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RECOMMENDATIONS ON THE UNDERGRADUATE MATHEMATICS PROGRAM FOR ENGINEERS AND PHYSICISTS.

BY- DURST, LINCOLN K.
COMMITTEE ON THE UNDERGRADUATE PROGRAM IN MATH.

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THIS REPORT PRESENTS A PROGRAM FOR THE UNDERGRADUATE MATHEMATICAL PREPARATION OF ENGINEERS AND PHYSICISTS. THE FOLLOWING STATEMENTS SERVE AS UNDERLYING ASSUMPTIONS FOR ANYONE EXPECTING TO MAKE USE OF THE RECOMMENDATIONS--(1) THE PROGRAM IS ONE FOR THE PRESENT AND THE PROPOSALS ARE SUCH THAT THEY CAN BE CONTINUALLY MODIFIED TO KEEP UP WITH FUTURE DEVELOPMENTS, (2) THE RECOMMENDED PROGRAM MAY SEEM EXCESSIVE IN THE LIGHT OF THE PRESENT STATUS OF PROGRAMS, BUT THE COMMITTEE BELIEVES THAT THIS IS THE MINIMUM AMOUNT OF MATHEMATICS APPROPRIATE FOR STUDENTS WHO WILL BE STARTING THEIR CAREERS WITHIN FIVE YEARS, AND (3) BEYOND THE COURSES REQUIRED OF ALL STUDENTS THERE MUST BE FLEXIBILITY TO ALLOW FOR VARIATIONS IN FIELDS AND IN THE QUALITY OF STUDENTS. THE RECOMMENDED PROGRAM FOR ENGINEERS INCLUDES--(1) COURSES TO BE REQUIRED OF ALL STUDENTS, (2) COURSES RECOMMENDED FOR STUDENTS INTENDING TO GO INTO RESEARCH AND DEVELOPMENT, (3) COURSES AVAILABLE FOR THEORETICALLY-MINDED STUDENTS CAPABLE OF EXTENDED GRADUATE STUDY, AND (4) COURSES OF POSSIBLE INTEREST TO SPECIAL GROUPS. ALSO INDICATED IS THE RECOMMENDED PROGRAM FOR PHYSICISTS. A DESCRIPTION OF EACH OF THE RECOMMENDED COURSES CONCLUDES THE REPORT. THIS DOCUMENT IS ALSO AVAILABLE WITHOUT CHARGE FROM CUPM CENTRAL OFFICE, P. O. BOX 1024, BERKELEY, CALIFORNIA 94701. (RP)

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MATHEMATICAL ASSOCIATION OF AMERICA

RECOMMENDATIONS

ON THE

UNDERGRADUATE

MATHEMATICS PROGRAM

FOR

ENGINEERS AND PHYSICISTS

COMMITTEE ON THE UNDERGRADUATE
PROGRAM IN MATHEMATICS
JANUARY, 1967

Panel on Mathematics for the Physical Sciences and Engineering

RECOMMENDATIONS ON THE UNDERGRADUATE MATHEMATICS PROGRAM FOR ENGINEERS AND PHYSICISTS

Revised: January 1967

COMMITTEE ON THE UNDERGRADUATE PROGRAM IN MATHEMATICS
Mathematical Association of America

The Committee on the Undergraduate Program in Mathematics is a committee of the Mathematical Association of America charged with making recommendations for the improvement of college and university mathematics curricula at all levels and in all educational areas. Financial support for CUPM has been provided by the National Science Foundation.

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CUPM Central Office
P. O. Box 1024
Berkeley, California 94701

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BACKGROUND (1962)

One reason for the current effort on the undergraduate program is the rapid change in the mathematical world and in its immediate surroundings. Three aspects of this change have a particular effect on undergraduate curricula in the physical sciences and engineering. The first is the work being done in improving mathematics education in the secondary school. Several programs of improvement in secondary school mathematics have already had considerable effect and can be expected to have a great deal more. Not only can we hope that soon most freshmen expecting to take a scientific program will have covered precalculus mathematics, but perhaps more important, they will be accustomed to care and precision of mathematical thought and statement. Of course, not all students will have this level of preparation in the foreseeable future, but the proportion will be large enough to enable us to plan on this basis. Students with poorer preparation may be expected to take remedial courses without credit before they start the regular program.

This improved preparation obviously means that we will be able to improve the content of the beginning calculus course since topics which take time in the first two years will have been covered earlier. More than that, however, it means that the elementary calculus course will have to take a more sophisticated attitude in order to keep the student from laughing at a course in college which is less careful mathematically than its secondary school predecessors.

The second aspect of change in mathematics which confronts us is the expansion in the applications of mathematics. There is a real "revolution" in engineering—perhaps "explosion" is an even better description than "revolution," because, as it turns out, several trends heading in different directions are simultaneously visible. One is a trend toward basic science. The mathematical aspect of this trend is a strengthening of interest in more algebraic and abstract concepts. An orthogonal trend is one toward the engineering of large systems. These systems, both military and nonmilitary, are of ever-increasing complexity and must be optimized with regard to such factors as cost, reliability, maintenance, etc. Resulting mathematical interests are linear algebra and probability-statistics. A further trend, in part a consequence of the preceding two, is a real increase in the variety and depth of the mathematical tools which interest the engineer. In general, engineers are finding that they need to use new and unfamiliar mathematics of a wide variety of types.

A third factor is the arrival of the electronic computer. It is having its effect on every phase of science and technology, all the way from basic research to the production line. In mathematics it has, for one thing, moved some techniques from the abstract to the practical field; for example, some series expansion, iterative techniques, and so forth. Then too, computers have led people to tackle problems they would never have considered before, such as large systems of linear equations, linear and nonlinear programming, and Monte Carlo methods. Many of these new techniques require increased sophistication in mathematics.

An additional factor entering from another direction must also be mentioned. Mathematicians in the United States have in recent years become much more closely involved with areas adjacent to their own research. Of the many factors which enter here, we may mention the greatly increased interest of mathematicians at all levels in education, the rapid growth of mathematical employment in industry, the spread of research and consulting contracts into the universities, and the development of a number of mathematical disciplines, such as information theory, that have many applications but are not classical applied mathematics. There is thus a real desire among mathematicians and scientists to cooperate in matters of education.

The conclusions above and the recommendations that constitute the body of this report were formulated by the Panel after extensive consultation with mathematicians, physicists, and engineers. In engineering, in particular, representatives of many fields and many types of institutions were consulted, as well as officials of the American Society for Engineering Education.* The recommendations for physicists were drawn up in close collaboration with the Commission on College Physics.

In considering the recommendations which follow, it is crucial to examine what has been our attitude toward certain ideas which inevitably occupy a central position in any discussion of mathematical education. Among these are mathematical sophistication and mathematical rigor, motivation, and intuition. Now it is a fact that mathematical rigor—by which we mean an attempt to prove essentially everything that is used—is not the way of life of the physicist and the engineer. On the other hand, mathematical sophistication—which means to us careful and clear mathematical statements, proofs of many things, and generally speaking a broad appreciation of the mathematical blocks from which models are built—is desired by, and desirable for, all students preparing for a scientific career. How does one choose what is actually to be proved?

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*Some of the results of a conference with engineers are embodied in four addresses delivered at a Conference on Mathematics in the Engineering Curriculum, held under the auspices of this Panel in March, 1961. These addresses were published in the Journal of Engineering Education, Volume 52, number 3, December, 1961, pp. 171-207.

It seems to us that this is related to the plausibility of the desired result. It is unwise to give rigor to either the utterly plausible or the utterly implausible, the former because the student cannot see what the fuss is all about, and the latter because the most likely effect is rejection of mathematics. The moderately plausible and the moderately implausible are the middle ground where we may insist on rigor with the greatest profit; the great danger in the overzealous use of rigor is to employ it to verify only that which is utterly apparent.

Let us turn next to the subject of motivation. Motivation means different things to different people, and thus requires clarification. One aspect of motivation is concerned with the difference between mathematics and the applications of mathematics, between a mathematical model and the real world. For many engineers and physicists motivation of mathematical concepts can be supplied by formulating real situations which lead to the construction of reasonable models that exhibit both the desirability and the usefulness of the mathematical concept. Thus, motion of a particle, or growth of a bacterial culture, may be used as physical motivation for the notion of a derivative. It is also possible, of course, to give a mathematical motivation for a new mathematical concept; the geometric notion of a tangent to a curve also leads to the notion of derivative and is quite enough motivation to a mathematician. Since each kind of motivation is meaningful to large groups of students, we feel that both should appear wherever relevant. It is certainly a matter of individual taste whether one or both motivations should precede, or perhaps follow, the presentation of a mathematical topic. In either case, however, it is necessary to be very clear in distinguishing the motivating mathematical or physical situation from the resulting abstraction.

Physical and mathematical examples which are used as motivation, as well as previous mathematical experience, help to develop one's intuition for the mathematical concept being considered. By "intuition" we mean an ability to guess both the mathematical properties and the limitations of a mathematical abstraction by analogy with known properties of the mathematical or physical objects which motivated that abstraction. Intuition should lead the way to rigor whenever possible; neither can be exchanged or substituted for the other in the development of mathematics.

A mathematics course for engineers and physicists must involve the full spectrum from motivation and intuition to sophistication and rigor. While the relative emphasis on these various aspects will forever be a subject for debate, no mathematics course is a complete experience if any of them is omitted.

INTRODUCTION TO THE REVISION (1967)

In the five years that have elapsed since the first publication of these recommendations several factors have emerged to affect the teaching of mathematics to engineers. The most striking of these is the widespread application of automatic computers to engineering problems. It is now a commonplace that all engineers must know how to use computers and that this knowledge must be gained early in their training and reinforced by use throughout it. We have accordingly included an introductory course in computer science as a requisite for all engineering students and have increased the amount of numerical mathematics in other courses wherever possible.

A second factor is the pretty general acceptance of linear algebra as part of the beginning mathematics program for all students. In the engineering curriculum this is tied in to the expansion in computing, since linear algebra and computers are precisely the right team for handling the large problems in systems analysis that appear in so many modern investigations. Five years ago there were only a handful of elementary texts on linear algebra—now treatments are appearing almost as fast as calculus books (with which they are often combined).

A development of particular interest to these recommendations is the appearance of the CUPM report A General Curriculum in Mathematics for Colleges (1965), referred to hereafter as GCMC. It is too early to judge how widely the GCMC will be adopted, but initial reactions, including those of teachers of engineering students, have been generally favorable. GCMC makes considerable use of material in the first version of these recommendations, and now we, in turn, borrow some of the courses in GCMC.

Minor changes in the content of courses and some rearrangement and changes of emphasis are the result of experience and discussions over the years.

Relatively little change has been made in the program for physicists. The only major one has been the inclusion of Introduction to Computer Science in the required courses. We do this in the conviction that all scientists (if not, indeed, all college graduates) should know something about the powers and limitations of automatic computers.

Applications of Undergraduate Mathematics in Engineering, written and edited by Ben Noble, published in January 1967 by the MAA and the Macmillan Co., is based on a collection of problems assembled as a joint project of CUPM and the Commission on Engineering Education. The book has five parts: Illustrative Applications of Elementary Mathematics, Applications of Ordinary Differential Equations, Applications to Field Problems, Applications of Linear Algebra, Applications of Probability Theory.

INTRODUCTION TO THE RECOMMENDATIONS

This report presents a program for the undergraduate mathematical preparation of engineers and physicists.

Since obviously no single program of study can be the best one for all types of students, all institutions, and all times, it is important that anyone expecting to make use of the present recommendations understand the assumptions underlying them. The following comments should make these assumptions clear and also explain some other features of the recommendations.

1. This is a program for today, not for several years in the future. Programs somewhat like this are already being given at various places, and the sample courses we outline are patterned after existing ones. We assume a good but not unusual background for the entering freshman.

Five or ten years from now the situation will undoubtedly be different: in the high schools, in research, in engineering practice, and in such adjacent areas as automatic computation. Such differences will necessitate changes in the mathematics curriculum, but a good curriculum can never be static, and it is our belief that the present proposal can be continually modified to keep up with developments. However, the material encompassed here will certainly continue to be an important part of the mathematical education needed by engineers and physicists.

2. The program we recommend may seem excessive in the light of what is now being done at many places, but it is our conviction that this is the minimal amount of mathematics appropriate for students who will be starting their careers four or five years from now. We recognize that some institutions may simply be unable to introduce such a program very soon. We would hope that such places regard the program as something to work toward. The members of the Panel are willing to be consulted on such matters and to give what advice they can as to a suitable modification of the basic program to fit special needs.

3. Beyond the courses required of all students there must be available considerable flexibility to allow for variations in fields and in the quality of students. The advanced material whose availability we have recommended can be regarded as a main stem that may have branches at any point. Also, students may truncate the program at points appropriate to their interests and abilities.

4. The order of presentation of topics in mathematics and some related courses is strongly influenced by two factors:

a. The best possible treatment of certain subjects in engineering and physics requires that they be preceded by certain mathematical topics.

b. Topics introduced in mathematics courses should be used in applications as soon afterwards as possible.

To attain these ends, coordination among the mathematics, engineering, and physics faculties is necessary, and this may lead to course changes in all fields.

5. The recommendations are of course the responsibility of CUPM. In cases where it seems of interest and is available, we have indicated the reaction of the groups of engineers and physicists who were consulted. For convenience we refer to them as "the consultants."

LIST OF RECOMMENDED COURSES

It is desirable that all calculus prerequisites, including analytic geometry, be taught in high school. At present it may be necessary to include some analytic geometry in the beginning analysis course, but all other deficiencies should be corrected on a noncredit basis.

The following courses should be available for undergraduate majors in engineering and physics:

1. Beginning Analysis (9-12 semester hours).

As far as general content is concerned this is a relatively standard course in calculus and differential equations. There can be many variations of such a course in matters of rigor, motivation, arrangement of topics, etc., and textbooks have been and are being written from several points of view.

The course should contain the following topics:

a. An intuitive introduction of four to six weeks to the basic notions of differentiation and integration. This course serves the dual purpose of augmenting the student's intuition for the more sophisticated treatment to come and preparing for immediate applications to physics.

b. Theory and techniques of differentiation and integration of functions of one real variable, with applications.

c. Infinite series, including Taylor series expansions.

d. A brief introduction to differentiation and integration of functions of two or more real variables.

e. Topics in differential equations, including the following: linear differential equations with constant coefficients and first order systems—linear algebra (including eigenvalue theory, see 2 below) should be used to treat both homogeneous and nonhomogeneous problems; first order linear and nonlinear equations, with Picard's method and an introduction to numerical techniques.

f. Some attempt should be made to fill the gap between the high school algebra of complex numbers and the use of complex exponentials in the solution of differential equations. In particular, some work on the calculus of complex-valued functions of a real variable should be included in items b and c.

g. Students should become familiar with vectors in two and three dimensions and with the differentiation of vector-valued functions of one variable. This material can obviously be correlated with the course in linear algebra (see below).

h. Theory and simple techniques of numerical computation should be introduced where relevant. Further comments on this point, applying to the whole program, will be found below (under course 3).

We feel that the above comments on beginning analysis sufficiently describe a familiar course. The remaining courses in our list are less generally familiar. Hence the brief descriptions of courses 2 through 12 are supplemented in the Appendix by detailed outlines of sample courses of the kind we have in mind.

2. Linear Algebra (3 semester hours).

A knowledge of the basic properties of n-dimensional vector spaces has become imperative for many fields of application as well as for progress in mathematics itself. Since this subject is so fundamental and since its development makes no use of the concepts of calculus, it should appear very early in the student's program. We recommend a course with strong emphasis on the geometrical interpretation of vectors and matrices, with applications to mathematics (see items 1-e and 1-g above), physics, and engineering. Topics should include the algebra and geometry of vector spaces, linear transformations and matrices, linear equations (including computational methods), quadratic forms and symmetric matrices, and elementary eigenvalue theory.

It may be desirable, for mathematical or scheduling purposes, to combine beginning analysis and linear algebra into a single coordinated course to be completed in the sophomore year.

The basic GCMC program starts with nine hours of beginning analysis and three hours of linear algebra, as described here. The linear algebra course outlined in Appendix A is one of the two versions described in GCMC. The outlines for courses in functions of several variables, functions of a complex variable, real variables, and algebraic structures, also agree with those in GCMC.

3. Introduction to Computer Science (3 semester hours).

The development of high-speed computers has made it necessary for the appliers of mathematics to know the path from mathematical theory through programming logic to numerical results. This course gives an understanding of the position of the computer along this path, the manner of its use, its capabilities, and its limitations. It also provides the student with the basic techniques needed for him to use the computer to solve problems in other courses.

An even more important part of the path must be provided by the student's program as a whole. All the courses discussed here should contain, where it is suitable and applicable, mathematical topics motivated by the desire to relate mathematical understanding to computation. It is especially desirable that the student see the possibility of significant advantage in combining analytical insight with numerical work. Indications of such opportunities are scattered throughout the course outlines in the Appendix.

4. Probability and Statistics (6 semester hours).

Basic topics in probability theory, both discrete and continuous, have become essential in every branch of engineering, and in many engineering fields an introduction to statistics is also needed. We recommend a course based on the notions of random variables and sample spaces, including, *inter alia*, an introduction to limit theorems and stochastic processes, and to estimation and hypothesis testing. Although this should be regarded as a single integrated course the first half can be taken as a course in probability theory. For ease of reference we designate the two halves 4a and 4b.

5. Advanced Multivariable Calculus (3 semester hours).

Continuation of item 1-d. A study of the properties of continuous mappings from E_n to E_m, making use of the linear algebra in course 2, and an introduction to differential forms and vector calculus based on line integrals, surface integrals, and the general Stokes' theorem. Application should be made to field theory, or to elementary hydrodynamics, or other similar topics, so that some intuitive understanding can be gained.

6. Intermediate Ordinary Differential Equations (3 semester hours).

This course continues the work of item 1-e into further topics important to applications, including linear equations with variable coefficients, boundary value problems, rudimentary existence theorems, and an introduction to nonlinear problems. Much attention should be given to numerical techniques.

7. Functions of a Complex Variable (3 semester hours).

This course presupposes somewhat more mathematical maturity than courses 5 and 6 and so would ordinarily be taken after them, even though they are not prerequisites as far as subject matter is concerned. In addition to the usual development of integrals and series, there should be material on multivalued functions, contour integration, conformal mapping, and integral transforms.

8. Partial Differential Equations (3 semester hours).

Derivation, classification, and solution techniques of boundary value problems.

9. Introduction to Functional Analysis (3 semester hours).

An introduction to the properties of general linear spaces and metric spaces, their transformations, measure theory, general Fourier series, and approximation theory.

10. Elements of Real Variable Theory (3 semester hours).

A rigorous treatment of basic topics in the theory of functions of a real variable.

11. Optimization (3 semester hours).

Linear, nonlinear, and dynamic programming, combinatorics, and calculus of variations.

12. Algebraic Structures (3 semester hours).

An introduction to the theory of groups, rings, and fields.

13. Numerical Analysis.

14. Mathematical Logic.

15. Differential Geometry.

The last three courses are topics that might well be of interest to special groups of students. Their lengths and contents may vary considerably. For sample syllabi see GCMC and the following:

Recommendations on the Undergraduate Mathematics Program for Work in Computing, CUPM (1964).

An Undergraduate Program in Computer Science - Preliminary Recommendations, Communications of the Association for Computing Machinery 8(1965) 543-552.

The above list of courses is the result of careful consideration by the Panel and the consultants. The brief description given here and the detailed sample outlines in the Appendix, while based on the mathematical structure of the topics themselves, reflect strongly the expressed interests of engineers and physicists. We realize that the nature of the institution and the requirements of other users of mathematics as well as of the mathematics majors may influence the specific offerings.

RECOMMENDED PROGRAM FOR ENGINEERS

A. *Courses to be required of all students.*

1. Beginning Analysis. This recommendation needs no comments.

2. Linear Algebra. The great majority of the consultants felt that this is important material that all engineers should have during the first two years.

3. Introduction to Computer Science. Developments of the last few years make it clear that engineering is strongly dependent on a knowledgeable use of computers.

4a. Probability. All students should have at least a three semester hour course in probability. The consultants agreed on the value of probability to an engineer, but there was considerable disagreement among the consultants as to the advisability of requiring it of all students. However, the members of our Panel are unanimously and strongly of the opinion that this subject will soon pervade all branches of engineering and that now is the time to begin preparing students for this development.

B. *Courses recommended for students intending to go into research and development.*

4b. Statistics.

5. Advanced Multivariable Calculus.

6. Intermediate Ordinary Differential Equations.

7. Functions of a Complex Variable.

The consultants agreed to the value of the material in courses 5, 6, and 7, and some preferred that it be completed within the junior year. The Panel is convinced that an adequate presentation requires a minimum of nine semester hours which could, of course, be taken in one year if desired. The order in which courses 5 and 6 are taken is immaterial except as they may be coordinated with other courses. If they are to be presented to the students in a fixed order, the instructor may wish to adjust the time schedules and choice of topics.

C. *Courses which should be available for theoretically minded students capable of extended graduate study.*

8. Partial Differential Equations.

9. Introduction to Functional Analysis.

10. Elements of Real Variable Theory.

Presumably a student would take either 9 or 10 but not both. 9 is probably more valuable but 10 is more likely to be available.

11. Optimization.

D. *Courses of possible interest to special groups.*

12. Algebraic Structures.

13. Numerical Analysis.

14. Mathematical Logic.

15. Differential Geometry.

RECOMMENDED PROGRAM FOR PHYSICISTS

A. *Courses to be required of all students.*

1. Beginning Analysis.

2. Linear Algebra. Like the engineers, the physicists felt that this material is essential.

3. Introduction to Computer Science.

5. Advanced Multivariable Calculus. This course should be taken in the sophomore year if possible, and in any event no later than the first part of the junior year.

6. Intermediate Ordinary Differential Equations.

B. *Additional courses, in order of preference.* Students contemplating graduate work should be required to take a minimum of three to nine semester hours of these courses.

7. Functions of a Complex Variable.

9. Introduction to Functional Analysis.

4a. Probability. The value of requiring this course in the undergraduate program of all physicists is not as well established as it is for engineers.

12. Algebraic Structures.

10. Elements of Real Variable Theory.

8. Partial Differential Equations.

APPENDIX

DESCRIPTION OF RECOMMENDED COURSES

While we feel strongly about the spirit of the courses outlined here, the specific embodiments are to be considered primarily as samples. Courses close to these have been taught successfully at appropriate levels, and our time schedules are based on this experience.

Some of these courses are sufficiently common that approximations to complete texts already exist; others have appeared only in lecture form. In general, the references which accompany an outline are not intended as texts for the course but as indications of possible sources of material to be moulded into the course.

2. Linear Algebra (3 semester hours).

The purpose of this course is to develop the algebra and geometry of finite-dimensional linear vector spaces and their linear transformations, the algebra of matrices, and the theory of eigenvalues and eigenvectors.

a. Linear equations and matrices (5 lectures). Systems of linear equations, equivalence under elementary row operations, Gaussian elimination, matrix of coefficients, row reduced echelon matrix, computations, solutions, matrix multiplication, invertible matrices, calculation of inverse by elementary row operations.

b. Vector spaces (6 lectures). Vector spaces defined abstractly. Examples. Linear dependence and independence. Linear bases and subspaces, dimension. Inner product, length, angle, direction cosines, applications to line and plane geometry.

c. Linear transformations (10 lectures). Transformations, linear transformations, kernel and image of a transformation. Rank of a transformation. The linear transformation associated with a matrix. Representation of linear transformations by matrices, composition of transformations and multiplication of matrices. Algebra of linear transformations and matrices. Change of basis in a linear transformation, similar matrices.

d. Determinants (4 lectures). Definition of a determinant of a square array, properties of determinants. Cramer's rule. Inverse of a matrix. Determinant of transposed matrix and of the product of two matrice .

e. Quadratic forms (4 lectures). Symmetric matrices and quadratic forms. Quadric surfaces. Effect of linear transformation. Rational reduction to diagonal. Invariance of index, positive definiteness. Inner product, orthogonal bases, Gram-Schmidt orthogonalization. Orthogonal expansions and Fourier rule. Orthogonal reduction of 2 x 2 quadratic form, application to plane conics. The general case, orthogonal reduction, eigenvalues and eigenvectors, invariance. Calculation of eigenvalues and eigenvectors. Cayley-Hamilton theorem, trace, discriminant and other scalar invariant functions. Applications to analytic geometry.

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For numerical methods see:

Faddeeva, V. N. COMPUTATIONAL METHODS OF LINEAR ALGEBRA. New York: Dover Publications, 1959.

Fox, L. AN INTRODUCTION TO NUMERICAL LINEAR ALGEBRA. New York: Oxford University Press, 1965.

3. Introduction to Computer Science (3 semester hours).

This course serves a number of purposes:

(1) It gives students an appreciation of the powers and limitations of automata.

(2) It develops an understanding of the interplay between the machine, its associated languages, and the algorithmic formulation of problems.

(3) It teaches students how to use a modern computer.

(4) It enables instructors in later courses to assign problems to be solved on the computer.

The syllabus below describes the lecture portion of a 2-hour lecture, 1-hour laboratory course. In addition to illustrating topics from the lectures the laboratory should implement items (3) and (4) above by teaching the students a suitable language. This need not be full-fledged FORTRAN, ALGOL, or PL/I but it should be close enough to one of these that the student can readily learn the senior language if he requires it later. It is important that the students have contact with the machine as early and as often as possible. They should be able to run a simple program by the second laboratory period and a minimum of five or six machine problems should be assigned in the course.

a. Introduction (1 lecture). Computer organization, one address logic, typical instructions.

b. Concept and organization of programs (2 lectures). Algorithms, loops, tests, initialization, address modification.

c. A simplified machine-type language (2 lectures). Instruction set, number representation, subroutines, index registers.

d. Number systems (3 lectures). Number representation and conversion, complement arithmetic, BCD coding schemes, fixed and floating representations, the algorithms for adding, multiplying, and dividing numbers in digital form.

e. Addressing (2 lectures). Relative, indirect; variable word length organization; 1, 2, 3, 4 address logic.

f. Hardware (2 lectures). Memory devices and their organization, cores, magnetic tapes, discs, drums; input and output techniques; speed, size, and cost considerations.

g. Software (6 lectures). Discussion and comparison of interpreters, assemblers, and compilers; advantages, uses, techniques for implementation; executive systems; comparison of ALGOL, COBOL, PL/I, etc. Actual machine assembly language: comparison with simplified one.

h. Computer applications (8 lectures). Characteristics of good applications; numerical computations, error generation; business data processing; simulation techniques, Monte Carlo; information organization and retrieval; remote access systems, time sharing.

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Marcovitz, A. B., and Schweppe, E. J. AN INTRODUCTION TO ALGORITHMIC METHODS USING THE MAD LANGUAGE. New York: Macmillan Co., 1966.

School Mathematics Study Group. ALGORITHMS, COMPUTATION AND MATHEMATICS. Stanford, California: Leland Stanford Junior University, 1966.

4. Probability and Statistics (6 semester hours).

This is a one year course presenting the basic theory of probability and statistics. Although the development of the ideas and results is mathematically precise, the aim is to prepare students to formulate realistic models and to apply appropriate statistical techniques in problems likely to arise in engineering. Therefore new ideas will be motivated and applications of results will be given wherever possible.

First Semester: Probability.

a. Basic probability theory (4 lectures). Different theories of probability (classical, frequency, and axiomatic). Combinatorial methods for computing probability. Conditional probability, independence. Bayes' theorem. Geometrical probability.

b. Random variables (5 lectures). Concept of random variable and of distribution function. Discrete and continuous types. Multidimensional random variables. Marginal and conditional distributions.

c. Parameters of a distribution (4 lectures). Expected values. Moments. Moment generating functions. Moment inequalities.

d. Characteristic functions (4 lectures). Definition, properties. Characteristic functions and moments. Determination of distribution function from characteristic function.

e. Various probability distributions (6 lectures). Binomial, Poisson, multinomial. Uniform, normal, gamma, Weibull, multivariate normal. Importance of normal distribution. Applications of normal distribution to error analysis.

f. Limit theorems (6 lectures). Various kinds of convergence. Law of large numbers. Central limit theorem.

g. Markov chains (4 lectures). Transition matrix. Ergodic theorem.

h. Stochastic processes (6 lectures). Markov processes. Processes with independent increments. Poisson process. Wiener process. Stationary processes.

Second Semester: Statistics.

a. Sample moments and their distributions (5 lectures). Sample, statistic. Distribution of sample mean. Student's distribution. Fisher's, Z distribution.

b. Order statistics (4 lectures). Empirical distribution function. Tolerance limits. Kolmogorov-Smirnov statistic.

c. Tests of hypotheses (5 lectures). Simple hypothesis against simple alternative. Composite hypotheses. Likelihood ratio test. Applications.

d. Point estimation (5 lectures). Consistent estimates. Unbiased estimates. Sufficient estimates. Efficiency of estimate. Methods of finding estimates.

e. Interval estimation (6 lectures). Confidence and tolerance intervals. Confidence intervals for large samples.

f. Regression and linear hypotheses (4 lectures). Elementary linear models. The general linear hypothesis.

g. Nonparametric methods (5 lectures). Tolerance limits. Comparison of two populations. Sign test. Mann-Whitney test.

h. Sequential methods (5 lectures). The probability ratio sequential test. Sequential estimation.

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Cramer, H. THE ELEMENTS OF PROBABILITY THEORY AND SOME OF ITS APPLICATIONS. New York: John Wiley & Sons, 1955.

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Mood, A. M., and Graybill, F. A. INTRODUCTION TO THE THEORY OF STATISTICS. New York: McGraw-Hill Book Co., 1963, second edition.

Owen, D. B. HANDBOOK OF STATISTICAL TABLES. Reading, Massachusetts: Addison-Wesley Publishing Co., 1962.

Parzen, E. MODERN PROBABILITY THEORY AND ITS APPLICATIONS. New York: John Wiley & Sons, 1960.

Tucker, H. G. AN INTRODUCTION TO PROBABILITY AND MATHEMATICAL STATISTICS. New York: Academic Press, 1962.

5. Advanced Multivariable Calculus (3 semester hours).

A study of the properties of continuous mappings from E_n to E_m, making use of the linear algebra in course 2, and an introduction to differential forms and vector calculus based on line integrals, surface integrals, and the general Stokes' theorem. Application should be made to field theory, or to elementary hydrodynamics, or other similar topics, so that some intuitive understanding can be gained.

a. Transformations (15 lectures). Functions (mappings) from E_n to E_m, for n, m = 1, 2, 3, 4. Continuity and implications of continuity; differentiation, and the differential of a mapping as a matrix valued function. The role of the Jacobian as the determinant of the differential; local and global inverses of mappings, and the implicit function theorem. Review of the chain rule for differentiation, and reduction to matrix multiplication. Application to change of variable in multiple integrals, and to the area of surfaces.

b. Differential forms (6 lectures). Integrals along curves. Introduction of differential forms; algebraic operations; differentiation rules. Application to change of variable in multiple integrals. Surface integrals; the meaning of a general k-form.

c. Vector analysis (4 lectures). Reinterpretation in terms of vectors; vector function as mapping into E_3; vector field as mapping from E_3 into E_3. Formulation of line and surface integrals (1-forms and 2-forms) in terms of vectors. The operations Div, Grad, Curl, and their corresponding translations into differential forms.

d. Vector calculus (8 lectures). The theorems of Gauss, Green, Stokes, stated for differential forms, and translated into vector equivalents. Invariant definitions of Div and Curl. Exact differential forms, and independence of path for line integrals. Application to a topic in hydrodynamics, or to Maxwell's equations, or to the derivation of Green's identities and their specializations for harmonic functions.

e. Fourier methods (6 lectures). The continuous functions as a vector (linear) space; inner products and orthogonality; geometric concepts and analogy with E_n. Best L^2 approximation; notion of an orthogonal basis, and of completeness. The Schwarz and Bessel inequalities. General Fourier series with respect to an orthonormal basis. Treatment of the case $\{e^{inx}\}$ and the standard trigonometric case. Application to the solution of one standard boundary value problem.

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Fleming, W. H. FUNCTIONS OF SEVERAL VARIABLES. Reading, Massachusetts: Addison-Wesley Publishing Co., 1965.

Goffman, C. CALCULUS OF SEVERAL VARIABLES. New York: Harper and Row, 1965.

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Munroe, M. E. MODERN MULTIDIMENSIONAL CALCULUS. Reading, Massachusetts: Addison-Wesley Publishing Co., 1963.

Purcell, E. M. ELECTRICITY AND MAGNETISM, Berkeley Physics Course, Volume II. New York: McGraw-Hill Book Co., 1965.

Spivak, M. CALCULUS ON MANIFOLDS. New York: W. A. Benjamin, 1965.

Woll, J. W., Jr. FUNCTIONS OF SEVERAL VARIABLES. New York: Harcourt, Brace and World, 1966.

6. Intermediate Ordinary Differential Equations (3 semester hours).

The presentation of the course material should include: (1) an account of the manner in which ordinary differential equations and their boundary value problems, both linear and nonlinear, arise; (2) a carefully reasoned discussion of the qualitative behavior of the solution of such problems, sometimes on a predictive basis and at other times in a *a posteriori* manner; (3) a clearly described awareness of the role of numerical processes in the treatment of these problems, including the disadvantages as well as advantages—in particular, there should be a firm emphasis on the fact that numerical integration is not a substitute for thought; (4) an admission that we devote most of our lecture time to linear problems because (with isolated exceptions) we don't know much about any nonlinear ones except those that (precisely or approximately) can be attacked through our understanding of the linear ones. Thus a thorough treatment of linear problems must precede a sophisticated attack on the nonlinear ones.

The distribution of time among items d through f cannot be prescribed easily or with universal acceptability. Only a superficial account of these topics can be given in the available time, but each should be introduced.

a. Systems of linear ordinary differential equations with constant coefficients (6 lectures). Review of homogeneous and nonhomogeneous problems; superposition and its dependence on linearity; transients in mechanical and electrical systems. The Laplace transform as a carefully developed operational technique without inversion integrals.

b. Linear ordinary differential equations with variable coefficients (10 lectures). Singular points, series solutions about regular points and about singular points. Bessel's equation and Bessel functions; Legendre's equation and Legendre polynomials; confluent hypergeometric functions. Wronskians, linear independence, number of linearly independent solutions of an ordinary differential equation. Sturm-Liouville theory and eigenfunction expansions.

c. Solution of boundary value problems involving nonhomogeneous linear ordinary differential equations (7 lectures). Methods using Wronskians, Green's functions (introduce δ functions), and eigenfunction expansions. Numerical methods. Rudimentary existence and uniqueness questions.

d. Asymptotic expansion and asymptotic behavior of solutions of ordinary differential equations (3 lectures). Essentially the material on pp. 498-500 and pp. 519-527 of Jeffreys and Jeffreys.

e. Introduction to nonlinear ordinary differential equations (6 lectures). Special nonlinear equations which are reducible to linear ones or to quadratures, elliptic functions (pendulum oscillations), introductory phase plane analysis (Poincaré).

f. Numerical methods (7 lectures). Step-by-step solution of initial value problems for single equations and for systems. Error analysis, roundoff, stability. Improper boundary conditions, discontinuities, and other pitfalls.

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Brouwer, D., and Clemence, G. M. METHODS OF CELESTIAL MECHANICS. New York: Academic Press, 1961.

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Jeffreys, H., and Jeffreys, B. S. METHODS OF MATHEMATICAL PHYSICS. New York: Cambridge University Press, 1956, third edition.

Kaplan, W. ORDINARY DIFFERENTIAL EQUATIONS. Reading, Massachusetts: Addison-Wesley Publishing Co., 1962.

Pontryagin, L. S. ORDINARY DIFFERENTIAL EQUATIONS. Reading, Massachusetts: Addison-Wesley Publishing Co., 1958.

Tricomi, F. G. DIFFERENTIAL EQUATIONS. New York: Hafner Publishing Co., 1961.

7. Functions of a Complex Variable (3 semester hours).

The development of skills in this area is very important in the sciences, and the course must exhibit many examples which illustrate the influence of singularities and which require varieties of techniques for finding conformal maps, for evaluating contour integrals (especially those with multivalued integrands), and for using integral transforms.

a. Introduction (4 lectures). The algebra and geometry of complex numbers. Definitions and properties of elementary functions, e.g., e^z, $\sin z$, $\log z$, z^ν.

b. Analytic functions (2 lectures). Limits, derivative, Cauchy-Riemann equations.

c. Integration (6 lectures). Integrals, functions defined by integrals. Cauchy's theorem and formula, integral representation of derivatives of all orders. Maximum modulus, Liouville's theorem, fundamental theorem of algebra.

d. Series (5 lectures). Taylor and Laurent series. Uniform convergence, term-by-term differentiation, uniform convergence in general. Domain of convergence and classification of singularities.

e. Contour integration (3 lectures). The residue theorem. Evaluation of integrals involving single-valued functions.

f. Analytic continuation and multivalued functions (4 lectures). Analytic continuation, multivalued functions and branch points. Technique for contour integrals involving multivalued functions.

g. Conformal mapping (5 lectures). Conformal mapping. Bilinear and Schwarz-Christoffel transformations, use of mapping in contour integral evaluation. Some mention should be made of the general Riemann mapping theorem.

h. Asymptotic evaluation of integrals (3 lectures). The methods of steepest descent and stationary phase. This is a good place to develop a detailed picture of the properties of some of the special functions (e.g., Bessel and gamma functions).

i. Boundary value problems (3 lectures). Laplace's equation in two dimensions and the solution of some of its boundary value problems, using conformal mapping.

j. Integral transforms (4 lectures). The Fourier and Laplace transforms, their inversion identities, and their use in boundary value problems.

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Ahlfors, L. COMPLEX ANALYSIS. New York: McGraw-Hill Book Co., 1966, second edition.

Carrier, G. F., Krook, F., and Pearson, C. E. FUNCTIONS OF A COMPLEX VARIABLE. New York: McGraw-Hill Book Co., 1966.

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Morse, P. M., and Feshbach, H. METHODS OF THEORETICAL PHYSICS. New York: McGraw-Hill Book Co., 1953, two parts.

Papoulis, A. THE FOURIER INTEGRAL AND ITS APPLICATIONS. New York: McGraw-Hill Book Co., 1962.

Sneddon, I. N. FOURIER TRANSFORMS. New York: McGraw-Hill Book Co., 1951.

Whittaker, E. T., and Watson, G. N. MODERN ANALYSIS. New York: Cambridge University Press, 1927, fourth edition.

8. Partial Differential Equations (3 semester hours).

This course is suitable for students who have completed a course in functions of a complex variable. The emphasis is on the development and solution of suitable mathematical formulations of scientific problems. Problems should be selected to emphasize the role of "time-like" and "space-like" coordinates and their relationship to the classification of differential equations. (It seems very useful to introduce the appropriate boundary conditions motivated by the physical questions and be led to the classification question by observing the properties of the solution.) The student should be led to recognize how few techniques we have and how special the equations and domains must be if explicit and exact solutions are to be obtained; he particularly must come to realize that the effective use of mathematics in science depends critically on the researcher's ability to select those questions which both fill the scientific need and admit efficient mathematical treatment. To accomplish this realization, the instructor should frequently introduce a realistic question from which he must retreat to a related tractable problem whose interpretation is informative in the context of the original question.

a. Derivation of equations (2 lectures). The derivation of mathematical models associated with many scientific problems. Review of heat conduction in a moving medium, the flow of a fluid in a porous medium, the diffusion of a solute in moving fluids, the dynamics of elastic structures, neutron diffusion, radiative transfer, surface waves in liquids.

b. Eigenfunction expansions (5 lectures). Eigenfunction expansions, in both finite and infinite domains (Titchmarsh).

c. Separation of variables (7 lectures). The product series solutions of partial differential equation boundary value problems. Integral transforms such as the Laplace, Fourier, Mellin, and Hankel transforms and their use. Copious illustration of these techniques, using elliptic, parabolic, and hyperbolic problems.

d. Types of partial differential equations (5 lectures). The classification of partial differential equations, characteristics; appropriate boundary conditions. Domains of influence and dependence in hyperbolic and parabolic problems. The use of characteristics as "independent" coordinates.

e. Numerical methods (8 lectures). Replacement of differential equations by difference equations; iterative methods; the method of characteristics. Convergence and error analysis.

f. Green's function and Riemann's function (9 lectures). Their determination and use in solving boundary value problems. Their use in converting partial differential equation boundary value problems into integral equation problems.

g. Similarity solutions (3 lectures).

h. Expansions in a parameter (3 lectures). Perturbation methods in both linear and nonlinear problems.

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Sommerfeld, A. PARTIAL DIFFERENTIAL EQUATIONS IN PHYSICS. New York: Academic Press, 1949.

Titchmarsh, E. C. EIGENFUNCTION EXPANSIONS ASSOCIATED WITH SECOND-ORDER DIFFERENTIAL EQUATIONS. New York: Oxford University Press, Part I, 1962, second edition; Part II, 1958.

van der Pol, B., and Bremmer, H. OPERATIONAL CALCULUS. New York: Cambridge University Press, 1964, second edition.

van Dyke, M. PERTURBATION METHODS IN FLUID MECHANICS. New York: Academic Press, 1964.

9. Introduction to Functional Analysis (3 semester hours).

The purpose of this course is to present some of the basic ideas of elementary functional analysis in a form which permits their use in other courses in mathematics and its applications. It should also enable a student to gain insight into the ways of thought of a practicing mathematician and it should open up much of the modern technical literature dealing with operator theory.

Prerequisite to this course is a good foundation in linear algebra, and in the concepts and techniques of the calculus of several variables. The material of this course should be presented with a strong geometrical flavor; undue time should not be spent on the more remote and theoretical aspects of functional analysis. Topics should be developed and first employed in mathematical surroundings familiar to the student. It would be very much in keeping with the intention of the course to emphasize the relationship between functional analysis and approximation theory, discussing (for example) some aspects of best uniform or best L^2 approximation to functions, and some error estimates in integration or interpolation formulas.

While some knowledge of measure theory and Lebesgue integration is needed for an understanding of this material, it is not intended that the treatment be as complete as that in a standard real analysis course. The intended level is that to be found in the treatment by Kolmogorov and Fomin, or perhaps that in Chapters III and IV of Royden. If there is additional time, students might be introduced to some of the elementary theory of integral equations, or to applications in probability theory, or to the study of a specific compact operator, or to distributions (cf. Halperin and Schwartz, Lorch, Riesz and Sz-Nagy).

a. Metric spaces (9 lectures). Basic topological notions, mappings and continuity, complete metric spaces; examples. The fixed point theorem for contraction mappings, and applications (e.g., the initial value problem for ordinary differential equations, nonlinear integral equations, the implicit function theorem). Study of the space of complex-valued functions on a compact metric space. Equicontinuity, and Arzela's theorem, with applications. The Stone-Weierstrass theorem.

b. Normed linear spaces (9 lectures). Examples, including sequence spaces. Hölder and Minkowski inequalities. Linear functionals. The Hahn-Banach theorem, and the principle of uniform boundedness. The dual space of a normed linear space, and representations of continuous functions on various sequence spaces. Weak convergence. Linear operators on normed linear spaces, boundedness and continuity; examples. Iteration methods for solving linear systems; inversion of linear operators which are close to the identity; examples. The spectrum and resolvent of an operator.

c. Hilbert space (7 lectures). Inner product spaces, orthogonality; projections and subspaces (should be used as a concrete example). Representation of continuous functionals. Orthonormal systems, and completeness (closure). Fourier expansion, Bessel's inequality. Examples of continuous linear operators; the adjoint of a linear operator.

d. Measure theory (14 lectures). Lebesgue measure on the line. Measurable functions; convergence a.e. The Lebesgue integral; the monotone, bounded and dominated convergence theorems. L^2 as a Hilbert space. Convergence (and summability) of Fourier series. Point measures; general measures. The Riesz representation theorem. Compact operators and the Fredholm alternative. Spectral decomposition for a self-adjoint compact operator.

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Cheney, E. W. INTRODUCTION TO APPROXIMATION THEORY. New York: McGraw-Hill Book Co., 1966.

Davis, P. J. INTERPOLATION AND APPROXIMATION. New York: Blaisdell Publishing Co., 1963.

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Kolmogorov, A. N., and Fomin, S. V. MEASURE, LEBESGUE INTEGRALS AND HILBERT SPACE. New York: Academic Press, 1961.

Lyusternik, L., and Sobolev, V. ELEMENTS OF FUNCTIONAL ANALYSIS. New York: Frederick Ungar Publishing Co., 1961.

Lorch, E. R. SPECTRAL THEORY. New York: Oxford University Press, 1962.

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Vulikh, B. Z. INTRODUCTION TO FUNCTIONAL ANALYSIS FOR SCIENTISTS AND TECHNOLOGISTS. Reading, Massachusetts: Addison-Wesley Publishing Co., 1963.

Williamson, J. H. LEBESGUE INTEGRATION. New York: Holt, Rinehart and Winston, 1962.

10. Elements of Real Variable Theory (3 semester hours).

a. Real numbers (3 lectures). Describe various ways of constructing them but omit details. Least upper bound property, nested interval property, denseness of the rationals.

b. Set theory (4 lectures). Basic notation and terminology; membership, inclusion, union and intersection, cartesian product, relation, function, sequence, equivalence relation, etc.; arbitrary unions and intersections. Countability of the rationals; uncountability of the reals.

c. Metric spaces (4 lectures). Basic definitions: metric, ball, boundedness, neighborhood, open set, closed set, interior, boundary, accumulation point, etc. Unions and intersections of open or closed sets. Subspaces. Compactness. Connectedness. Convergent sequence, subsequence, uniqueness of limit. A point of accumulation of a set is a limit of a sequence of points of the set. Cauchy sequence. Completeness.

d. Euclidean spaces (4 lectures). R^n as a normed vector space over R. Completeness. Bolzano-Weierstrass and Heine-Borel-Lebesgue theorems. Topology of the line. Outline of the Cauchy construction of R. Infinite decimals.

e. Continuity (5 lectures). Functions into a metric space: Limit at a point, continuity at a point, inverses of open or closed sets; uniform continuity. Functions into R: A function continuous on a compact set attains its maximum; intermediate value theorem.

f. Differentiation (3 lectures). Review of previous information, including sign of the derivative, mean value theorem, L'Hospital's rule, Taylor's theorem with remainder.

g. Riemann-Stieltjes or Riemann integration (5 lectures). Functions of bounded variation (if the Riemann-Stieltjes integral is discussed), basic properties of the integral, the fundamental theorem of the calculus.

h. Series of numbers (8 lectures). Tests for convergence, absolute and conditional convergence. Monotone sequences, lim sup, series of positive terms.

i. Series of functions (3 lectures). Uniform convergence, continuity of uniform limit of continuous functions, integration and differentiation term by term.

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Bartle, R. G. THE ELEMENTS OF REAL ANALYSIS. New York: John Wiley & Sons, 1964.

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Spiegel, M. R. THEORY AND PROBLEMS OF ADVANCED CALCULUS. New York: Schaum Publishing Co., 1963.

Widder, D. V. ADVANCED CALCULUS. Englewood Cliffs, New Jersey: Prentice-Hall, 1961, second edition.

11. Optimization (3 semester hours).

Attempts to determine the "best" or "most desirable" solution to large-scale engineering problems inevitably lead to optimization studies. Generally, the appropriate methods are highly mathematical and include such relatively new techniques as mathematical programming, optimal control theory, and certain combinatorial methods, in addition to more classical techniques of the calculus of variations and standard maxima-minima considerations of the calculus.

This three semester hour course in optimization is planned to provide a basic mathematical background for such optimization studies. It will, in addition, acquaint the student with references and illuminate directions for further study.

a. Simple, specific examples of typical optimization problems (3 lectures). Minimization with side conditions (Lagrange multipliers, simple geometrical example). Linear program (diet problem). Nonlinear program (least squares under inequality constraints, delay line problem). Combinatorial problem (marriage or network). Variational problem (brachistochrone). Control problem (missile). Dynamic program (replacement schedule).

b. Convexity and n-space geometry (6 lectures). Convex regions, functions, general definition (homework: use definition to show convexity (or nonconvexity) in nonobvious cases, as Chebyshev error over simple family of functions). Local, global minima. Convex polyhedra (review matrix, scalar product geometry). Geometric picture of linear programming.

c. Lagrange multipliers and duality (6 lectures). Classical problem with equality constraints. Kuhn-Tucker conditions for inequality constraints. Linear programs. Dual variables as Lagrange multipliers. Reciprocity, duality theorems.

d. Solution of linear programs—simplex method (3 lectures).

e. Combinatorial problems (6 lectures). Unimodular property. Assignment problem (Hall theorem, unique representatives). Networks (min cut - max flow).

f. Classical calculus of variations (7 lectures). Stationarity. Euler's differential equation, gradient in function space. Examples, especially Fermat's principle and brachistochrones.

g. Control theory (8 lectures). Formulation. Pontryagin's maximum principle (Lagrange multipliers again).

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Saaty, T. L., and Bram, J. NONLINEAR MATHEMATICS. New York: McGraw-Hill Book Co., 1964.

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12. Algebraic Structures (3 semester hours).

This course is intended to introduce the basic algebraic properties of groups, rings, and fields, culminating in the fundamental theorem of Galois theory and some indication of its uses. The material from course 2 on linear transformations and matrices is used mainly to provide examples of groups and rings.

a. Introduction (2 lectures). Sets, relations, functions, operations, etc. Algebraic systems: integers, rationals, matrices, etc. Isomorphism and examples. Equivalence classes.

b. Groups (10 lectures). Definitions and examples: algebraic examples, geometric transformations, permutations, matrices. Subgroups, cyclic groups, basic theorems, Lagrange's theorem. Homomorphism, normal subgroup, quotient group. The isomorphism theorems. Composition series, the Jordan-Hölder theorem.

c. Rings (7 lectures). Definition and examples: integers, matrices, polynomials, etc. Integral domains, fields, quotient field. Homomorphism, ideals, residue class rings. Isomorphism theorems.

d. Unique factorization domains (7 lectures). Examples of failure of unique factorization. Euclidean domains, integers, Gaussian integers, and polynomials over a field as examples. Division algorithm, highest common factor, unique factorization in Euclidean domains. If R has unique factorization, so has $R[x]$.

e. Fields (6 lectures). Prime fields and characteristics. Extension fields. Simple algebraic extensions, minimal polynomial, structure of simple extension. Finite extensions, degree, structure.

f. Galois theory (7 lectures). Galois group. Splitting field. Separable extensions. Normal extensions. Subfield and subgroup. The fundamental theorem. Finite fields. Solvability in radicals (mainly discursive).

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