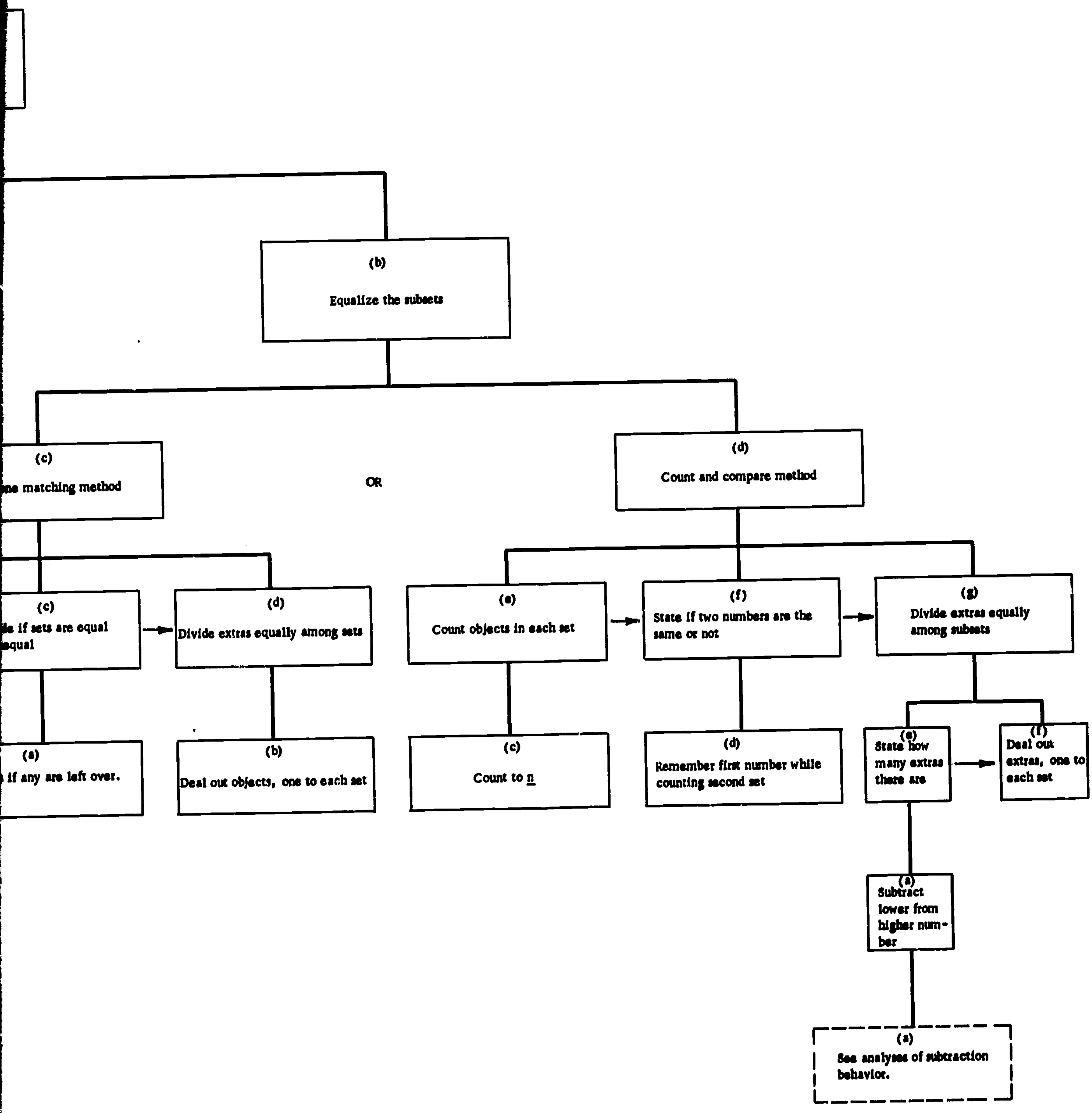
Once a range of objectives for inclusion in the instructional program has been identified, the next step in curriculum design is to sequence the objectives in a manner which facilitates the child's moving smoothly through the curriculum. This entails identifying prerequisite skills and concepts for each objective; that is, skills and concepts which the child must already command before he can successfully learn the new objective. An effective technique for doing this is "behavior" or "component analysis." (cf. Gagne, 1962a, 1962b, Mechner, 1967, Resnick, 1963). This technique, under the name "task analysis," has been widely used in the development of industrial and military training programs (Miller, 1965), but has seen very little use in designing educational programs for children. Component analysis begins with any desired instructional objective, behaviorally stated, and asks, in effect, "To perform this behavior, what prerequisite or component behaviors must the child be able to perform?" Several new behaviors are identified in this way. For each behavior so identified, the same question is asked, thus generating a hierarchy of objectives based on behavioral prerequisites.

Component analysis is a highly flexible technique in that it can be applied to any objective at any level of the curriculum. To begin the analysis an objective at any "grade level" may be chosen: the analysis will always specify objectives that should come <u>earlier</u> in the curriculum. The initial objective itself may have been derived from an earlier analysis of a higher-level objective. Similarly, any sub-objective identified in a given analysis may serve as the basis for a new analysis. The importance of the backward analytic procedure for an early learning curriculum is that it affords a method of identifing critical <u>early</u> behaviors — ones that may not have been identified before, or not identified as related to school performance. Purely logically, the process can be continued indefinitely. In practice, however, a component analysis ceases when the new behaviors identified are ones the investigator believes can be safely assumed in the student population. Thus, component analysis not only provides ordered sets of skills for inclusion in the curriculum; it also specifies the skills a student needs to successfully enter the curriculum.

The skill hierarchies generated through the process of component analysis are best represented in the form of tree diagrams such as appear in Figures 1 through 5. As a first example, Figure 1 displays the analysis of a familiar early mathematics objective, dividing a set into halves or thirds. The behavior to be analyzed, or "terminal objective," appears at the top of the tree diagram. To accomplish this objective the child must be able to divide a set into the appropraite number of subsets and to make certain that the sets are of equal size. These behaviors appear in level I of the diagram, and components of these behaviors appear still lower. Thus, to equalize subsets (box Ib), the child can use either a one-to-one matching method (IIc) or a count and compare method (IId). If he uses the former, he must be able to pair objects, drawing one from each sets, to decide whether the sets are equal, and to equalize the sets if they are not (III b, c, d). Prerequisites for these skills include recognizing whether objects are left over after pairing, and dealing out objects equally among sets (IVa, b). If he uses the count and compare method, the child must count the objects in each set separately and then state whether the two numbers are the same (III e, f). To accomplish this, he must be able to recall the first number after he has finished counting the second subset (IVd),

FIGURE 1


Divide a set of objects into halves or thirds
I
(a)
Divide into 2 (3) subsets
II
(a)
State rule: "1/2 (1/3) is one of 2 (3) equal subsets"
(b)
State how many subsets exist
(c)
One-to-one matching method
III
(a)
Count objects to 3
(b)
Pair objects, one from each set
(c)
Decide if sets are equal or unequal
IV
(a)
Decide if any are left over.
V
VI



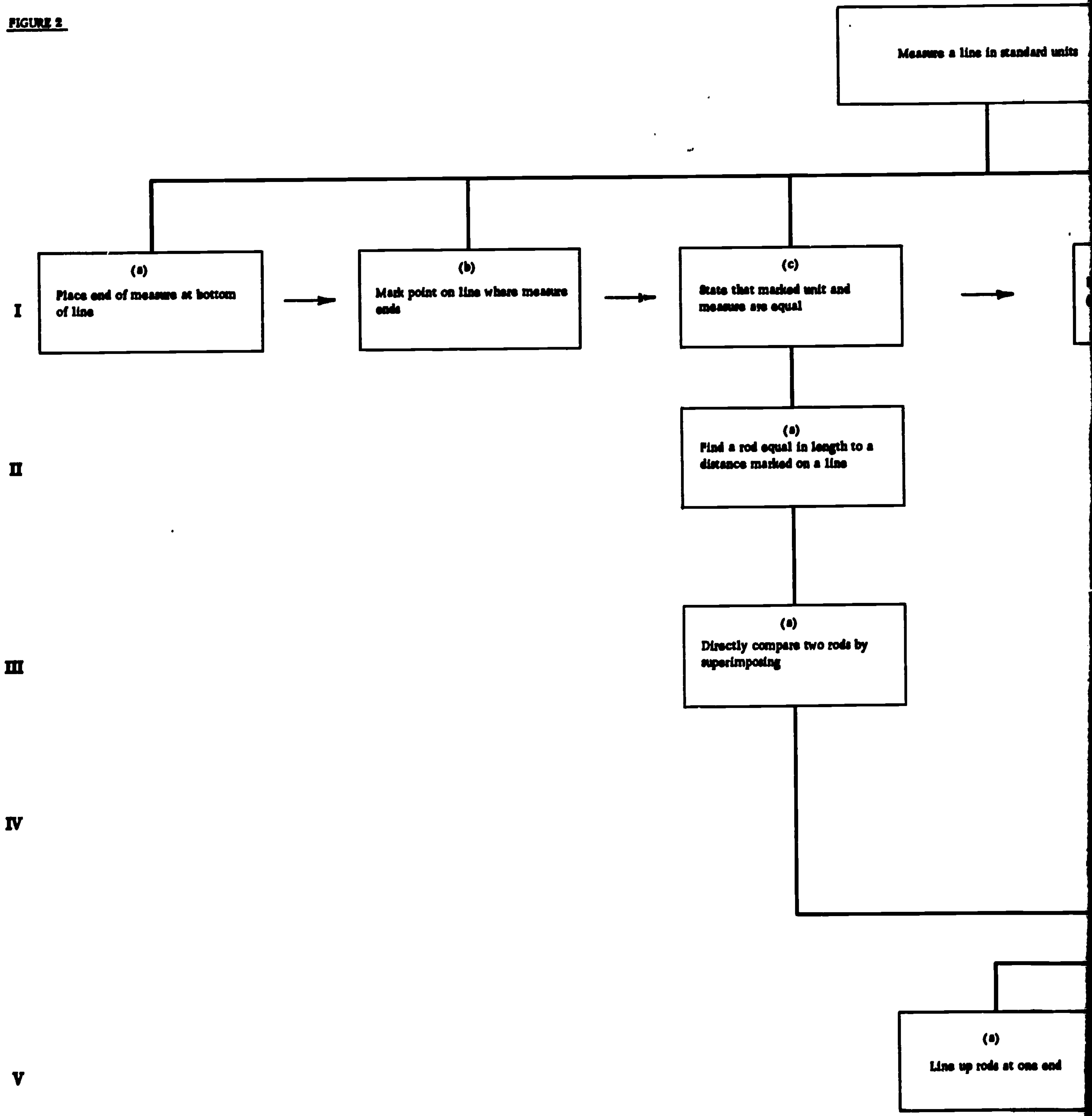
(b)
Equalize the subsets
(c)
ne matching method
OR
(d)
Count and compare method
(c)
de if sets are equal
equal
(d)
Divide extras equally among sets
(e)
Count objects in each set
(f)
State if two numbers are the same or not
(g)
Divide extras equally among subsets
(a)
if any are left over.
(b)
Deal out objects, one to each set
(c)
Count to n
(d)
Remember first number while counting second set
(e)
State how many extras there are
(f)
Deal out extras, one to each set
(a)
Subtract lower from higher number
(a)
See analyses of subtraction behavior.

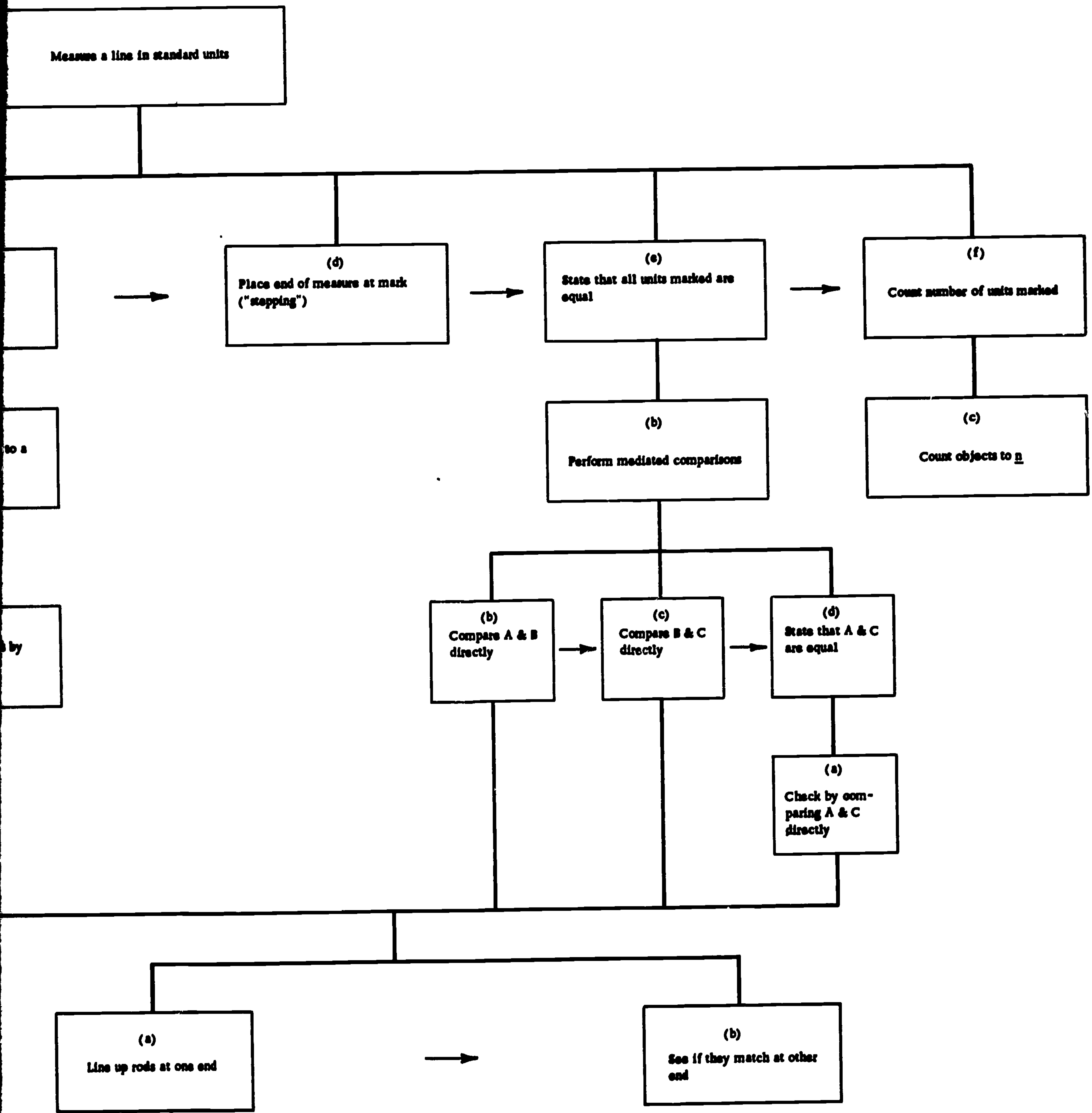
a memory task of considerable difficulty since it requires retaining a response in the face of competing behavior -- i.e., counting the new set. Dividing the extras, under the counting method (IIIg) requires deciding how many extras there are (IVe), which in turn requires subtraction (Va). Rather than repeat a complex analysis of subtraction behavior, the diagram refers to other analyses (VIa).

To derive a teaching sequence from this hierarchy, it is necessary to observe the basic constraint that no objective be taught until the learner has met all prerequisites for that objective. Thus, dealing out objects, one to each set (box IVb), must be taught before dividing extra objects in the equalization process (IIId) occurs; and counting objects to three (IIIa) must be well established before the establishment of a fixed number of subsets (IIb) is taught. Where there is a choice of methods, as between "one-to-one matching" and "count and compare," only one of the methods <u>must</u> be taught, although both may be. In this case, since the one-to-one matching method involves fewer prerequisite behaviors, it would probably be chosen for the child's earliest work in establishing equal sets.

Figure 2 shows a component analysis of a skill, measuring in standard units, to which Piaget and others (1960) have attached considerable importance as an indicator of the child's stage of intellectual development. The terminal objective for this hierarchy is typically among the initial objectives in first-grade mathematics and science curricula. The prerequisites identified in the analysis are rarely taught explicitly, and as if to emphasize this fact, Piaget discusses the skills involved under the title "spontaneous measurement" (Piaget et al., 1960, ch. 2). Yet it is evident that a curriculum in measurement skills for young

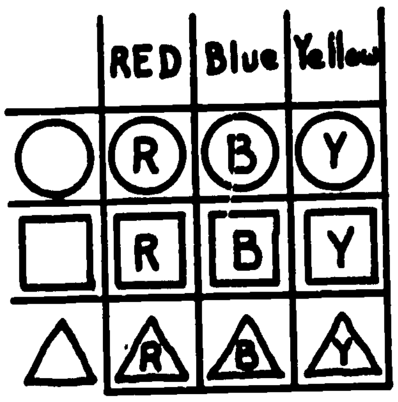
FIGURE 2


Measure a line in standard units
I
(a)
Place end of measure at bottom of line
(b)
Mark point on line where measure ends
(c)
State that marked unit and measure are equal
II
(a)
Find a rod equal in length to a distance marked on a line
III
(a)
Directly compare two rods by superimposing
IV
V
(a)
Line up rods at one end


Measure a line in standard units
(d)
Place end of measure at mark ("stepping")
(e)
State that all units marked are equal
(f)
Count number of units marked
to a
(b)
Perform mediated comparisons
(c)
Count objects to n
by
(b)
Compare A & B directly
(c)
Compare B & C directly
(d)
State that A & C are equal
(a)
Check by comparing A & C directly
(a)
Line up rods at one end
(b)
See if they match at other end

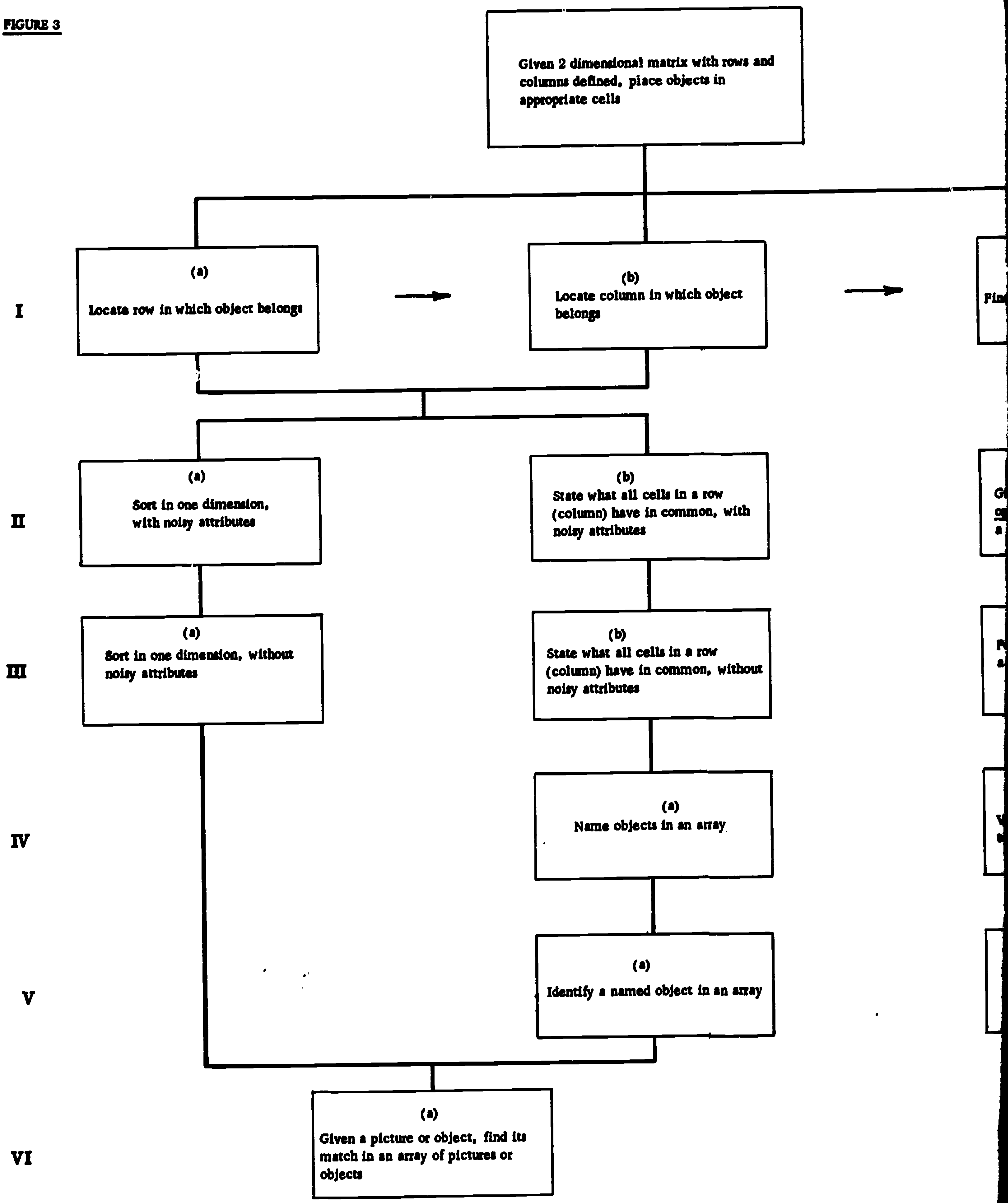
children could be developed simply by systematically teaching the prerequisites displayed in this hierarchy. Such a curriculum sequence would probably begin with teaching the child to directly compare two rods for length (boxes IIIa and IVa) by lining the rods up at one end (Va), and observing whether they were equal at the other end (Vb). This is the basic operation of length measurement, and, as is shown in the diagram, is prerequisite to virtually every step in the process of measuring in standard units. Once this behavior is established, the process of mediated comparison can be taught (IIIb, c, d) and then the process of marking units and "stepping" the measuring unit along the line (Ia–e). Counting (IIc), which is behaviorally independent of the other skills, can be taught concurrently with the basic measuring skills, and later integrated into the measuring operation.

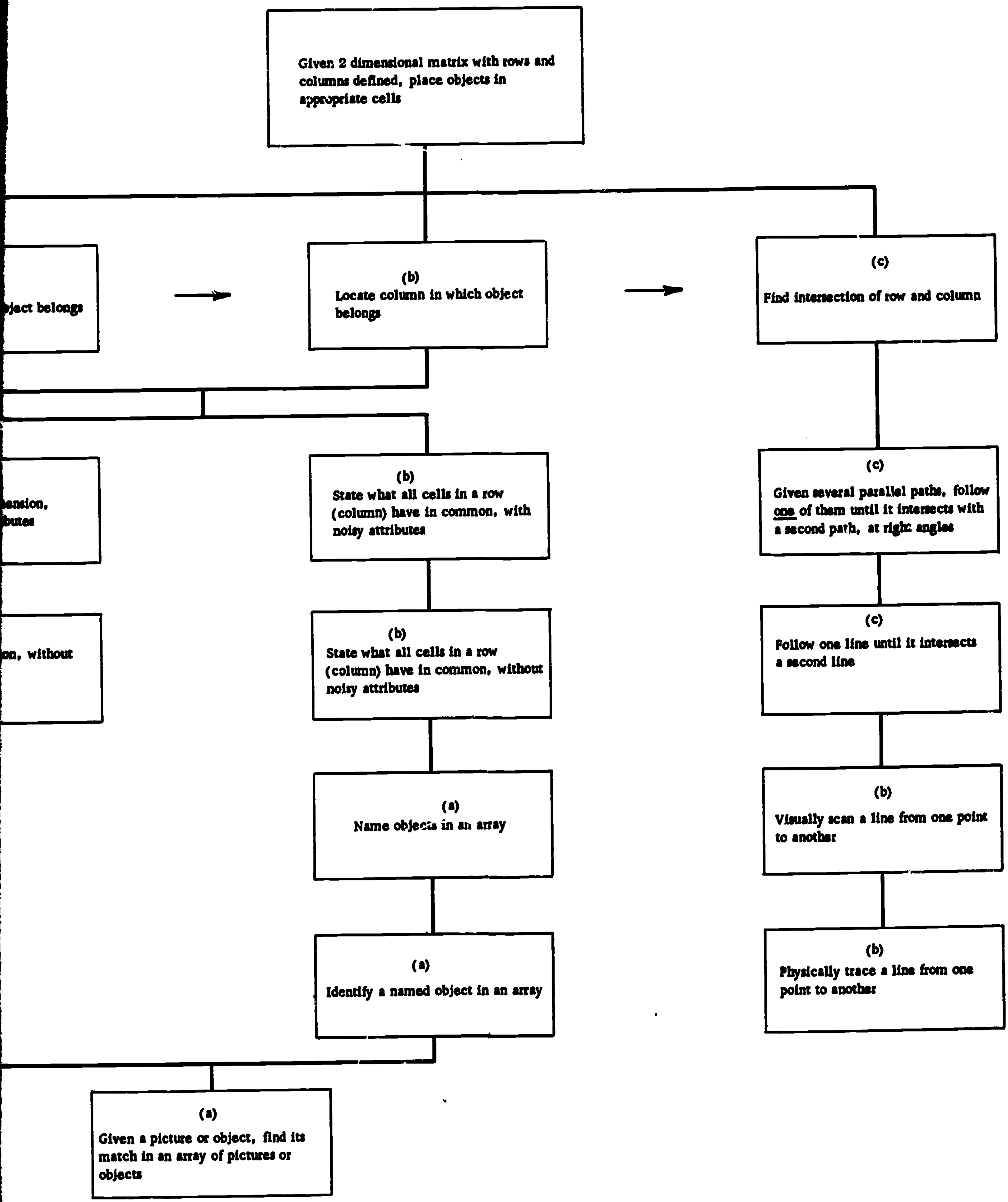
In Figures 3 and 4 component analyses of two related classification skills appear. Both involve the ability to classify objects in two dimensions simultaneously, using a matrix format. For instance, a set of geometric figures in several shapes and colors can be arranged in a matrix whose rows are defined by shape and whose columns are defined by color:




The terminal objective for the first analysis (Figure 3) requires the child to place objects in the appropriate cells of the matrix. The terminal behavior for

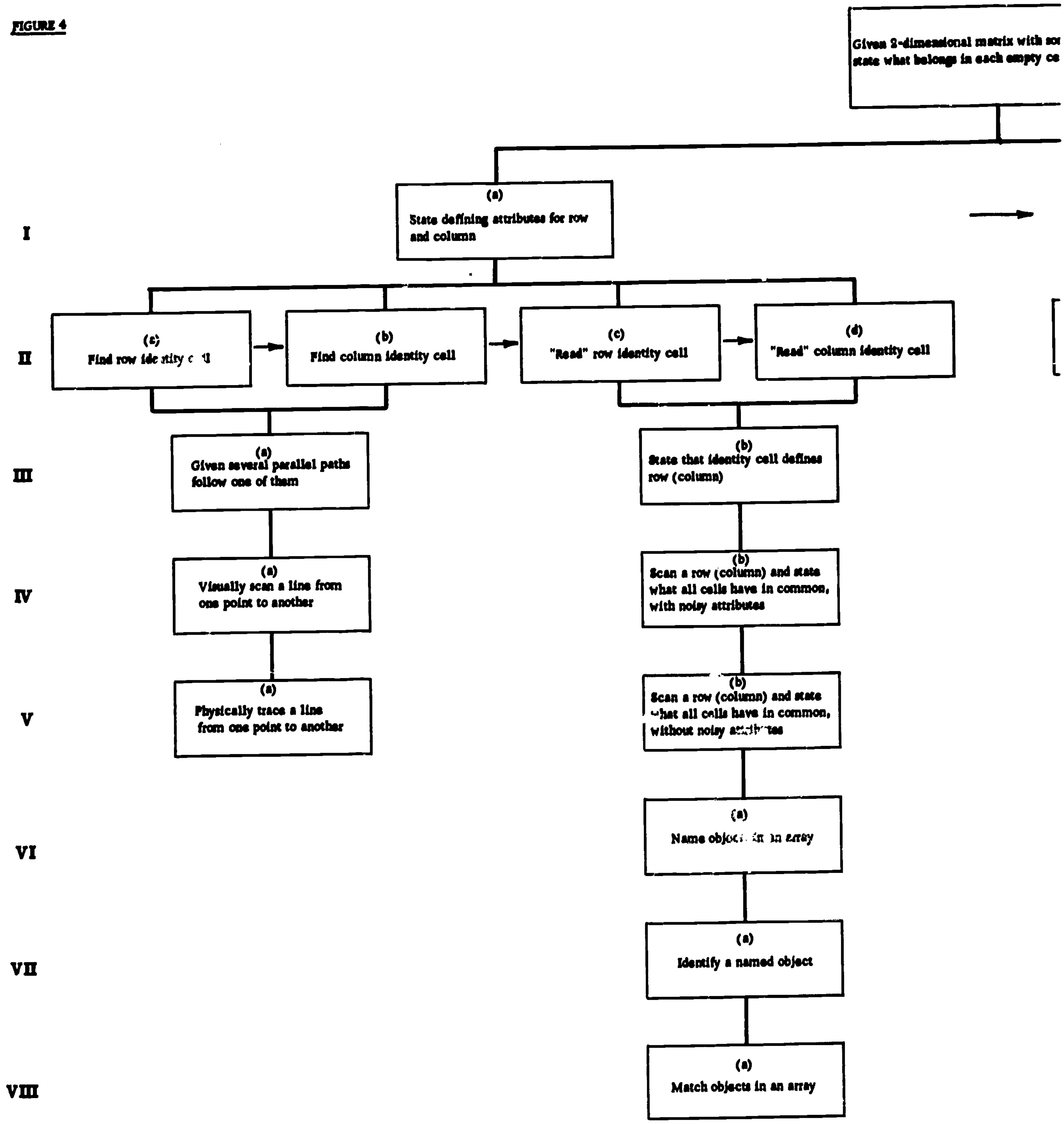
FIGURE 3


Given 2 dimensional matrix with rows and columns defined, place objects in appropriate cells
I
(a) Locate row in which object belongs
(b) Locate column in which object belongs
II
(a) Sort in one dimension, with noisy attributes
(b) State what all cells in a row (column) have in common, with noisy attributes
III
(a) Sort in one dimension, without noisy attributes
(b) State what all cells in a row (column) have in common, without noisy attributes
IV
(a) Name objects in an array
V
(a) Identify a named object in an array
VI
(a) Given a picture or object, find its match in an array of pictures or objects

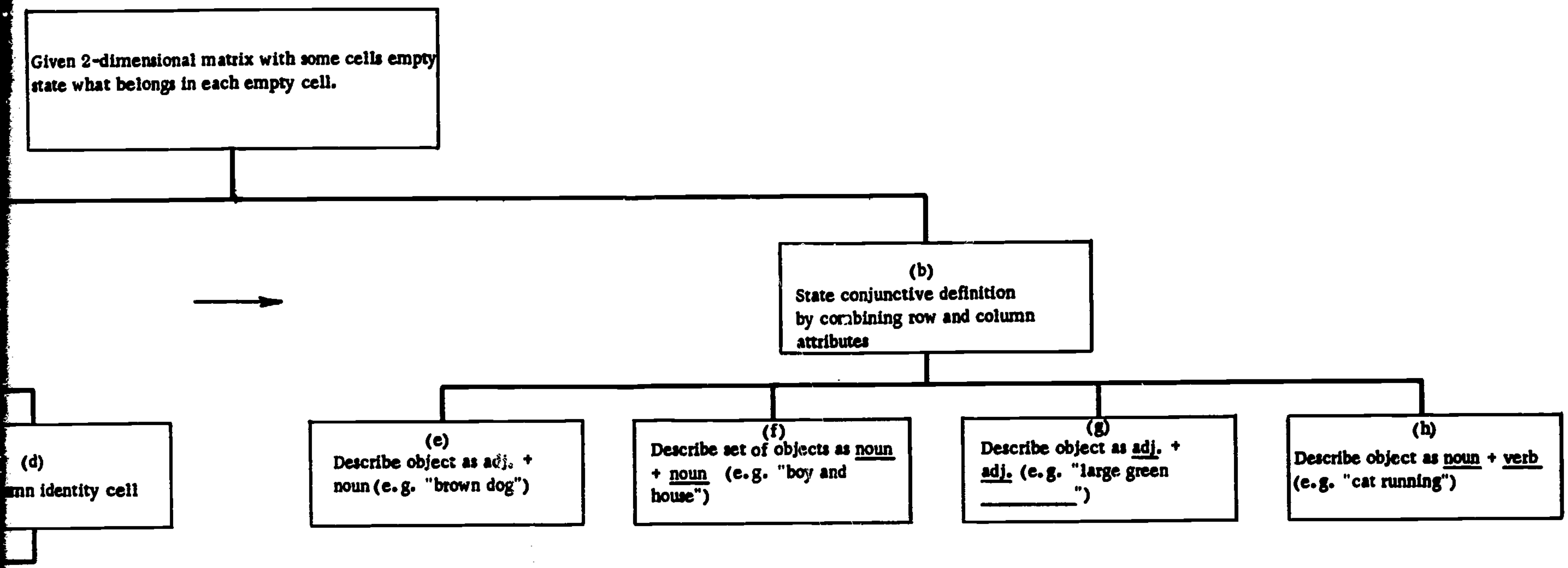


Given 2 dimensional matrix with rows and columns defined, place objects in appropriate cells
ject belongs
(b)
Locate column in which object belongs
(c)
Find intersection of row and column
ension,
butes
(b)
State what all cells in a row (column) have in common, with noisy attributes
(c)
Given several parallel paths, follow one of them until it intersects with a second path, at right angles
on, without
(b)
State what all cells in a row (column) have in common, without noisy attributes
(c)
Follow one line until it intersects a second line
(a)
Name objects in an array
(b)
Visually scan a line from one point to another
(a)
Identify a named object in an array
(b)
Physically trace a line from one point to another
(a)
Given a picture or object, find its match in an array of pictures or objects

FIGURE 4



Given 2-dimensional matrix with so
state what belongs in each empty ce
I
(a)
State defining attributes for row and column
II
(a)
Find row identity cell
(b)
Find column identity cell
(c)
"Read" row identity cell
(d)
"Read" column identity cell
III
(a)
Given several parallel paths follow one of them
(b)
State that identity cell defines row (column)
IV
(a)
Visually scan a line from one point to another
(b)
Scan a row (column) and state what all cells have in common, with noisy attributes
V
(a)
Physically trace a line from one point to another
(b)
Scan a row (column) and state what all cells have in common, without noisy attributes
VI
(a)
Name objects in an array
VII
(a)
Identify a named object
VIII
(a)
Match objects in an array

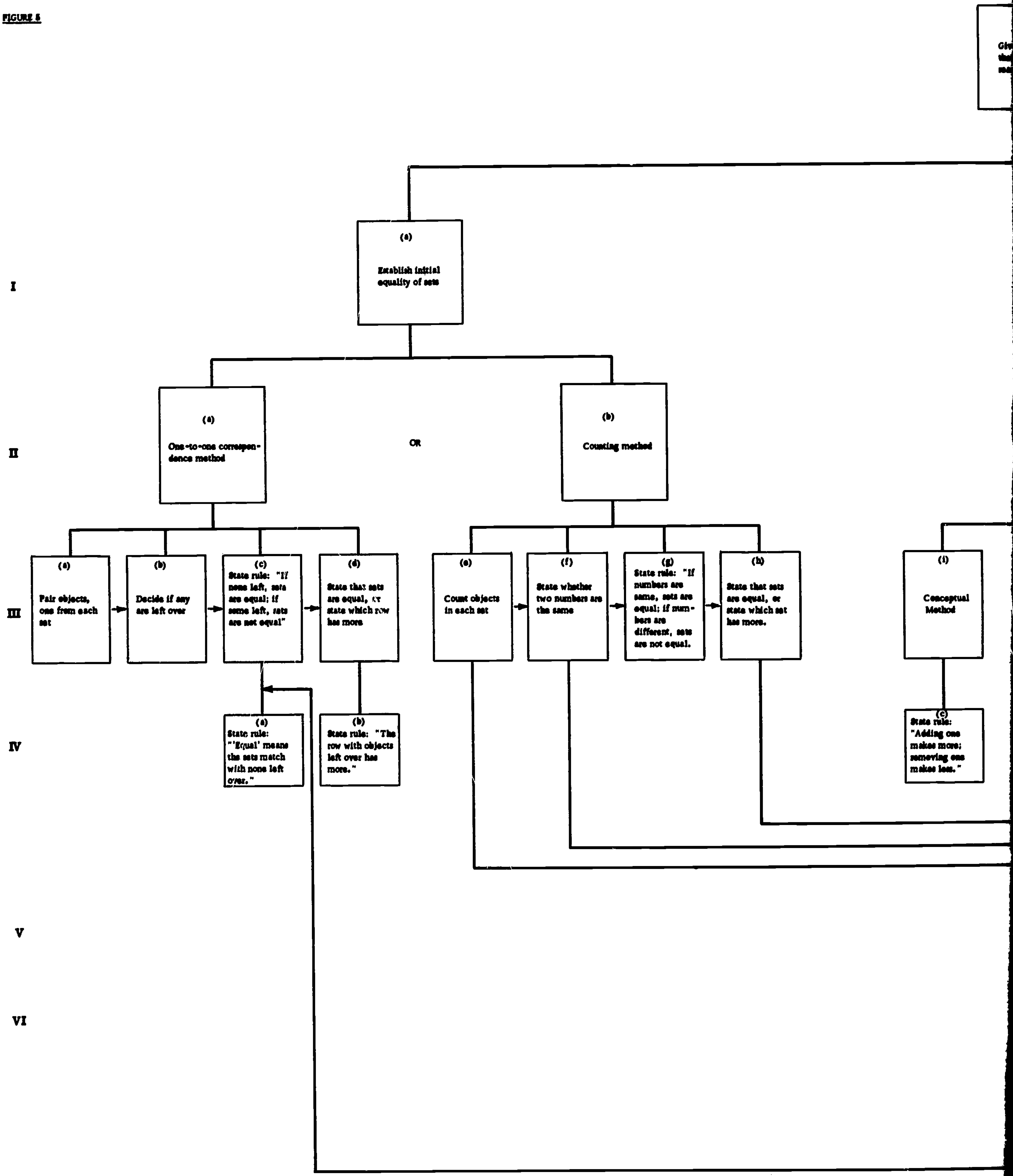

Given 2-dimensional matrix with some cells empty state what belongs in each empty cell.
(b)
State conjunctive definition by combining row and column attributes
(d)
mn identity cell
(e)
Describe object as adj. + noun (e.g. "brown dog")
(f)
Describe set of objects as noun + noun (e.g. "boy and house")
(g)
Describe object as adj. + adj. (e.g. "large green ______")
(h)
Describe object as noun + verb (e.g. "cat running")

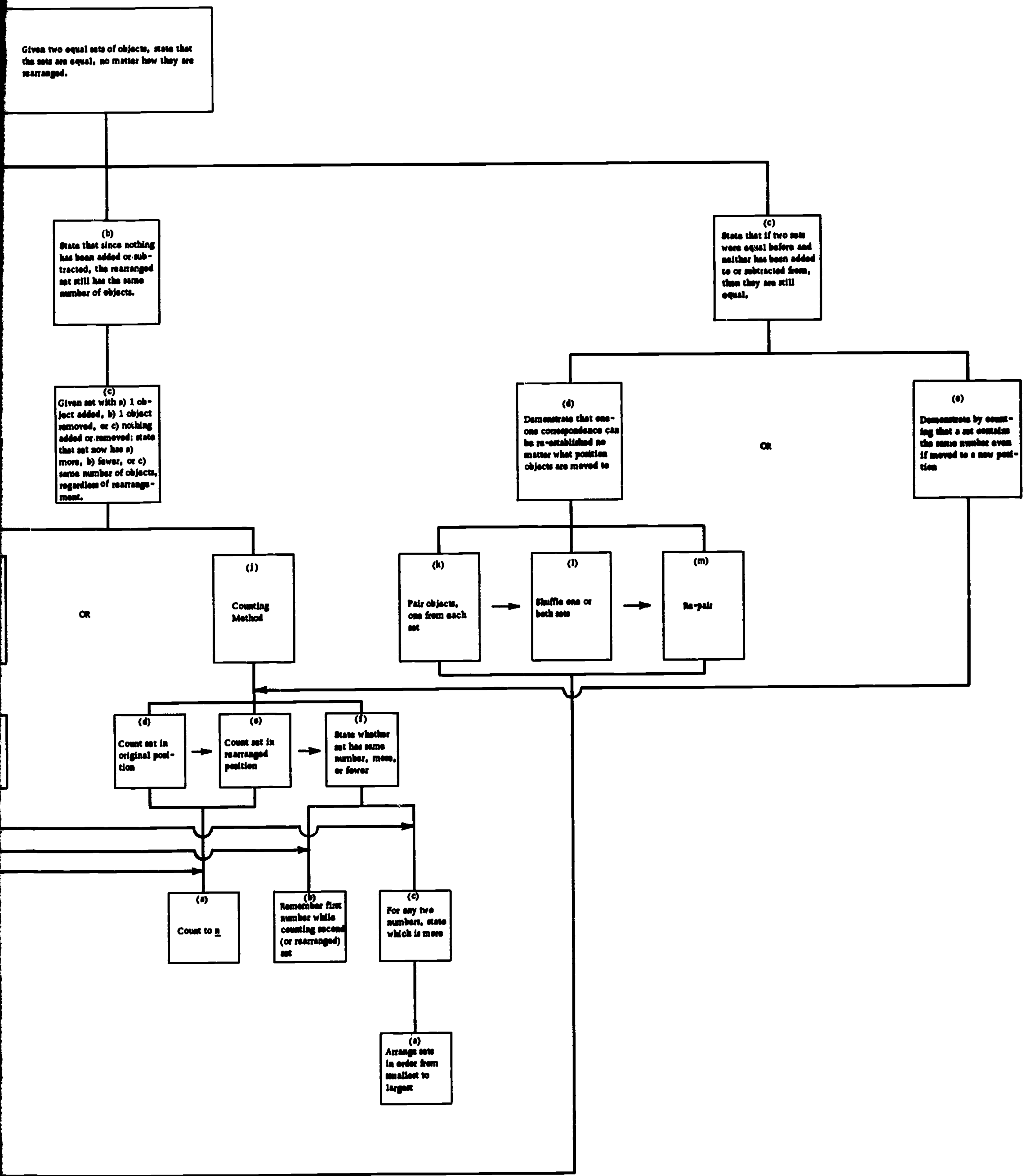
the second analysis requires the child to describe the kind of object that belongs in an empty cell of the matrix.

Since virtually any attributes can be used to define the rows and columns of a matrix of this kind, ability to form and use such matrices would provide the child with a powerful tool for organizing and using many kinds of information, and thus for learning from his environment. These are sophisticated skills which are not usually taught in a systematic way in school, but which are used in a variety of subject-matter domains. The component analyses identify prerequisite skills such as sorting, stating what groups of objects have in common, describing objects in terms of two characteristics, and identifying and naming objects in an array. In addition, they identify a set of skills enabling one to use a matrix or grid to locate positions in space. All of these prerequisite skills are appropriate for inclusion in an early learning curriculum.

A final example of component analysis, appearing in Figure 5, demonstrates how the technique can be applied to what is one of the most fundamental mathematical concepts according to current cognitive theory. The concept of "conservation" of number or quantity was first offered by Piaget (1965) and has since been the subject of extensive experimentation in the United States (e. g., Bruner and Olver, 1967). In a standard conservation problem the child is shown two rows of objects, each row containing the same number of objects, and arranged so that objects from the two rows are paired, as in Diagram A (p 12). The child is asked which row has more objects. At a very young age most children will say that the two rows are equal. One of the rows is then elongated (Diagram B) or contracted (Diagram C), but with no objects added or subtracted.

FIGURE 5

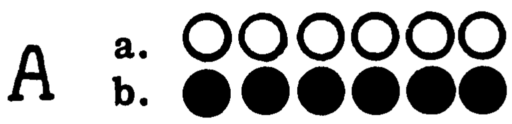

I
II
III
IV
V
VI
(a)
Establish initial equality of sets
(a)
One-to-one correspondence method
OR
(b)
Counting method
(a)
Pair objects, one from each set
(b)
Decide if any are left over
(c)
State rule: "If none left, sets are equal; if some left, sets are not equal"
(d)
State that sets are equal, or state which row has more
(e)
Count objects in each set
(f)
State whether two numbers are the same
(g)
State rule: "If numbers are same, sets are equal; if numbers are different, sets are not equal."
(h)
State that sets are equal, or state which set has more.
(i)
Conceptual Method
(a)
State rule: "'Equal' means the sets match with none left over."
(b)
State rule: "The row with objects left over has more."
(c)
State rule: "Adding one makes more; removing one makes less."



Given two equal sets of objects, state that the sets are equal, no matter how they are rearranged.
(b) State that since nothing has been added or subtracted, the rearranged set still has the same number of objects.
(c) State that if two sets were equal before and neither has been added to or subtracted from, then they are still equal.
(c) Given set with a) 1 object added, b) 1 object removed, or c) nothing added or removed; state that set now has a) more, b) fewer, or c) same number of objects, regardless of rearrangement.
(d) Demonstrate that one-one correspondence can be re-established no matter what position objects are moved to
OR
(e) Demonstrate by counting that a set contains the same number even if moved to a new position
OR
(j) Counting Method
(k) Pair objects, one from each set
(l) Shuffle one or both sets
(m) Re-pair
(d) Count set in original position
(e) Count set in rearranged position
(f) State whether set has same number, more, or fewer
(a) Count to n
(b) Remember first number while counting second (or rearranged) set
(c) For any two numbers, state which is more
(a) Arrange sets in order from smallest to largest

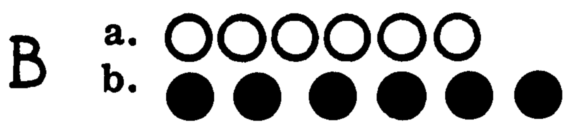
Most children below the age of six will now say that the longer row has more objects. When a child recognizes that the two rows still have the same number of objects, regardless of their physical arrangement, he is said to be able to conserve quantity.

Diagram A



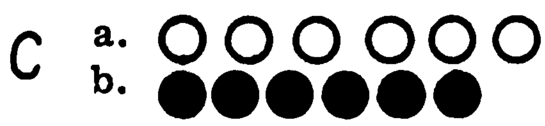

Child says row a and b have the same number.

Diagram B




Non-conserving child says row b has more. Conserving child says the rows still have the same number.

Diagram C




Non-conserving child says row a. has more. Conserving child says the rows still have the same number.

Most attempts to teach conservation have thus far failed. Examination of the training procedures in these studies, however, shows that they have usually been short-term efforts, and that each has concentrated on a single skill thought to be related to conservation. A detailed analysis of conservation behavior, as presented in Figure 3, reveals that there are several component behaviors, all of which must be well established before conservation can occur. Each of these behaviors can be taught, but each is relatively complex and will probably require extended periods of practice on the child's part. This analysis suggests, then, that a definitive test of whether conservation can be taught has yet to be made.

## Curriculum Sequences and Their Use in the Classroom

Component analyses identify prerequisite behaviors and systematically reveal the hierarchical relationships among learning objectives. On the basis of these analyses, formal curriculum sequences can be prepared in which objectives are ordered so as to insure that all prerequisites are met before instruction in any given objective is begun. These sequences specify the steps through which the child must pass. Tests or observational procedures can be designed for each step in the sequence, and teaching materials prepared for them. Thus, formal curriculum sequences are an effective means of translating the results of component analysis into a form that is more readily usable by teachers, who will rarely possess either the time or the training necessary for studying behavior analysis diagrams and deriving teaching sequences from them.

Examples of curriculum sequences derived from some of the component analyses presented in this paper appear in Appendices B, C and D. Appendix B contains two behaviorally independent sequences, one for sorting and classifying behaviors and one for locating positions with a matrix. Together these can be expected to lead to the ability to sort objects into the appropriate cells of a two-dimensional classification matrix, as analyzed in Figure 3. An alternative strategy for teaching the same skill is exemplified in Appendix C. This sequence is aimed at the objectives analyzed in both Figures 3 and 4. That is, the child learns not only to place items in the appropriate cells of a matrix, but also to state what items should appear in empty cells of the matrix. The authors of this sequence have chosen to combine the sorting and classifying skills with the positional matrix behaviors, thus creating a single curriculum sequence. This sequence, in slightly modified form, is currently being successfully used to teach five year olds in a tutorial setting. Appendix D contains a sequence derived directly from the component analysis of number conservation appearing in Figure 5.

A formal curriculum sequence has no necessary implications for the style of classroom organization. The sequence can be used, as in Bereiter and Engelmann's (1966) programs for the disadvantaged, in a manner that requires the participation of all children in all drills with little room for choice on the part of either child or teacher. However, it can be equally well used in a classroom whose organization is free and open as in the Montessori model or some modification of the progressive nursery school. In the latter situations, the child is free to move within the room, to choose his own activities within certain limitations, and to spend as long as he wishes at a given task. The curriculum sequence need not be evident to him as a constraint. However, the teacher can use the sequence to keep careful track of each child and how he is progressing through the hierarchy of skills.

Used in this way, a formal curriculum sequence can actually permit a greater degree of individualization than is possible without the detailed statement of objectives. This is because the sequence tells the teacher what observations to make in assessing children's learning and what constraints must apply in permitting or encouraging the child to progress to the next level. Thus, for example, a child who can already use grids for locational purposes, and identify common attributes of sets may be taught classification in a matrix format while another child who is still having difficulty categorizing in a single dimension will be given matching and sorting tasks and still another may be given training in simple spatial and locational skills. Put in another way, the detailed curriculum sequence permits the possibility of a "match" between the child's present conceptual structure and the kinds of lesson materials that are presented to him (cf Hunt, 1961, p. 267ff) on an individual rather than an age group basis.

The formal curriculum sequence also provides a mechanism by which current redefinitions of "readiness" for learning can be applied. Consider Bruner's formulation:

> The "curriculum revolution" has made it plain even after only a decade that the idea of "readiness" is a mischevious half-truth. It is a half-truth largely because it turns out that one teaches readiness or provides opportunities for its nurture, one does not simply wait for it. Readiness, in these terms consists of mastery of those simpler skills that permit one to reach higher skills. Readiness for Euclidian geometry can be gained by teaching intuitive geometry or by giving children an opportunity to build increasingly elaborate constructions with polygons. Or, to take the aim of the new "second-generation" mathematics projects, if you wish to teach the calculus in the eighth grade, then begin it in the first grade by teaching the kinds of ideas and skills necessary for its mastery later. (Bruner, 1966, p. 29)

The formal curriculum sequence, because it is based on a skill hierarchy, specifies the simpler or preparatory skills for each objective in the curriculum. Thus, it defines at each step, what <u>must</u> be taught when one "teaches readiness."

## Validation of Curriculum Sequences

Curriculum sequences based on rigorous component analysis are far more likely to work effectively with children than casually or intuitively devised sequences. Nevertheless, even these formally derived sequences can not be regarded as authoritative until there is sufficient empirical evidence for their validity. Since most component analyses -- as has been demonstrated in the case of the classification skills analyzed in this paper -- reveal not one, but several possible curriculum sequences, it is necessary to determine whether the sequence chosen is a workable one for the type of child in question. Furthermore, the component analysis iteself may be incomplete, having failed to identify all necessary component and prerequisitve behaviors. Thus, following detailed analysis and ordering of objectives, the next step in curriculum design is to arrange for an appropriate empirical validation of the curriculum sequence.

Validation of an analysis and the resulting curriculum sequence can sometimes be accomplished through a carefully controlled testing program. More often, however, validation requires actually designing and trying teaching materials and techniques, while carefully recording each individual's responses and dificulaties. This is a time-consuming but productive task, for it often results not only in a reordering of objectives, but also in the discovery of prerequisite or component behaviors that had not been identified earlier and which need to be included in the curriculum sequence. On the basis of tests and trials of this kind, revised curriculum sequences can be prepared and tested. This cycle -- of analysis, testing, reordering, and retesting -- lies at the heart of curriculum research. It is only through such research that we can effectively move from the psychological laboratory to the psychologically valid curriculum, from subject-matter expertise to effective education.

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# APPENDIX A

## SKILLS FOR INCLUSION IN AN EARLY LEARNING CURRICULUM

by

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# Table of Contents

## Appendix A

## 1. Orienting and Attending Skills

1.0 Attention span. The child can work at a given task for increasingly extended periods of time.

2.0 Focussing attention. The child can discover and attend to relevant aspects of a stimulus.

3.0 Task completion. The child normally completes a task before moving on to another one.

4.0 Persistence. The child can continue working at a task even in the face ot distraction or frustration.

5.0 Impulse control. The child can:
- 5.1 wait for instruction on how to proceed
- 5.2 wait for his turn to respond
- 5.3 refrain from handling materials that are not intended for his use

6.0 Delay and character of rewards.
- 6.1 The child will work for a reward that does not come immediately upon task completion.
- 6.2 The child will work for "abstract" rather than "concrete" rewards -- e.g., verbal praise, "points," etc.
- 6.3 The child is increasingly rewarded by the pleasure of doing and completing the task itself (i.e. by "intrinsic reinforcements").

7.0 Self-confidence.
- 7.1 The child will try a new task, even when there is risk of not succeeding.
- 7.2 A single failure does not cause the child to cease working at a task.

8.0 Direction-following. The child can follow instructions on how to perform a task (see Language Skills I and II).

9.0 Competition. The child can:
- 9.1 compete with others in simple games and find reward in "winning"
- 9.2 compete with himself, in the sense of trying to pass a previously set standard

10.0 Social skills. The child can:
- 10.1 work with other children on a task
- 10.2 ask for help from other children or teacher
- 10.3 give help when asked

## 2. Gross Motor Skills

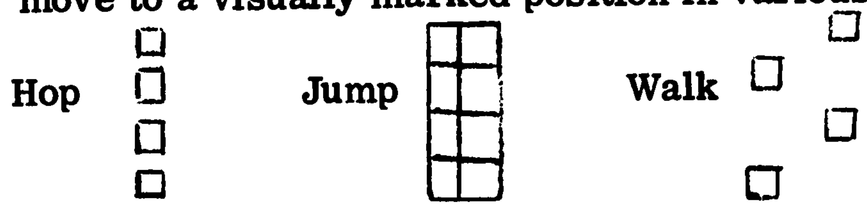
These are the skills that indicate a child's ability to use his body efficiently and move comfortably in his environment.

1.0 Basic forms of movement. The child can:
- 1.1 walk at various rates, at an even pace
- 1.2 jump, landing simultaneously on both feet
- 1.3 hop, on either foot
- 1.4 skip, gallop, run, etc.

2.0 Elaborated forms of movement. The child can walk, jump, hop, etc.
- 2.1 backward
- 2.2 sideways
- 2.3 at various rates
- 2.4 high and low
- 2.5 in one-half and quarter turns
- 2.6 etc.

3.0 Alternation of sides of body. The child can:
- 3.1 climb stairs, using alternation pattern, at an even pace
- 3.2 beat out simple rhythms alternately with right and left hands (or feet)

4.0 Directional movement. The child can:
- 4.1 move to a visually marked position in various patterns; e.g.,




- 4.2 move in a verbally described direction (e.g., "high," "low," "forward," "backward")

5.0 Body control.
- 5.1 Given a path defined by two rows of benches (later, lines on the floor), child can move along the path without touching sides.
- 5.2 Given ten pins set up with some space between them, child can walk through them without knocking them over.
- 5.3 Given an obstacle course involving climbing, crawling, and other movements, the child can:
  - 5.3.1 move through it, performing the same movements as a "leader"
  - 5.3.2 move through it following verbal directions

6.0 Balance.
- 6.1 The child can balance on his toes, on one or both feet.
- 6.2 While balancing on one foot, the child can raise or swing his other foot.
- 6.3 Given a balance beam, the child can move forward, backward, sideways, with eyes open or closed, while carrying things, etc.

Gross Motor Skills - Continued

7.0 Aim. The child can:

7.1 throw (roll, kick, bat, punch) a ball or beanbag to increasingly narrow visual targets in various positions in relation to his body

7.2 throw (etc.) ball to a verbally described position (e.g. "near," "far," "in front of you," "next to the desk," "under the table")

## 3. Positioning Skills

These are skills that develop the child's awareness of his own body and its position in space.

1.0 Parts of his own body. The child can:
- 1.1 move various parts of his body independently (or in combination) following the leader, and later on verbal command
- 1.2 when teacher points to part of body on a doll or picture, touch same part of his own body
- 1.3 touch various parts of his body on verbal command; later name them
- 1.4 adopt a pose from a model (or a picture)

2.0 Parts of the body - general. The child can:
- 2.1 assemble a person or face from cut-out pieces, and name parts
- 2.2 complete a drawing of a person or face

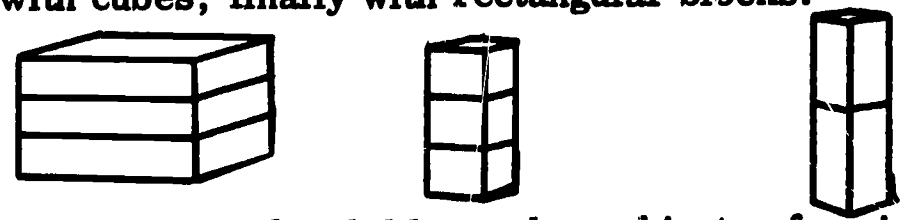
3.0 Right-left discrimination. The child can:
- 3.1 identify his own right and left hand and foot
- 3.2 turn to the right or left upon command
- 3.3 identify the right or left hand (foot) on another person, in various positions in relation to child
- 3.4 given two objects placed before him, identify the right-hand (left-hand) objects
- 3.5 given two objects placed before him, state which objects <u>would</u> be on right (left) if he were on the other side

4.0 Path-following.
- 4.1 Given a path with various branches, drawn on the floor, and with visual markers at certain points, the child can:
  - 4.1.1 move along the path to a given marker (visually presented), or past a series of markers
  - 4.1.2 move along the path to a point specified verbally
- 4.2 Given a two-dimensional model or a map with pathways marked, the child can trace a path with finger, to a visual marker or to place specified verbally.
- 4.3 Given a matrix drawn on the floor, the child can walk across rows, up and down columns, etc., according to visual patterns and verbal instructions.

## 4. Fine Motor Skills

These are the manipulative skills required for good performance in other parts of the curriculum.

1.0 Stacking objects. The child can build a tower, first with flat blocks, then with cubes, finally with rectangular blocks.



2.0 Placing objects. The child can place objects of various shapes into correspondingly shaped holes, and later onto drawings of the same size and shape as the object.

3.0 Alignment. The child can:
- 3.1 line up blocks or cards in "trains "
- 3.2 place two rods so that marks on each of them are lined up
- 3.3 place a rod so that one end is at a mark on a piece of paper
- 3.4 set a dial to a premarked position

4.0 Fasteners and locks. The child can operate:
- 4.1 buttons
- 4.2 zippers
- 4.3 snaps
- 4.4 key locks
- 4.5 combination locks (simplified)
- 4.6 door knobs
- 4.7 screw-top jars
- 4.8 etc.

5.0 Tools. The child can use:
- 5.1 a hammer, first on pegs, later on small nails
- 5.2 pliers
- 5.3 a screwdriver
- 5.4 wrench
- 5.5 nuts and bolts
- 5.6 etc.

6.0 Pouring. The child can pour rice (or other gross material) then sand, then water
- 6.1 from and into variously shaped containers
- 6.2 up to a marked line

7.0 Drawing and writing. The child can:
- 7.1 hold and use a crayon, then a pencil, comfortably
- 7.2 draw a line,staying inside the boundaries of a path
- 7.3 trace, using stencil cutouts

Fine Motor Skills - Continued

7.4 trace a drawing
7.5 copy simple patterns or letters
7.6 fill in between lines

8.0 Threading. The child can:
8.1 thread a rigid object onto a rigid pole
8.2 thread an object with two or more holes onto two or more appropriately placed poles
8.3 thread a rigid object onto flexible wire
8.4 string beads
8.5 complete a punched sewing card, alternating direction (i.e., start from above, then from below, etc.)
8.6 lace shoes

9.0 Hand co-ordination. The child can:
9.1 use one hand to hold an object in place while the other works (as in hammering, drawing)
9.2 alternate use of hands in simple tasks
9.3 use hands independently or together as appropriate

## 5. Sensory Skills

These skills involve the sense of sight, hearing and touch in various combinations. They require the child to make the sensory discriminations that underlie virtually all conceptual functioning.

1.0 Visual Discrimination

- 1.1 Discrimination of shape; e.g., given an array of plane or solid figures, the child selects the one that matches a model. The array may be different in size, color and material from the model; and discriminations will become progressively finer as the child's skill develops.
- 1.2 Discrimination of color. Given an array of colored objects, the child selects one that exactly matches a sample.
- 1.3 Discrimination of size in one, two, and three dimensions. Given an array of sticks of various lengths, circles of various diameters, or solid geometric figures, the child selects the one that is the same size as a model. Eventually the child should be able to select the matching item with very little trial and error.
- 1.4 Discrimination of position and orientation. Given several objects arranged in a pattern, select another set of the same objects arranged in exactly the same position. Or, given a sample drawing plus examples of the same drawing in other positions (including "reversals" and "mirror images"), select the example that matches the sample.
- 1.5 Recognition of representations. Given an array of objects and an array of pictures of those objects, in various perspectives, child can match pictures with corresponding objects.

2.0 Auditory Discrimination

- 2.1 Discrimination of pitch. Given two tones, child can identify the higher (or lower) one. Or, given a tone, child can match it (e.g. by finding a tone bell or a step on a xylophone that is the same).
- 2.2 Discrimination of sound-intensity. Given two sounds, the child can state which is louder (or softer).
- 2.3 Discrimination among types (i.e. sources) of sounds. Given a sound, child can state what is producing the sound. Or, child can identify a sound that is the same as a sample. This can involve discrimination between different musical instruments, or between other sources of sound (e.g. table being moved, water running, pots being washed, etc.)
- 2.4 Discrimination of direction and distance of sounds. A pitch-pipe is blown or someone moves and the child, blindfolded, can tell in what part of room the sound was made.
- 2.5 Discrimination of duration of sound. Give two continuous sounds, presented simultaneously or successively, the child can state which one lasted longer.

Sensory Skills -Continued

2.6 Discrimination and production of rhythmic patterns. Child can match and copy rhythmic patterns of various kinds. Also, child can label different types of rhythmic patterns (fast-slow; regular-irregular; syncopation; waltz-time, etc.)

2.7 Discrimination and production of stress (beat) patterns in speech and in music; e.g., child can state which note is accented in a rhythmic pattern, or child can repeat a line of poetry accenting different words.

3.0 Tactile Discrimination

3.1 Discrimination of textures. Given an array of fabric samples, child pairs identical samples while blindfolded.

3.2 Discrimination of shape. Blindfolded, the child matches plane and solid geometric figures, or completes a simple inset puzzle.

3.3 Identification of objects on the basis of touch. Blindfolded, child names a number of objects in front of him, or moves about the room describing where he is.

3.4 Discrimination of temperature. Given two glasses of water of different temperatures, child states which is warmer, which colder.

4.0 Cross-modal Discrimination. Eg., given an array of objects which the child can touch but not see, and another arry which he can see but not touch, child matches objects.

## 6. Classification Skills

All individuals respond to objects and events as members of classes or categories of similar events. This is necessary merely to exist and maneuver in a complicated environment. When this classification can be performed and recorded systematically, a tool of enormous intellectual power has been acquired. The skills listed here are all involved in systematic classification behavior.

1.0 One-dimensional sorting, without noisy attributes. Given an array of objects which differ in only one attribute (e.g. color or function or texture, etc.) sort them into separate categories on the basis of that attribute.

2.0 One-dimensional sorting, with noisy attributes. Given an array of objects which differ in more than one attribute (e.g. shape, size and color), sort on the basis of any one of those attributes. Then resort the entire array on the basis of a different attribute with given and self-selected attributes.

3.0 Sorting to form conjuctive classes. Given an array of objects which differ in several attributes, sort on the basis of co-occurrence of two attributes at a time (e.g. small blue objects form class A; large red objects form class B, etc.)

4.0 Two dimensional sorting, with and without noisy attributes. Given an array of objects which differ in at least two dimensions; sort in a matrix format, using two dimensions, e.g. by color and shape.

5.0 Three-dimensional sorting, with and without noisy attributes. Given an array of objects which differ in at least three-dimensions; sort in a three-dimensional matrix format, using three dimensions (format would resemble three-dimensional tic tac toe game).

6.0 Using matrices to locate relevant examples. Having sorted in a two or three-dimensional matrix form, quickly pick out objects described.

7.0 Hierarchical sorting. Given an array of items which differ in several dimensions; sort on one dimension. Then take each class and sort it, separately from the other classes, on a second dimension.

8.0 Sorting to form disjunctive classes. Given an array of objects which differ in several attributes, sort on the basis of occurrence of one or the other of two attributes (e.g. if either blue or small, object belongs in class A; if either large or red, belongs in class B).

Classification Skills -Continued

9.0 Using verbal description to guide classification. Given a verbal description of a class or several classes, sort an array of objects into the described classes.

10.0 Giving verbal descriptions of classification systems. Given an array of objects sorted into several classes, describe the bases of classification.

11.0 Select dimensions for sorting. In each of the problems above, the child chooses the dimensions on which he will classify and then sorts or describes the system (as opposed to having the teacher set the categories or dimensions).

## 7. Skills in Using Examples

Examples are used in almost all teaching, as a means of defining concepts or as models for children to match or copy. Yet many children do not know how to learn from examples. The skills below are among those that will permit children to use examples effectively. They are, of course, closely related to classification skills.

1.0 Same-different discrimination. Given two objects or pictures, state whether they are identical or not identical.

2.0 Matching to sample, with no noisy attributes. Given a model (e.g., large red circle) select from an array another item that is exactly the same (large red circle).

3.0 Matching to sample, with noisy attributes. Given a model (e.g., large red circle) select from an array another item that is the same in one specified dimension, ignoring the other dimensions (e.g., small blue <u>circle</u>; or large green <u>circle</u>, etc.)

4.0 Oddity problem. Given an array of objects, select the one that is different from all the others; or different from a sample. This may be done with and without noisy attributes.

5.0 Compare and contrast. Given two non-identical examples, state how they are the same and how they are different.

6.0 Extracting common properties of an array. Given a set of non-identical examples, state what they all have in common.

7.0 Scanning for the basis of discrimination. Given two classes of examples, state how the two classes differ.

8.0 Selecting relevant examples for a particular problem. Given a problem (e.g., "Are all green items the same weight?") select from an array the examples that should be studied.

## 8. Spatial and Locational Skills

Building on awareness of his own body, these are skills which permit the child to visualize and describe spatial relations, and to use models and maps for planning and direction.

1.0 Locating points in the real environment (cf. Positioning Skills). The child can:
- 1.1 Describe a location by stating what objects are there; also, find a location so described
- 1.2 Describe a location by stating positional relation to other objects (near, above, below, left of, etc.); also, find location so described
- 1.3 Follow a described route, passing designated places in designated order; also, describe a route a person has followed
- 1.4 Name all points that would be passed in a given route (from <u>x</u> to <u>y</u>)

2.0 Using a three-dimensional model. The child can:
- 2.1 Perform each of the behaviors in 1.0 using a three-dimensional model of the space
- 2.2 Relate three-dimensional model of space to real space by:
  - 2.2.1 placing a doll at the point in the model where a real person is standing (or vice-versa)
  - 2.2.2 naming and pointing to objects in model that corresponds to real space
  - 2.2.3 arranging model furniture, etc. to correspond to real-space arrangement

3.0 Using a two dimensional map. The child can:
- 3.1 Perform each of the behaviors in 1.0 using a two-dimensional map
- 3.2 Relate two-dimensional drawing to three-dimensional model and to real space in the ways listed in 2.2

4.0 Perspective and distortion. The child can:
- 4.1 Estimate relative size and distance of two objects in various perspectives (e.g. both large, one close and one far; one large and one small, both far) etc
- 4.2 Given views of objects from various orientations (front, back, side, top, bottom, various angles, and distances, etc.) identify which are the same object, which are different (i.e. "object conservation" )

Spatial and Locational Skills - Continued

4.3 Discriminate mirror from direct images and reversal from non-reversal patterns
4.4 Predict distortion of objects in different types of mirrors and projections. Also, recognize objects so distorted
4.5 Locate the same point in maps of different scale or projection

## 9. Plan-Following and Pattern-Recognition

The ability to follow non-verbal plans and use recursive patterns of various kinds is critical in a technological society. We believe there are a number of generalizable skills which will permit children to use plans and patterns effectively in a variety of substantive areas. These skills are closely related to some of the spatial skills and also to the use of examples in learning.

1.0 Direct copying.
- 1.1 One-dimensional. E.g., given a string of beads, the child makes a string containing the same beads in the same order.
- 1.2 Two-dimensional. E.g., given a parquetry block layout, the child constructs the same layout, using a second set of blocks; or given a matrix with various objects in the cells, the child copies the matrix, using a second set of materials.
- 1.3 Three-dimensional. E.g., given a tinkertoy, or block, construction, child reproduces it, using a second set of materials.

2.0 Copying involving some "translation."
- 2.1 Change of scale. E.g., given a small-scale picture of a tile design, child constructs the design using large tiles.
- 2.2 Use of two-dimensional diagrams for constructing three-dimensional figures. E.g., given a picture of a three-dimensional construction, and the necessary materials, the child produces the appropriate construction.

3.0 Extending recursive patterns.
- 3.1 Visual patterns in one, two and three-dimensions. E.g., given a string of beads, a matrix array or a three-dimensional construction that repeats itself in two or more sections, the child can add one more section that duplicates the pattern.
- 3.2 Temporal patterns. E.g., given a recurrent pattern of flashing lights, child can predict which light will flash next.

Note: The patterns and constructions discussed here can be made up of discrete elements (e.g. beads, tiles, blocks) or continuous elements (e.g. line drawings, clay). To handle continuous constructions, the child must not only have the necessary motor control; he must also be capable of laying a grid over the model (either actually or in imagination) and then copying the squares in the grid.

## 10. Language Skills I. Functional Use of Language

The child must be able to use language for a number of different purposes. These purposes are listed here, while the specific linguistic content is considered in the next section.

1.0 Requesting and providing assistance. The child can ask for and respond to requests for:
- 1.1 Objects
- 1.2 Help in performing an action
- 1.3 Simple information (e.g., "What is this?" "How many are there?" "When do we go out?")

2.0 Giving and following directions.

3.0 Describing objects and events. The child can:
- 3.1 Name objects and identify objects named
- 3.2 Identify and describe objects in terms of:
  - 3.2.1 physical characteristics
  - 3.2.2 function
  - 3.2.3 location
- 3.3 Classify events and objects according to various criteria given orally
- 3.4 Narrate real or fictional events, in sequence

4.0 Expressing and describing feelings and emotions.

5.0 Discussing past events and plan for future events.

6.0 Mediating his own problem-solving activity. The child can:
- 6.1 Ask himself questions
- 6.2 State rules to himself
- 6.3 State a logical deduction to himself
- 6.4 See Problem-Solving Skills (p. 37)

7.0 Engaging in discussion. The child can:
- 7.1 Persuade someone to do something
- 7.2 Prove a point
- 7.3 Request or provide examples
- 7.4 Request and give clarification or definition
- 7.5 Request and give reasons for a statement

## 11. Language Skills II. Linguistic Accuracy

In this section the specific linguistic content of the child's language is considered.

1.0 Phonology. The child can:
- 1.1 produce the full range of English phonemes
- 1.2 discriminate words that differ in a single phoneme
- 1.3 echo words of various phonological characteristics

2.0 Vocabulary. The child can:
- 2.1 name or identify a variety of
  - 2.1.1 objects or classes (nouns)
  - 2.1.2 actions (verbs)
  - 2.1.3 qualities (adjectives and adverbs)
- 2.2 appropriately use and respond to relational terms, including
  - 2.2.1 time relation terms, mainly adverbs (e.g. so, before, after, during, etc.)
  - 2.2.2 space relation terms, mainly prepositions (e.g. up, down, inside, outside, left, right, etc.)
  - 2.2.3 logical relation terms (e.g. so, therefore, because, still, although, of, unless, both, either, not, etc.)
  - 2.2.4 tense aspects of verbs
  - 2.2.5 pronouns
  - 2.2.6 reflexives

3.0 The child can use and respond to:
- 3.1 Basic sentence patterns including
  - 3.1.1 active
  - 3.1.2 passive
  - 3.1.3 negative
  - 3.1.4 interrogative
- 3.2 Syntactic structures of increasing complexity. For example,
  - 3.2.1 Nominals
    - 3.2.1.1 Noun with preceding modifiers, usually adjectives (The two large red balls)
    - 3.2.1.2 Noun modified by prepositional phrase (the red ball in the box)
    - 3.2.1.3 Noun modified by relative clause (the children who visited yesterday)
    - 3.2.1.4 Noun modified by participal phrase (the birds sitting on the fence)
  - 3.2.2 Verbal constructions with various modifiers
  - 3.2.3 Participal phrases used in various ways
  - 3.2.4 Infinitive constructions
  - 3.2.5 Adverbials in various sentence positions

## 12. Memory Skills

Psychological studies of memory functions make it clear that there are certain behaviors that can increase the amount of material retained and the period of time during which it is retained.

1.0 Visual memory.
- 1.1 Child is shown picture or object, which is then removed. He selects matching picture from an array.
- 1.2 Child is shown an array of objects. While he isn't looking, one is removed. He states which object has been removed.
- 1.3 Child copies 1, 2, or 3 dimensions in a situation in which he cannot view model and his own work at the same time.

2.0 Auditory memory.
- 2.1 Child repeats a sequence of directions or events in a story.
- 2.2 Child repeats a sequence of words or numbers.
- 2.3 Child echoes a sentence accurately.
- 2.4 Child reproduces a pitch or other sound after a delay.

3.0 Coding skills for increasing memory span.
- 3.1 Given a number of objects to remember, the child groups them into categories, then recalls members of each category in sequence.
- 3.2 Given an object to remember, child names it and uses the name as an aid in recall.
- 3.3 Child learns or constructs a poem, rhyme, or other easily remembered mnemonic to aid in recall.

4.0 Strategies for memorizing.
- 4.1 Backward chaining. If a verbal or other sequence is to be remembered in order, learn the last two or three items first, then the next-to-last plus the last, and so forth until the entire sequence has been memorized.
- 4.2 Using rhythm and other aids. Recite material in a fixed rhythmic pattern, or set it to a tune, to increase the number of "cues" for recall.
- 4.3 Grouping for efficient memorization.
  - 4.3.1 Separate material to be memorized into several related classes.
  - 4.3.2 Identify items most likely to become confused with one another and pay extra attention to those subsets in memorizing.

## 13. Problem-Solving Skills

Listed here are some of the skills useful in solving problems of various kinds. Systematic instruction in skills of this kind, together with extensive practice in solving real problems, should result in improved problem-solving performance by children.

1.0 Strategies for searching for an object in space. For example,
- 1.1 searching an entire area systematically
- 1.2 keeping track of areas already searched
- 1.3 quick scanning over a general area
- 1.4 reducing possibilities

2.0 Strategies for identifying an object in an array. For example,
- 2.1 questioning to narrow possibilities
- 2.2 ranking possibilities according to probability
- 2.3 using "hints "

3.0 Strategies for finding materials. For example,
- 3.1 sorting objects by possible function
- 3.2 predicting possible difficulties in use of certain materials
- 3.3 thinking of unusual uses of things

4.0 Strategies for information gathering. For example,
- 4.1 formulating questions relevant to a problem
- 4.2 asking appropriate people
- 4.3 rejecting irrelevant information

5.0 Hypothesizing and predicting. For example,
- 5.1 predicting probable outcomes
- 5.2 stating what would be the case if X were true

6.0 Strategies for testing and verification. For example,
- 6.1 trying out a given solution on original problems
- 6.2 trying solution on a range of related problems

7.0 Strategies for analyzing problems. For example,
- 7.1 finding similar problems and deciding whether solutions to those would work for new problems
- 7.2 identifying elements of a problem and deciding which are solved and which need new solutions
- 7.3 using analogies

8.0 Strategies for analyzing solutions. For example,
- 8.1 generating several possible solutions
- 8.2 ranking alternatives where no clear test of correctness of solution is available
- 8.3 stating conditions under which a given solution could apply

## 14. Quantitative Skills

These are early mathematical skills exclusive of geometry and measurement, which appear separately.

1.0 Sets
- 1.1 Equivalence of sets. Given two sets of objects, the child can state whether the sets are of equal size.
- 1.2 Comparing and ordering sets.
  - 1.2.1 Given two unequal sets, the child can state which has more (less) objects.
  - 1.2.2 Given an array of sets of various sizes, the child can order them from smallest to largest, or vice versa.
- 1.3 The empty set. The child can identify the empty set as a set containing no objects.
- 1.4 Conservation of number. The child demonstrates that the number of objects in a set remains the same regardless of the spatial arrangement of the objects.
- 1.5 Union and partitioning of sets.
  - 1.5.1 Given a set, the child can partition it into two or more subsets and indicate the quantitative relations between the original set and the subsets.
  - 1.5.2 Given two or more sets, the child can combine them to form a larger set, and indicate the quantitative relations between the original sets and the combined set.
- 1.6 The rectangular array. Given a set of objects, the child can arrange them in a matrix format and demonstrate that the set is made up of _n_ subsets of _m_ objects.

2.0 Numbers and Numeration
- 2.1 Cardinal numbers. The child can state how many objects are in a set (including the empty set).
- 2.2 Ordinal numbers. Given an ordered set, the child can identify the 1st, 2nd, 3rd, etc. items.
- 2.3 Counting. The child can count to 10 and beyond by 1's, 2's, 3's, 5's, 10's, etc.
- 2.4 Numerals.
  - 2.4.1 The child can read and write one-digit and multi-digit numerals.
  - 2.4.2 The child can match numerals with sets of the appropriate number.
- 2.5 Base system.
  - 2.5.1 The child can interpret a 2 digit numeral as _n_ tens plus _n_ ones.
  - 2.5.2 Given a set of more than ten objects, the child can partition it into _n_ subsets of 10 plus _n_ subsets of 1.

Quantitative Skills - Continued

3.0 Operations of numbers
- 3.1 The child can perform addition and subtraction as a form of
  - 3.1.1 union (or partioning) of sets
  - 3.1.2 successive movements along a number line
- 3.2 The child can perform multiplication and division as a form of
  - 3.2.1 operation on a rectangular array
  - 3.2.2 successive addition or subtraction
- 3.3 Word problems. The child can solve verbally stated problems involving addition, subtraction, multiplication and division.

4.0 Equations and the concept of the equal sign. Given a statement in equation form (but not necessarily involving numerical quantities) the child can perform various operations (adding, subtracting, rearranging, etc.) and still maintain the equivalence of the two sides of the equation.

5.0 Fractions of sets
- 5.1 Given a set of objects, the child can divide it into halves, thirds, fourths, etc.
- 5.2 The child can read fractional numerals and match fractional numerals with the appropriate subsets of a set.
- 5.3 The child can order fractions from smallest to largest.
- 5.4 The child can demonstrate the equality of certain fractional expressions (e.g., $2/6 = 1/3$).

## 15. Geometric Skills

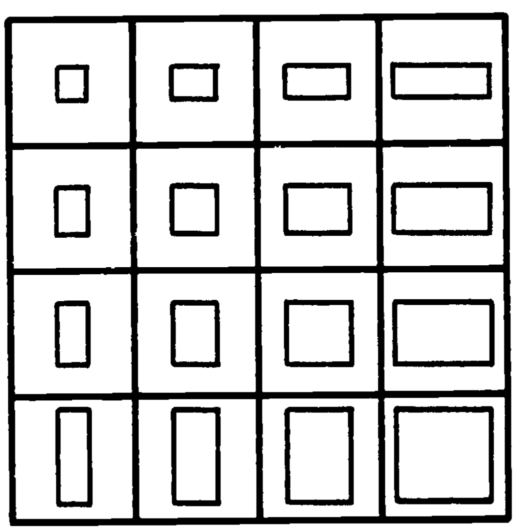
1.0 Shape recognition.
   1.1 Given an array of plane or solid geometric figures, child can name or identify all of the basic shapes.
   1.2 Child can identify straight and curved line segments.
   1.3 Child can identify open and closed curves.

2.0 Line relationships.
   2.1 Child can identify parallel, perpendicular and oblique lines.
   2.2 Child can identify right, acute and obtuse angles.

3.0 Construction and relationships of shapes.
   3.1 Given a set of plane geometric figures, child assembles them into a figure.
   3.2 The child cuts or draws lines to divide two-dimensional figure into other basic figures.
   3.3 The child can construct other basic figures (e.g., a cube, or other three-dimensional form) out of smaller forms.

4.0 Congruence of geometric forms.
   4.1 Child can superimpose one figure on another to determine whether they are the same size.
   4.2 Child can reconstruct one figure to permit direct comparison with another.
   4.3 Finally, child can use standard units to determine equality of area or volume (see measurement skills).

5.0 Seriation of forms in two (later, three) dimensions simultaneously. E.g., given an array of rectangles, child can construct a length-by-width matrix, as in the following diagram:



Geometric Skills _Continued

6.0 Conservation of area and volume. Given a wide, short, figure, child can determine whether it has the same area as a narrow, tall one. (Note: This assumes seriation in two or more dimensions and comparison of figures for equality.)

7.0 Fractions of wholes.

7.1 Given a plane (later solid) geometric figure, the child identifies 1/2, 1/3, 1/4, etc. of it.

7.2 Given a figure with some part shaded, the child states what fractional part has been identified.

8.0 Geometric visualization. E.g.,

8.1 Child predicts (before trying) which geometric figures can be constructed from a set of parts.

8.2 Child predicts which figures will exactly fit an outline drawing.

8.3 Child predicts whether lines will touch when extended.

8.4 Child can identify and construct symmetrical and asymmetrical designs.

## 16. Measurement Skills

These skills are intimately involved in both mathematics and science study, but are of sufficient general importance to warrant separate consideration.

1.0 Direct comparison. E.g., given two rods, child holds them together to decide whether they are equal or which is longer. Direct comparison can be performed in 1- 2- or 3-dimensions. Weight, temperature, duration, hue intensity, etc. can all be compared in addition to length, area and other spatial dimensions.

2.0 Mediated comparison. E.g., given two lines which the child must compare for length and which he cannot superimpose, child finds a rod exactly equal in length to one of the lines, then compares the rod with second line. Again, many qualities can be compared in this way.

3.0 Mediated comparison where only a part of the measuring stick is used. E.g., given two lines to compare, the child finds a rod longer than line A, makes a mark on the rod to indicate the length of A, then uses the marked rod for comparison with line B.

4.0 Mediated comparison using standard units. E.g., given two lines to compare, child uses a rod considerably shorter than line A and steps it along, counting the number of steps he makes. He then steps the same rod along line B, and compares the number of steps.

# APPENDIX B

# CURRICULUM SEQUENCE FOR TEACHING CLASSIFICATION IN A MATRIX FORMAT. I

by

Lauren B. Resnick, University of Pittsburgh

## Terminal Objective

| Given: | The child can: |
|---|---|
| A two-dimensional matrix with identity cells filled, and an array of objects | Place each object in the appropriate cell. |

N.B. Two independent sequences are given here. These can be taught simultaneously. Together, they establish the prerequisites for the "terminal behavior."

### A. Teaching Sequence: Sorting and Classifying Behaviors

| Given: | The child can: |
|---|---|
| 1.0 An array of objects made up of several sets of identical objects and a row (or column) of sorting boxes. | 1.0 Sort, putting identical objects into the same box. |
| 2.0 A set of identical objects placed in a row (column) of boxes, one to each box. | 2.0 State what the boxes have in common, in the form, "These are all ____," or, "These all have _____." |
| 3.0 Several rows (columns) of identical objects, each row containing a different object. | 3.0 State what each row (column) has in common. |
| 4.0 An array of objects differing in two or more attributes (e.g., size and shape, color and number) and a row (column) of sorting boxes. | 4.0 Sort, using <u>one</u> of the attributes as the basis for establishing classes and ignoring the others. |
| 5.0 A set of objects identical in one attribute, but varying in other attributes (e.g. circles of several colors and sizes), arranged in a row (column). | 5.0 State what the set has in common. |
| 6.0 Several rows (columns) of objects, each row with a common attribute. | 6.0 State what each row (column) has in common. |
| 7.0 A filled two-dimensional matrix. | 7.0 State what each row (column) has in common. |
| 8.0 A two-dimensional matrix with identity cells filled, and an array of objects. | 8.0 For each object in the array, state which row it belongs in and which column it belongs in. |

## B. Positional Matrix Behaviors

1.0 A simple straight path drawn on paper.

1.0 Draw a line along the path from beginning to end.

2.0 A single straight path, but with squares marked off, and each square differently "labelled" (i.e., by color, a drawing, etc.)

2.1 Draw a line along the path from beginning to end.

2.2 Draw a line along the path from any cell to any other.

3.0 Several straight paths, with cells as in 2.0, arranged in parallel rows or columns.

3.1 Draw a line along any one of the paths from beginning to end.

3.2 Draw a line along any one of the paths from any cell to any other.

4.0 A single straight path with cells marked off each containing a picture or object.

4.0 Name the objects in the cells, in order, from left to right or top to bottom.

5.0 Several straight paths with cells, each containing a picture, arranged in parallel rows or columns.

5.0 Name the objects in any row or column, in order.

6.0 A straight path with several other paths crossing it, each path a different color, and at each intersection a different picture.

6.1 Draw a line along the main path to the point where the green (e.g.) path crosses it.

6.2 Name the object at the point where the green path crosses the main path.

| | |
|---|---|
| 7.0 A matrix of paths, each path a different color, and at each intersection a different picture. | 7.0 Start at the beginning of any row (column) and draw a line to the point where it intersects any column (row). |
| 8.0 A matrix of paths, each path a different color, and at each intersection a different picture. | 8.0 Name the object at the point where any row and column intersect. |

# APPENDIX C

# CURRICULUM SEQUENCE FOR TEACHING CLASSIFICATION IN A MATRIX FORMAT. II

by

Esther Kresh, Pittsburgh Public Schools

Alexander Siegel, University of Pittsburgh

Sheila Levine, University of Pittsburgh

## Terminal Objective

| Given: | The child can: |
|---|---|
| A two dimensional matrix with identity cells filled in. | 1. Place objects from an array in the appropriate cells. |
| | 2. Describe object that belongs in any given cell. |

## Teaching Sequence

| Given: | The child can: |
|---|---|
| 1.0 An array containing three different objects, several of each. | 1.0 Sort the objects into three boxes, arranged in a row or column. |
| 2.0 A one-dimensional matrix with an array of objects sorted into the identity boxes. | 2.0 Place each object from the identity boxes into a separate box in the row (column). |
| 3.0 A one-dimensional matrix with the objects placed in the rows (or columns). | 3.0 Name the appropriate object when asked, "What is in every box in this row (column)?" |
| 4.0 A one-dimensional matrix with objects placed in each identity cell and a separate array of objects identical to those in the identity cells. | 4.0 Name the object that belongs in each row (column) and place objects correctly. |
| 5.0 A matrix with two arrays of objects sorted into identity boxes on the x and y axes. | 5.0 Place each object from the row identity boxes into a separate box in the row, and each object from the column identity boxes in a separate box in the column (so that each box in the matrix finally has two objects in it). |
| 6.0 A two-dimensional matrix with the interior cells each filled with two objects, and the identity cells empty. | 6.0 Name the object common to any column or row and construct the identity cells. |
| 7.0 A two dimensional matrix with objects in the identity cells, and the interior cells empty. | 7.0 Name the two objects that belong in any given cell or identify pictures of the two objects for any given cell. |

| | |
|---|---|
| 8.0 A two dimensional matrix with objects in the identity cells and an array of cards containing pictures of the two objects which would belong in each cell. | 8.0 Place each card in the appropriate cell. |
| 9.0 A sample of a colored object to be matched, plus two sets of choice items. | |
| 9.1 same objects, varying in color | 9.1 Select object that is same color as the sample. |
| 9.2 different objects, all the same color. | 9.2 Select same object as sample, ignoring color. |
| 10.0 A two-dimensional matrix, the identity cells empty, and the interior cells filled with objects in different colors. | 10.0 Name the object or color common to each row or column and construct the identity cell. |
| 11.0 A two dimensional matrix with objects in the identity cells of one axis and color swatches in the identity cells of the other axis; and an array of colored objects. | 11.0 Identify the appropriate row and column for each object and place it in the appropriate cell. |
| 12.0 A two dimensional matrix with objects placed in the identity cells of one axis and color swatches in the identity cells of the other axis. | 12.0 Name the color and object that belong in any given cell. |
| 13.0 A sample of an object with two attributes and two sets of choice items | |
| 13.1 Same in attribute a, varying in attribute b | 13.1 Select object that matches sample in attribute b. |
| 13.2 Same in attribute b, varying in attribute a | 13.2 Select object that matches sample in attribute a. |
| 14.0 Conditions of 10.0, 11.0, and 12.0, but using objects varying in two attributes (e.g. size and shape, color and size, etc.) | 14.0 Perform behaviors 10.0, 11.0, and 12.0 |

# APPENDIX D

# CURRICULUM SEQUENCE FOR NUMBER CONSERVATION

by

Jerome D. Kaplan, Teachers College, Columbia University

Lauren B. Resnick, University of Pittsburgh

## Terminal Objective

| Given: | The child can: |
|---|---|
| Two equal sets of chips | State that the sets have the same number, regardless of how they are rearranged or how much space they take. |

N.B. The sequence below contains the following subsets of objectives: a) establishing equality and inequality of sets by the "matching" or one-to-one correspondence method (nos. 1-5); b) demonstrating the retention of equality by showing that rearranged sets can be repaired (nos. 6-8); c) defining "more," "less," and "same" as, respectively, adding something, removing something, and neither adding nor removing, that is by the "conceptual" method (nos. 9-11); and d) several versions of the terminal behavior (nos. 12-14).

## Teaching Sequence

| Given: | The Child can: |
|---|---|
| 1.0 Two sets of chips arranged in parrallel rows, and paired in the following types of arrangements: | |
| 1.1 ○ ○ ○ ○ ○<br>● ● ● ● ● | 1.1 Say that both rows have the same number of chips. |
| 1.2 ○ ○ ○ ○ ○<br>● ● ● ● | 1.2 Say that the top row has more chips. |
| 1.3 ○ ○ ○<br>● ● ● ● ● | 1.3 Say that the bottom row has more chips. |
| 2.0 Two sets of various kinds of objects, arranged as in 1.0. | 2.0 Respond as in 1.0. |
| 3.0 A set of chips (objects) arranged in a row, and a second set of chips (objects) arranged in a pile. | 3.0 Pair one chip (object) from the second set with each chip (object) of the first set and state which set has more (or state that the two sets are equal). |
| 4.0 Two piles of chips (objects). | 4.0 Pair the chips, one from each set, in parallel rows and then state which set has more (or state that the two sets are equal). |

Given:

5.0 Given two beakers of the same size and shape and given an even number of beads (rods, cubes, etc.)

The child can:

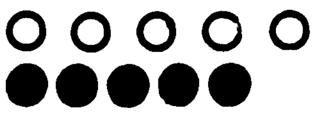
5.1 Establish equal sets by picking up two beads at a time, one in each hand, and placing one in each beaker.

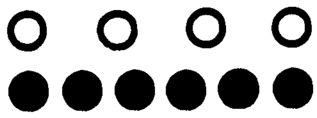
5.2 Then state that the two beakers have the same number of beads because "I counted them out," or "I put one in each," or "I made them equal" or equivalent reason.

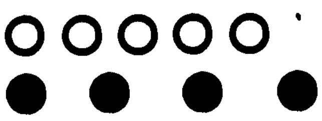
6.0 Two equal sets of chips (objects) arranged in parallel rows and paired; and then given one of the sets re-arranged into a pile.

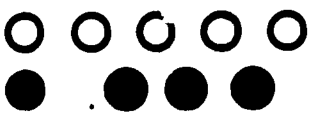
6.0 Place the second set back into their original (paired) positions, and then state that the sets are still equal.

7.0 Two sets of chips (objects)in the following kinds of arrangements:

7.1 

7.2 

7.3 

7.4 

7.0 Pair the chips (objects) and state that:

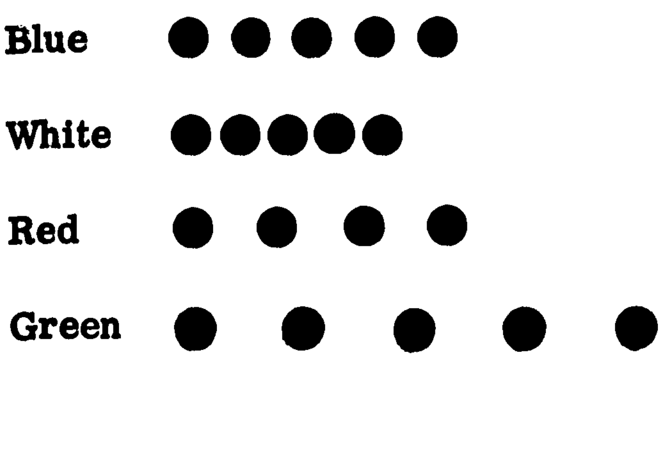
7.1 The sets have the same number.

7.2 The bottom set has more.

7.3 The top set has more.

7.4 The top set has more.

8.0 A row of blue chips and several rows of other chips, arranged as follows:




8.1 Pair each other set with the blue, if necessary, and

8.2 State that the white and green rows have the same number of chips as the blue.

| Given: | The child can: |
|---|---|
| 9.0 A set of chips, arranged in a row, to which teacher: | |
| 9.1 adds a chip, with child watching | 9.1 State that there are now more chips. |
| 9.2 removes a chip, with child watching | 9.2 State that there are now fewer (less) chips. |
| 9.3 neither adds nor removes a chip. | 9.3 State that there are the same number of chips as before. |
| 10.0 A set of chips in a pile, to which teacher: | 10.0 Respond as in 9.0 |
| 10.1 adds a chip, with child watching | |
| 10.2 removes a chip, with child watching | |
| 10.3 neither adds nor removes a chip. | |
| 11.0 A set of objects arranged in a row, teacher rearranges into a pile (or circle), with child watching, and | 11.0 Respond as in 9.0 |
| 11.1 adds an object | |
| 11.2 removes an object | |
| 11.3 neither adds nor removes an object. | |
| 12.0 An equal number of eggcups and eggs (or cups and saucers, red and blue chips) | |
| 12.1 arranged so that the eggs and eggcups are paired | 12.1 State that there are the same number of eggs and eggcups. |
| 12.2 the eggs then bunched in a pile | 12.2 State that there are the same number without re-pairing because "none were added or taken away," "I could put them back in the eggcups," or equivalent reason. |

| Given: | The child can: |
| --- | --- |
| 13.0 Two equal sets of chips (rods, beads) | |
| 13.1 arranged in parallel rows of the same length (i.e. paired) | 13.1 State that the two sets have the same number of objects. |
| 13.2 then, one of the two rows is elongated (1 1/2 times other row) | 13.2 State that the two sets still have the same number, without re-pairing, and state reason (cf. 12.2) |
| 14.0 Two beakers, one tall and thin, the other short and wide, and an even number of beads (rods, cubes, etc.) | |
| | 14.1 Establish equal sets by picking up two beads at a time, one in each hand, and placing one in each beaker, |
| | 14.2 then state that the two beakers have the same number of beads and state "why" or "how he knows" (cf. objective 5.0) |