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QUALIFICATIONS FOR A COLLEGE FACULTY IN MATHEMATICS.

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IN THIS REPORT BY THE COMMITTEE ON THE UNDERGRADUATE PROGRAM IN MATHEMATICS (CUPM) ATTENTION IS FOCUSED ON THE TRAINING AND QUALIFICATIONS OF TEACHERS NEEDED IN OUR COLLEGES AND UNIVERSITIES IN ORDER TO EFFECT THE REQUIRED CHANGES IN THE UNDERGRADUATE CURRICULUM NECESSITATED BY THE CURRICULUM REFORMS IN MATHEMATICS AT THE ELEMENTARY AND SECONDARY LEVELS DURING THE PAST DECADE. THE PRINCIPAL PURPOSE OF THIS REPORT IS TO SET FORTH APPROPRIATE QUALIFICATIONS FOR TEACHERS RESPONSIBLE FOR COURSES RECOMMENDED BY CUPM IN ITS REPORT "A GENERAL CURRICULUM IN MATHEMATICS FOR COLLEGES" (GCMC) IN TERMS OF A TEACHER'S OWN ACADEMIC BACKGROUND. DESIRED PREPARATION IN MATHEMATICS INCLUDES A STRONG UNDERGRADUATE MATHEMATICS MAJOR, LOWER LEVEL GRADUATE COURSES IN MATHEMATICS, AND ADVANCED GRADUATE COURSES IN MATHEMATICS. SUPERVISED TEACHING AT THE COLLEGE LEVEL ALSO IS RECOMMENDED. THIS DOCUMENT IS ALSO AVAILABLE WITHOUT CHARGE FROM CUPM CENTRAL OFFICE, P. O. BOX 1024, BERKELEY, CALIFORNIA 94701. (RP)

COMMITTEE ON THE UNDERGRADUATE
PROGRAM IN MATHEMATICS

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FOR A

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IN

MATHEMATICS

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OFFICE OF EDUCATION

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QUALIFICATIONS FOR A COLLEGE FACULTY IN MATHEMATICS

Report of the
Ad Hoc Committee on the Qualifications of
College Teachers

COMMITTEE ON THE UNDERGRADUATE PROGRAM
IN MATHEMATICS
Mathematical Association of America

January 1967

The Committee on the Undergraduate Program in Mathematics is a committee of the Mathematical Association of America charged with making recommendations for the improvement of college and university mathematics curricula at all levels and in all educational areas. Financial support for CUPM has been provided by the National Science Foundation.

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BACKGROUND

The Committee on the Undergraduate Program in Mathematics of the Mathematical Association of America has directed its attention for several years to the course material in mathematics that should be taught in colleges and universities. Curricular recommendations have been published for undergraduate mathematics programs in the following areas: applied mathematics; the biological, management, and social sciences; computer science; and engineering and physics. CUPM has also published recommendations for preparation of elementary and secondary school teachers, and for preparation for graduate study in mathematics.* Following the publication of these reports, which were devoted to the mathematical content of various specific curricula, CUPM addressed its attention to the problem of constructing a general mathematics program, simple enough to be within the means of small colleges having limited staffs, yet flexible enough to fit the requirements of each of the major special programs discussed in the earlier reports. The resulting report, *A General Curriculum in Mathematics for Colleges,* referred to as the GCMC report, was published in 1965.

The curriculum reforms in mathematics at the elementary and secondary school levels during the past decade have necessitated immediate programs of support to provide the quality of teaching needed in those schools. Today we are beginning to notice many changes in college mathematics courses stemming from the CUPM recommendations. Hence it is now time to focus attention on the training and qualifications of teachers needed in our colleges and universities in order to effect the required changes in the undergraduate curriculum.

The recent dramatic growth of mathematical research activity, combined with the growing demands of industry and government for people with mathematical training, has created a severe shortage of mathematics teachers who have doctoral degrees. The rapidly increasing mathematics enrollments within a growing college population, and the expansion of areas of application of mathematics, have

* A list of CUPM curricular reports will be found on page 16.

left many college mathematics departments seriously understaffed, greatly overworked, and quite unprepared to initiate urgently required modifications of their course offerings. It is imperative that decisions for curriculum changes, as well as for the other critical problems facing mathematics departments, be made and carried out by people with the highest possible mathematical qualifications.

The simple traditional requirement of many colleges, and of some junior colleges, that new appointments to the mathematics faculty be awarded only to people with a Ph.D. degree is, at the present time, quite unrealistic. Recipients of new Ph.D.'s in mathematics are simply not available in the required numbers. For example, in 1964-1965 barely more than one quarter of the new full-time mathematics teachers employed by four year colleges had Ph.D.'s. The shortage is likely to continue, and junior colleges and four year colleges will, of necessity, continue to use teachers whose academic preparation is intermediate between the bachelor's degree and the doctor's degree.

Our principal goal in this report is to set forth appropriate qualifications for teaching the courses recommended by CUPM in its report *A General Curriculum in Mathematics for Colleges* (GCMC) in terms of a teacher's own academic background. As a further task, we consider the distribution of training within a mathematics faculty today which makes it possible for the department to teach effectively the program recommended in the GCMC Report.

It should be understood that no academic program or degree in itself qualifies an individual to teach effectively at any level unless this preparation is accompanied by a genuine interest in teaching and by professional activities reflecting continuing mathematical growth. These activities may assume the form of several of the following:

(a) taking additional course work,

(b) reading and studying to keep aware of new developments and to explore new fields,

(c) engaging in research for new mathematical results (even when unpublished),

(d) developing new courses and new ways of teaching,

(e) publishing expository or research articles,

(f) participating in the activities of professional mathematical organizations.

The preceding list reflects our conviction that an effective teacher must maintain an active interest in the communication of ideas and have a dedication to studying, learning, and understanding mathematics at levels significantly beyond those at which he is teaching.

A college mathematics department, whose staff members are engaged in activities such as those described above and have the academic qualifications to be described below, should have confidence in its ability to provide the quality of teaching required of it.

THE TRADITIONAL ROLE OF THE Ph.D. DEGREE

Colleges and universities have come to place considerable emphasis on the doctor's degree as a necessary requirement for college teaching. This emphasis is quite understandable, since the Ph.D. is the most advanced degree offered by American universities and is therefore a symbol of maximal academic achievement. Unfortunately, the relevance of the doctoral degree in the qualification of a college teacher is often misunderstood, and the resulting confusion has, in many cases, led to serious abuses. We have in mind such abuses as the preferential treatment frequently assured the holder of a doctoral degree over an otherwise well qualified teacher who lacks a Ph.D.; or unrealistic emphasis at some institutions on the number of doctoral degrees, regardless of origin, held by members of the faculty. We shall examine the requirements for a Ph.D. in mathematics in order to analyze the relevance of each of them in evaluating the qualifications of a college teacher.

The Ph.D. in mathematics is by long tradition a research degree —and research in mathematics has meant the creation of new mathematics and not, as in many fields, the scholarly analysis or synthesis of previous work. A mathematics student working toward a Ph.D. is expected to spend a considerable portion of his time, in the later undergraduate and early graduate years, acquiring a broad general background in mathematics. The breadth of his knowledge is usually tested by special examinations after one, two, or more years of graduate study. After these examinations, the student's work becomes highly specialized with seminars, independent study, and thesis work penetrating in depth some area of particular interest.

The earlier years of graduate study provide a breadth of knowledge essential to a college teacher. The subsequent, very specialized, graduate study is equally essential for research work.

An institution that has, or that aspires to have, a legitimate graduate program must necessarily have a substantial number of research

mathematicians on its faculty, to implement the program and to provide the necessary leadership. This is a condition which obviously should not be changed. Even in an undergraduate college which does not offer a graduate program in mathematics, there are good reasons for wanting faculty members to have the kind of preparation required for the Ph.D. Both the nature and the content of undergraduate courses in mathematics must undergo frequent revision to reflect the rapid developments in mathematics and in related fields. The competent teacher of undergraduate mathematics must be able to master new material independently, to prepare new courses involving material which he never studied in his own course work, and frequently he must guide independent study by gifted undergraduates. These challenges call for a degree of mathematical maturity which comes only with extended effort. A confident approach to new material is made possible not alone by the amount of knowledge a teacher may have; it requires in addition a broad understanding and a deep appreciation of the nature of mathematics. A significant research experience such as that demanded for the Ph.D. dissertation is perhaps the best guarantee that a person actually has the kind of maturity we have in mind. The research work itself may not provide the prospective teacher with the necessary breadth of knowledge, but it provides him with maturity which should enable him to continue his mathematical education independently and indefinitely.

THE FORMAL EDUCATION OF COLLEGE TEACHERS OF MATHEMATICS

There are a number of levels of mathematical preparation which are appropriate for teaching the various courses described in the GCMC report *A General Curriculum in Mathematics for Colleges.* In discussing these levels we shall begin with a prospective college teacher's undergraduate program and then indicate the teaching responsibilities compatible with successive components of his additional mathematical education.

A. STRONG UNDERGRADUATE MATHEMATICS MAJOR PROGRAM

Mathematics major programs differ widely from one institution to another. For present purposes, we shall refer to a major program based on courses described in the GCMC report.* This report suggests that a mathematics major program for students preparing for graduate work in mathematics should include the lower division analysis courses 1, 2, 4, 5, the lower division probability course 2P, the lower division linear algebra course 3, and the upper division courses in algebra (6), analysis (11, 12, 13), and applied mathematics (10). The report adds that, where possible, a stronger major is desirable, with options to be selected from the courses in probability and statistics (7), numerical analysis (8), and differential geometry (9). The CUPM Panel on Pregraduate Training, in reviewing these recommendations in their report *Preparation for Graduate Study in Mathematics,* observed that most graduate departments desire an incoming student to be especially well grounded in algebra and analysis. Consequently they recommended, and described in outline, a year course in algebra to replace course 6, as well as the content for the year course in real analysis (11, 12) which they considered essential to preparation for graduate study.

* These courses will be cited below using the numbers given them in the report *A General Curriculum in Mathematics for Colleges*; all of them are semester courses.

We do not favor special undergraduate curricula for prospective college teachers. Instead, we recommend a *strong mathematics major program* which begins with the mathematics major as described in GCMC and includes the analysis courses 11, 12 outlined by the Pregraduate Panel, the additional work in algebra recommended by the Pregraduate Panel, and two additional courses selected from probability and statistics (7), numerical analysis (8), and differential geometry (9). We firmly believe that applications should be presented in all mathematics courses and that, where possible, students also should have some courses in fields where mathematics is applied (for example, theoretical physics or mathematical economics).

While the strong mathematics major program which we have described is certainly desirable for a college teacher, one must expect and encourage wide variation in the undergraduate programs which students actually encounter. Indeed it is to be expected that *strongly motivated research oriented students will be advised to proceed to graduate work without some of the undergraduate courses we have listed.* There are institutions where this strong major will be completed by many students at the time they receive the bachelor's degree. On the other hand, some students, including many in other disciplines or in training programs for secondary school teachers, will not encounter some of the more advanced upper division courses until they reach graduate school.

Graduate students who have completed a strong undergraduate mathematics major program with distinction and who have a definite interest in teaching are qualified to assist more mature teachers in teaching elementary courses at the college level. Completion of this strong mathematics major should not be considered permanent qualification for a teacher of even the most elementary college courses. As we pointed out, continued intellectual growth is an essential qualification for sustained competence as a teacher. (In junior colleges, or at other institutions where remedial mathematics courses are offered, there could be some justification for outstanding teachers with training equivalent to that of a strong mathematics major being members of the faculty and responsible for these courses.)

B. FIRST GRADUATE COMPONENT

In this section we describe the additional graduate work which a prospective college teacher, who has completed the strong major program, will need in order to acquire the mathematical background necessary to teach the lower division curriculum of GCMC (and hence the mathematics courses for junior college students who plan to transfer to a college or university). Those who complete both the strong major program and this first graduate component will also have the technical qualifications needed to teach some of the upper division courses of the GCMC program.

We must emphasize that the courses to be described are not meant to be minimal introductions to their subject matter. The courses demand a serious involvement with graduate mathematics. Where questions of substance arise, mathematics departments should tend in the direction of the recommendations of the CUPM Pregraduate Panel's report *Pregraduate Preparation of Research Mathematicians.*

The time required to complete the first graduate component will vary considerably; obviously a student who achieves only minimal success in his course work or whose undergraduate training has fallen short of the strong mathematics major will require more than the usual amount of time to reach the necessary level of mathematical maturity. We have found, incidentally, that the programs of many Academic Year Institutes bring the student to a level only slightly beyond that of a strong mathematics major.

Hopefully a student who completes the first graduate component will have developed a mathematical maturity that will enable him to bring to his classes an awareness of the fact that the mathematics taught in lower division courses is a part of the basic fabric of applied mathematics. He should be able to present illustrations from outside of mathematics including both the physical and the behavioral sciences, where appropriate. It would be desirable, but it is not necessary, that he have made a serious study of some field of application (as represented, for example, by a year's course work), but it would also be possible for him to broaden his appreciation for the

applications of mathematics by supplementary reading outside of his regular course assignments.

The first graduate component, which is an essential part of the preparation of a college mathematics teacher, and for which a master's degree would be suitable recognition, includes:

1. The completion of the strong mathematics major, if it has not been completed by the time the student begins graduate work.

2. At least two of the following three items:

   a. A substantial year's work in modern algebraic theory building on the earlier courses which presented the fundamental concepts of algebra.

   b. A year's work in analysis designed to follow the undergraduate analysis courses 11, 12, 13 of GCMC.

   c. A full year of "geometry" from a topological point of view following an undergraduate geometry course such as 9 of GCMC. This should include a semester of general topology and at least an introduction to algebraic topology.

3. At least one semester, preferably two, of teaching a class of undergraduate mathematics under the close supervision of an experienced teacher. Serious special attention should be devoted to the pedagogical problems involved in developing mathematical material for an immature audience. If possible this teaching experience should also be accompanied by a proseminar designed to give students experience in articulating mathematical concepts before a critical audience.

We have repeatedly stated that a college teacher must continue his mathematical growth throughout his career. While the early graduate years are themselves a period of growth, it is also desirable that

the student review college mathematics from the more advanced point of view of his graduate courses. There are many books by distinguished mathematicians which can help in this review, and provide a wealth of illustrations to enrich his teaching.

C. ADVANCED GRADUATE COMPONENT

In this section we describe a program of study which, when offered in a graduate department having an established Ph.D. program in mathematics, should provide the prospective college mathematics teacher with the mathematical background and with the maturity he will need to be prepared to teach all of the courses in the four year GCMC program. Successful completion of both the first graduate component and the advanced graduate component should also provide a sound basis for the continued professional and intellectual growth which a college teacher requires in order to qualify, in due course, for promotion, tenure, and administrative responsibility in his department—whether or not he subsequently earns an advanced degree. Some of the work which we include in the advanced graduate component is intended specifically for prospective college teachers and to this extent it complements regular graduate programs designed to prepare research mathematicians.

The work of the advanced component builds on that of the first component, and consists of the satisfactory completion of the following:

1. A year course in any of the three fields, algebra, analysis, topology-geometry, not included in satisfying Recommendation 2 of the first graduate component.
2. A second year of graduate study in at least one of the three fields mentioned above as well as additional graduate courses in mathematics representing areas of special interest to the faculty.
3. A graduate research seminar designed to bring the student into active contact with the creative efforts of a member of the research faculty.

4. A seminar or reading course designed to provide a critical review of the relationship of the student's graduate courses to the undergraduate courses he might be called upon to teach: briefly, a form of "Elementary Mathematics from an Advanced Viewpoint."

5. A general examination designed to test the breadth of knowledge essential to a college mathematics teacher. It would cover each of the major areas of mathematics in which the student has taken courses at the graduate level.

6. A lecture project designed to test the student's ability to prepare and deliver a seminar talk, and to provide him an opportunity to develop his expository ability. We suggest that the topic assigned for the lecture be one outside of the student's field of specialization, in order that he may also demonstrate his competence to pursue mathematics on his own initiative.

While the time and the course work required to complete the advanced graduate component will vary a great deal among individuals and institutions, it should be clear that a candidate who reaches this level must have a strong personal commitment to mathematics, and that he will have successfully completed at least two or three years of serious full-time graduate study beyond the strong major program.

The depth of understanding, the breadth of knowledge, and the mathematical maturity attested to by the successful achievement of the advanced graduate component are essential for the effective teaching of the various courses in mathematics offered at the college level. We believe that such achievement should be recognized by appropriate certification. Recent action of the faculties at Michigan, at Yale, and on the Berkeley Campus of the University of California, seems to indicate a growing sentiment in favor of some such formal recognition.

D. THE DOCTORATE

Although we have asserted that a Ph.D. degree in mathematics should not be regarded as an absolute necessity for the academic qualification of a college teacher of mathematics, we certainly would not suggest that the work and the study required to earn a Ph.D. are not important, or that they would not enhance the effectiveness of any college teacher. The significant difference between the advanced graduate component and the Ph.D. degree consists of research seminars and independent reading in the candidate's field of specialization, leading to an original contribution to mathematical knowledge reported in the thesis. Making an original contribution to mathematical knowledge is extremely valuable for the college teacher for, by engaging in research, he becomes a participating member of the mathematical profession, and thus is able to transmit to his students, both in the classroom and outside of it, the knowledge and the stimulation that come from the experience of creating new ideas.

It is our intention that the successful completion of the advanced graduate component when followed by an appropriate thesis should be worthy of a doctor's degree. Thus we believe that it should be offered only in those departments which already have established Ph.D. programs in mathematics: only in the vital research atmosphere of such a department can the required quality be attained. We also believe that graduate schools should be encouraged to seek ways of increasing the opportunities for qualified college teachers of mathematics to earn the Ph.D. after some years of teaching.

THE COMPOSITION OF AN UNDERGRADUATE DEPARTMENT

It is clear from the preceding discussion that we consider it neither necessary nor desirable to specify a single standard to be applied to all college teachers of mathematics. A very effective department can be composed of staff members with different levels of preparation and experience. Of course, there is no such thing as being too highly qualified to teach any course: higher qualifications can always be translated into more effective teaching, the design of an improved course, the preparation of better materials, and so on. However, the critical shortage of mathematics teachers requires that the available staff be used as effectively as possible, both in the individual college and in the country as a whole.

Let us consider reasonable academic qualifications for the mathematics faculty of a small college, one with a mathematics staff of six. We assume that the college has no graduate students and hence no graduate teaching assistants. At least two-thirds of the teaching load is likely to be in lower division courses. We believe that if three or four of the six staff members are at or near the level of the first graduate component and the others are either at the level of the advanced graduate component or have Ph.D.'s in mathematics, the department will have the technical qualifications needed to do an excellent job. Care must be exercised in the selection of staff members to assure that advanced study is not concentrated in only one area of mathematics. For example, the GCMC courses in applied mathematics, numerical analysis, and probability and statistics require special attention: there should be members of the staff who have had graduate work in these areas.

We do not suggest that all the lower division courses ought to be taught by teachers in the first group and all the advanced courses by the others. On the contrary, we consider it essential that some of the most highly qualified teachers be involved in the elementary courses; just as we believe that many of the less well-prepared teachers can be expected to do excellent work in some of the more advanced

courses. Indeed, one very effective way for any teacher to increase his knowledge is for him to give an advanced course in which he may learn as he teaches. The level of qualification of any staff member cannot be regarded as permanent or fixed. Since continued intellectual growth is required for good teaching, every staff member at whatever level must be considered as on his way to higher qualifications. This applies just as much to a man with a Ph.D. in mathematics as it does to any other teacher in the department.

Finally, we do not wish to imply that rank or salary should depend entirely on the levels of academic preparation we have described. In general, rank should correspond to professional competence and achievement, as indicated by all professional activities and by teaching effectiveness, as well as by earned degrees.

Our suggestions are, of course, subject to modification to fit the needs of individual institutions. We predict, for example, that for the foreseeable future the first graduate component should represent adequate preparation for teaching transfer students in junior colleges, provided the teacher continues to remain "intellectually alive." At universities, and at colleges near universities, it is certainly appropriate to make use of teaching assistants who have reached only the level of a strong mathematics major, or who have not yet completed the first graduate component, provided that the teaching is adequately supervised and that there is clear evidence of progress toward the next level.

FINAL REMARKS

We have repeatedly stated our conviction that continued intellectual and professional growth is essential to continued competence as a teacher. One needs to move forward in order not to fall behind. A significant reason for recognizing the Ph.D. as a meaningful and desirable level of qualification for college teachers is that it is both evidence of an individual's ability to continue his mathematical growth by himself and is an indication of momentum in that direction. However, for reasons of isolation or inadequate training, many college teachers are unable to provide for their own professional growth. For them, and for college teachers who do not have even minimal academic qualifications for the responsibilities they are asked to assume, there is an urgent need for expanded programs of external stimuli for improvement: guidance, financial assistance, and easily accessible and attractive study programs. Institutes, internships and and new forms of retraining need to be explored and developed. We must recognize, however, that the intellectual growth of college teachers depends primarily not on opportunities of this kind but on the conditions of their daily work. If their teaching and administrative duties leave them no time or energy for study and reflection then it cannot be expected that their scientific qualifications will improve from year to year, or even that they will be maintained.

We have no definite advice to offer for solving these problems. We can only call attention to them, and suggest that the difficulties involved in upgrading many of our present teachers and in stimulating continued growth in others provide some of the most important and pressing problems faced by the mathematical community.

CUPM CURRICULAR PUBLICATIONS

A General Curriculum in Mathematics for Colleges (1965)

A Curriculum in Applied Mathematics (1966)

Recommendations on the Undergraduate Mathematics Program for Engineers and Physicists (1962, Reprinted 1965, Revised 1967)

Recommendations on the Undergraduate Mathematics Program for Work in Computing (1964)

Mathematical Engineering: A Five Year Program (1966)

Pregraduate Preparation of Research Mathematicians (1963, Reprinted 1965)

Preparation for Graduate Study in Mathematics (1965)

Tentative Recommendations for the Undergraduate Mathematics Program for Students in the Biological, Management, and Social Sciences (1964)

Recommendations for the Training of Teachers of Mathematics (1961, Revised 1964, Revised 1966)

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