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A CURRICULUM IN APPLIED MATHEMATICS.
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COMMITTEE ON THE UNDERGRADUATE PROGRAM IN MATH.

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REPORTED IS THE DEVELOPMENT OF UNDERGRADUATE MATHEMATICS COURSES PROPERLY REFLECTING THE MATHEMATICAL NEEDS OF STUDENTS IN THE RAPIDLY DEVELOPING ENGINEERING, PHYSICAL, AND SOCIAL SCIENCES. THE PURPOSE OF THIS UNDERGRADUATE PROGRAM IS TO PERMIT STUDENTS TO DEVELOP AND NURTURE INTERESTS IN APPLIED MATHEMATICS AT AN EARLY STAGE SO THAT SIGNIFICANT INCREASES IN THE NUMBER AND QUALITY OF APPLIED MATHEMATICIANS WOULD RESULT. THE PROGRAM DESCRIBED IS PRE-GRADUATE, IN THE SENSE THAT ITS GOAL IS TO PREPARE STUDENTS FOR GRADUATE WORK IN APPLIED MATHEMATICS. PHILOSOPHY, CONTENT, AND IMPLEMENTATION OF THE PROGRAM ARE PRESENTED IN THE MAIN BODY OF THE REPORT. THIS DOCUMENT IS ALSO AVAILABLE WITHOUT CHARGE FROM CUPM CENTRAL OFFICE, P. O. BOX 1024, BERKELEY, CALIFORNIA 94701. (RP)

MATHEMATICAL ASSOCIATION OF AMERICA

A CURRICULUM IN APPLIED MATHEMATICS

COMMITTEE ON THE UNDERGRADUATE PROGRAM IN MATHEMATICS

1966

A CURRICULUM IN APPLIED MATHEMATICS

Report of

AD HOC SUBCOMMITTEE ON APPLIED MATHEMATICS

to

Committee on the Undergraduate Program in Mathematics
January 24, 1966

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PREFACE

The Committee on the Undergraduate Program in Mathematics has always had as one of its major concerns the development of undergraduate mathematics courses properly reflecting the mathematical needs of students in the rapidly developing engineering, physical, and social sciences. The ad hoc Subcommittee on Applied Mathematics of CUPM was appointed in 1964 by Professor William L. Duren, Jr., then Chairman of CUPM. One of its charges was to suggest appropriate undergraduate programs for students planning careers in applied mathematics.

The Subcommittee finds that, in this country, there is an almost total lack of formal undergraduate programs, comparable in stimulation and content to those in pure mathematics, whose specific intent is the preparation of mathematics students for graduate work in applied mathematics. It is strongly convinced that a well-formulated undergraduate program which would permit students to develop and nurture interests in applied mathematics at an early stage in their mathematical education would result in significant increases in the number and quality of applied mathematicians.

In this report the Subcommittee describes an undergraduate program in applied mathematics for mathematics majors. It is a pre-graduate program in the sense that its goal is to prepare students for graduate work in applied mathematics — it is not be be construed as a terminal undergraduate program. The philosophy, the content, and the implementation (an extremely important aspect in the Subcommittee's view) are set out in the following pages.

INTRODUCTION

The primary aim of applied mathematics is the understanding of a wide spectrum of scientific* phenomena through the use of mathematical ideas, abstractions, methods and techniques. The applied mathematician is at once a mathematical specialist and a versatile scientist, whose interests and motivations derive from a strong desire to confront highly complex or descriptive situations with mathematical analysis and ideas. In essence, then, in his research and teaching activities the applied mathematician contributes to the development of both mathematics and science by bringing these disciplines into closer relationship with one another.

The success of an applied mathematician is highly dependent upon his ability to formulate idealized but relevant mathematical models of scientific situations, and to pose precise and cogent mathematical questions of the models which, on the one hand, have a likelihood of being answered and, on the other hand, may be pertinent to an understanding of the original situation. In this process he frequently must exercise careful scientific judgment and sophisticated mathematical insight. Ultimately, the applied mathematician seeks to abstract the essential mathematical features of a given model in the hope of making it more generally applicable. Frequently this leads him into purely mathematical research.

The education of an applied mathematician, therefore, should contain the following three basic ingredients:

(1) A substantial knowledge of the concepts and methods of the various branches of modern mathematics and a considerable expertise in those mathematical areas most closely related to his particular applied interests.

(2) An understanding in depth of the principles, methods, and practice of some scientific areas.

(3) The development of the desire and the ability to confront scientific situations with mathematical ideas and analysis.

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* In this report the words <u>science</u> and <u>scientific</u> are used in a very broad sense. They refer not only to the classical physical and engineering sciences but also to the social, computing, biological, medical, and management sciences.

The amount and diversity of knowledge that these criteria imply make it clear that the formal education of an applied mathematician must extend well beyond the undergraduate level. In point of fact, almost all organized educational programs in applied mathematics in this country are graduate school activities. Little effort has been expended at the undergraduate level to prepare and mold the undergraduate student in mathematics for graduate work in applied mathematics. Mostly, such students have been left to their own devices to pursue a course of study alternating between pure mathematics and specific fields of science. Many students who may have been motivated initially toward applied matnematics lose that motivation because they are not exposed to the spirit and practice of applied mathematics soon enough. Moreover, it is not unusual for students to be unaware of the possibility of pursuing a career in applied mathematics.

STATEMENT OF THE PROGRAM

The undergraduate program in applied mathematics set out here is a program for mathematics majors and is meant to be preparatory for graduate work in applied mathematics and allied fields. The essential feature of the program is the introduction of courses in applied mathematics at an early stage in the undergraduate program. These courses are designed to develop, stimulate, and nurture the attitudes and practice of applied mathematics. Their main themes are the construction, analysis, and interpretation of mathematical models for significant and interesting phenomena and situations.

The program has three principal aspects:

(a) A pregraduate, undergraduate major in mathematics, slightly modified and a bit more rigidly prescribed than the CUPM pregraduate recommendations outlined in the report Preparation for Graduate Study in Mathematics [C].* It is important that the student in this program obtain a broad knowledge and a firm mathematical grounding in pure mathematics, especially in the basic concepts, logical structure and techniques of analysis and algebra, so as to qualify him for the study of advanced mathematics in graduate courses. An important educational desideratum is that the student be made aware of the important role of computers in the applications of mathematics.

(b) A study in some depth of one or two particular fields of application. For this purpose, we recommend two to four semesters of upper division or beginning graduate courses in such fields. These courses should stress fundamental principles and analytical methodology. Naturally, the student in his first two years will take the basic elementary science courses prerequisite for such study. Indeed, we view this collateral science study as an undergraduate minor for the applied mathematics student. We have appended a list — by title only — of such depth courses in the section on course descriptions. Further remarks concerning such courses appear in the section on implementation of the program.

(c) A positive and stimulating study of the practice of applied mathematics. As stated earlier, we propose to accomplish this part of the program through courses given in the mathematics department on mathematical model building and analysis. They should illustrate the vital interplay between mathematics and science. (The CUPM report A General Curriculum in Mathematics for Colleges [A]

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* See Bibliography, page 27.

contains an excellent description of the mathematical model building process, pp 66-67.) Such courses are crucial to the program: they form the vehicle for developing the motivations and the abilities of the student in applied mathematics. It is, therefore, essential that these courses be given with the same high standards and intellectual excitement as the pure mathematics courses. Only an applied mathematician, or a mathematician with sympathetic and knowledgeable interests in applied mathematics, should offer such courses.

The recommendation of the Subcommittee is that the equivalent of a year's course of the type mentioned above be required of the applied mathematics undergraduate. While this course should appear as early as possible in the undergraduate program, it is obviously necessary that the student already possess a measure of mathematical sophistication and familiarity with some basic science. We feel strongly that the junior year is the most appropriate time for this course.

The model building, analysis, and interpretation aspects of these courses are very important, perhaps more so than the specification of the scientific or mathematical topics covered. Certainly the exact scientific topics covered will be dictated, in large measure, by the interests and experience of the instructor. However, the aspect of model building should lead but not dominate the mathematical analysis and interpretation aspects. In particular, such a course should not consist merely of a large collection of unrelated models requiring only very trivial or elementary mathematics for their analysis. The selected topics and models should lead to the development of a substantial body of related mathematics. The mathematical aspects of such a course should be of the same high standards as any other mathematics course; the introduction of new concepts should, whenever feasible, be motivated by the applications under discussion. We emphasize as strongly as we can that these courses are not to be pure methodology courses or, for that matter, mathematical service courses for other disciplines. Indeed, pure mathematics students should also be urged to take such courses to give them a view of the manner in which mathematics interacts with science.

In spite of the fact that the actual topics covered in such model building are less important than the spirit, some coherency is desirable. For this reason, we have outlined two such two-semester courses: one oriented toward the physical and engineering sciences, in which analysis and differential equations play a dominant role; and the other oriented toward the social and management sciences, in which linear algebra, probability and statistics are dominant. These outlines, with appropriate comments, are to be found in the course description section. Certainly other themes — e.g., the computer sciences — are feasible.

IMPLEMENTATION OF THE PROGRAM

To implement the Subcommittee's undergraduate program in applied mathematics at a given university or college it is necessary that (1) the mathematics department be capable of offering a pregraduate mathematics program as outlined in the CUPM recommendations [A] and [C]; (2) that there exist properly qualified instructors for the crucial model building courses; (3) that there exists at the given institution educational activity of an advanced nature in at least one area of application, e.g., the physical, engineering, biological, computing, social or management sciences.

In connection with (1), we note that the Subcommittee's program in applied mathematics is an option in mathematics and, therefore, does not require two distinct tracks or types of mathematics courses: one for students of pure mathematics and a counterpart for students of applied mathematics. We should like to call attention, indeed, to one of the objectives of the pregraduate curriculum [C]; namely that every student preparing for graduate work in mathematics should, in his mathematics courses, "be (made) aware of the applicability of mathematics and of the constructive interplay between mathematics and other disciplines."

We emphasize as strongly as we can that the success of the Subcommittee's program is very much dependent upon maintaining the proper intellectual attitudes both in material and instruction for the applied mathematics model building courses. The program should only be attempted where there are willing and competent instructors for such courses. At present, there exists little comprehensive text material in the spirit in which we envisage these courses; this material must, to a great extent, be created by the instructors.

Although the depth courses in fields of application will generally be offered by departments other than mathematics, the mathematics department must be actively concerned about them. First of all, at a particular school the only fields which should be considered as possible areas of interest for the applied mathematics student are those which offer suitable advanced courses, of a fundamental and analytical nature, without long strings of technical prerequisites. Secondly, it is desirable that such fields be closely related to some of the applied mathematical interests within the mathematics department. It must be the responsibility of the mathematics department to determine which fields of application and which courses are appropriate for the applied mathematics program, and it should list such courses in the description of its applied mathematics program so that the student may more effectively plan his educational goals.

Finally, we believe that the success of such an applied mathematics program is very much dependent upon the intellectual atmosphere and guidance provided for the applied mathematics student by the mathematics department. This atmosphere and guidance should be comparable to that provided for the pure mathematics student.

DESCRIPTION OF COURSES

(a) Pregraduate mathematics courses. In these recommendations for mathematics courses for the applied mathematics undergraduate, we rely on the course descriptions in the CUPM reports A General Curriculum in Mathematics for Colleges [A] and Preparation for Graduate Study in Mathematics [C]. The course numbering used here is that in [A]. All GCMC courses are semester courses.

In the first two years, the student should have a set of calculus courses containing material equivalent to the four semesters of GCMC 1, 2, 4, 5. This sequence should provide the student with a good intuitive notion of the limit concept, an appreciation of mathematical rigor and proof, a firm knowledge of the techniques of the calculus, and the ability to use the methods and ideas of the calculus to formulate, solve, and interpret problems in areas of application. Moreover, the sequence should provide an introduction to differential equations and an introduction to vector and multivariate calculus, including some applications to other fields of mathematics and science.

No later than the first semester of his second year, the student should have a course (such as GCMC 3) in linear algebra and its applications. In addition, it would be desirable for the applied mathematics student to have some exposure to elementary probability and statistics during his first two years; GCMC 2P would serve this purpose. In the event that GCMC 2P is not available, then an upper division course including material from GCMC 2P and GCMC 7 should be taken in the third or fourth year.

As early as possible the applied mathematics student should have an introduction to computer science; either as a formal course or in connection with his other early courses. An outline of a one-semester introductory course in computer science is given in the report on computing [E] prepared by CUPM's Panel on Physical Sciences and Engineering.

The upper division mathematics courses might be as varied for a particular applied mathematics student as for a pure mathematics student. Nevertheless, all applied mathematics students should continue their studies of analysis and algebra by taking a real analysis course equivalent to GCMC 11, an algebraic structures course such as GCMC 6, and the numerical analysis course GCMC 8. Students whose interests point toward the physical or engineering sciences should take the complex variable course GCMC 13 and possibly an intermediate differential equations course which concentrates on boundary value and eigenvalue problems for linear ordinary differential equations and on the corresponding application of these ideas to partial differential equations. Students primarily interested in applications to the management, social and computing sciences should take a probability course which includes some material on stochastic processes (GCMC 7 is such a course). In addition a semester course, in which some of the elementary notions of normed linear spaces and of functional analysis are introduced, should be included for these students. Any additional work that may be necessary to complete a major in mathematics would be on an elective basis.

(b) Depth courses. We list below titles of possible depth courses as illustrations. In each case, the mathematics department should determine the suitability of such courses from the applied mathematics point of view: Does it stress fundamental principles and the analytical aspects of the field? In many cases — at least in the physical sciences area — excellent courses with the titles mentioned below are generally available to students who have good backgrounds in freshman and sophomore physics. From two to four semesters' work in one or two areas of concentration is suggested.

The following list gives a general idea of the kind of subject matter recommended for depth courses in the physical sciences: Celestial Mechanics; Analytical Mechanics; Fluid Mechanics (hydrodynamics, aerodynamics); Elasticity and Plasticity; Quantum Mechanics; Statistical Mechanics; Thermodynamics; Control Theory; Information Theory. Courses in these subjects should be chosen which have a high degree of mathematical content.

For the social and management sciences, courses such as the following are recommended: Theory of Games; Theory of Linear Inequalities; Linear Programming; Non-Linear Programming; Dynamic Programming; Queueing Theory; Systems Analysis; Optimization in General Systems; Production and Inventory Control; Automatic Process Control.

(c) Model building or applied mathematics courses. Broad outlines for two possible year courses in applied mathematics are given here. It is recommended that these courses be offered in the junior year. The first is slanted toward the engineering and physical sciences, while the second is concerned with applications to the social and management sciences. In both cases more material is mentioned than can be covered in a year, thus enabling an instructor to make a suitable choice of topics depending upon his and the students' interests. It is emphasized that the mere listing of mathematical or applied topics and references is not sufficient to describe such courses. The various topics become meaningful only when they are interpreted in the following sense: The depth to which any particular topic is penetrated is that which is necessary to demonstrate the vital interaction between the model building and analysis process and the mathematical concepts and techniques involved. The instructor must achieve a delicate balance between the desire to pursue new mathematical ideas that arise for their own sake and contenting himself with illustrating the depth of such ideas by well-chosen examples and applied problems.

The two outlines differ in their organization. The first, which involves the physical sciences, lists a number of "classical" topics which might be said to fall under the general categories of differential equations on the mathematical side and particle and continuum mechanics on the physical side. Material for such topics, though not accessible in any one text at present, can be found in a variety of well-known books. However, what is conspicuously lacking in the literature are good sources containing the appropriate intertwining of the mathematics and the applications. It is hoped that suitably integrated material, written in the spirit we have in mind, will appear in the textbook literature before too long.

The second course, aimed at the social and management sciences, is organized somewhat differently. A variety of topics is listed under two general headings: Deterministic Models and Stochastic Models. The material contained in this course is relatively new and, therefore, source material is included along with relevant comments for each topic. No single text (or pair of texts) is available for this course, and the Subcommittee again expresses the hope that such material will soon appear in suitable texts.

INTRODUCTION TO APPLIED MATHEMATICS
Physical Science Option

The purpose of this course is to demonstrate the strong interdependence between mathematics and the physical sciences and engineering. This is to be accomplished through the construction, analysis and interpretation of mathematical models for several interesting and significant physical problems. This course should be offered in the junior year. Mathematical prerequisites are the introductory analysis and linear algebra courses: GCMC 1 through 5. In addition the student should have studied physics in college and be familiar with the basic laws of physics, especially mechanics.

The mathematical applications in this course are primarily in particle and continuum mechanics. The course outline has been divided into five parts according to the mathematical techniques required for the construction of the mathematical models considered. Although many other areas of mathematical analysis might have been included here, they had to be omitted to keep the list within reasonable bounds. The topics that are included center around the theory of differential equations; hence they are closely related to the student's elementary work in calculus and physics.

The spirit in which the course is presented is of utmost importance: It is essential to maintain a vital and significant interplay between the applications and the mathematical developments. Although the various topics should be presented with appropriate mathematical soundness and should be properly integrated, care should be taken to insure that much of the motivation for the mathematics stems from the applications.

a. Ordinary differential equations. Although most applications in this section are to particle dynamics, it is important to consider problems from diverse areas of science which lead to ordinary differential equations as mathematical models. Excellent examples are: circuit theory, chemical reactors, biological systems, celestial mechanics, and others. The mathematical theory of systems of linear differential equations should be done in matrix form; general representation formulas for the solution of the initial value problem for such systems should be derived. The treatment of nonlinear problems should be primarily qualitative, involving systems with one degree of freedom, and properly illustrated with well-chosen examples. Throughout this section numerical methods should play an important role and, if possible, a computer should be utilized.

Linear systems with constant coefficients. Normal modes. Resonance. General first order linear systems, representation formula for solutions of the initial value problem. Conservative systems of particles, small vibrations. Pendulum motion and particle motion in a nonlinear resistive medium. Statement and proof of the basic theorems for initial value problems: local existence, uniqueness, and differentiable dependence on parameters. Linearization, local stability, and simple phase-plane geometry of trajectories. Self-sustained oscillations of a nonlinear (nonconservative) system. Forced oscillations of a nonlinear system (e.g. $\ddot{x} + k^2(x-\mu x^3) = A \cos \omega t$) and the corresponding resonance phenomenon. A simple singular perturbation problem illustrating boundary layer phenomena. Numerical schemes for solution of initial value problems, stability analysis of such schemes.

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b. Diffusion processes. The classical model for heat conduction is continuous. Here we generalize considerably to include discrete models as well as the limiting cases of continuous models. The amount of probability theory needed is minimal and can be covered easily. Within this framework one can consider a simple linear ordinary differential equation system forced by a random input (one dimensional Brownian motion is a good example).

One dimensional diffusion: $u(x,t_{n+1}) = \int u(x+\xi,t_n)\, dF_n(\xi)$

where $F_n(\xi)$ are either pure jump or absolutely continuous probability distributions. The notion of a generating function and the use of asymptotic expansions. One dimensional Brownian motion. Heat conduction and neutron diffusion as limits of discrete diffusion processes. Initial value problem for the heat equation. The fundamental solution and its probabilistic interpretation. Heat conduction in a finite rod and in a homogeneous sphere; boundary conditions. Regular eigenvalue problems for second order linear ordinary differential equations arising from separation of variables. Uniqueness theorems. Numerical methods for the solution of problems involving the heat equation; the von Neumann stability criterion.

A simple two-phase problem for the heat equation in which the moving boundary between the phases is to be determined is an interesting problem to consider numerically on a computer: e.g., a model for the freezing of a lake or for geological strata.

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Uhlenbeck, G., and Ford, G. W., with Montroll, E. W. LECTURES IN STATISTICAL MECHANICS. (Lectures in Applied Mathematics, Volume I) Providence, Rhode Island: American Mathematical Society, 1963.

c. Partial differential equations. This is not meant to be a general treatment of partial differential equations. Rather, a few simple equations, such as the Laplace and the wave equation, should be studied as models for a wide range of applications in continuum mechanics, fluid mechanics, theory of sound, electrostatics, etc. Although, at this level, the principal technique for solving such problems will be the method of separation of variables, stress should be placed on general features such as the fundamental notion of a Green's function and its use in understanding the behavior of general solutions, and the notion of characteristics and their important role in propagation phenomena. At least one simplified nonlinear model should be considered.

Derivation of the equations of motion for a fluid and/or an elastic body from general integral conservation principles. Special problems in plane elasticity, plane incompressible isentropic flow, electrostatics, which lead to the Laplace equation. Separation of variables. Poisson's formula for the disk and the half-plane.

Newtonian potentials, simple pole and dipole distributions. Green's function and Green's formula. Maximum principle for harmonic functions. Some numerical methods for the solution of boundary value problems for the Laplace equation. Wave propagation (sound, light, etc.) along a line and in space. Characteristics. Plane waves, dispersion, scattering. Vibration of strings, membranes, rods. A qualitative study of a simplified nonlinear problem as a model for shock waves in a tube, water waves in a shallow sea, or diffusion with a nonlinear transport term.

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d. Calculus of variations. The material of this section is a rather brief excursion into the calculus of variations with a view toward obtaining global mathematical models for a number of applications discussed in the previous sections. Indeed, this material may well be incorporated in those sections. Courant-Hilbert, Volume I, has an excellent introductory chapter on the calculus of variations.

The brachistochrone. The Euler-Lagrange equations. Hamilton's principle and the Hamilton-Jacobi equations; planetary motion, geometrical optics. The use of variational problems to derive appropriate boundary conditions. Energy principles for elastic bodies. Min-max characterization of natural frequencies. Rayleigh-Ritz computations for simple torsion and buckling problems. A simple control problem or "rocket programming" problem.

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e. Fourier analysis. Although Fourier series have already been used in previous sections, the topic is of sufficient importance in applied mathematics to warrant systematic treatment, and of all the topics in this course it is perhaps the easiest to treat formally at this level. In several fields of application, such as circuit theory and information theory, spectral (frequency) models are intimately connected with the basic physical concepts. The (L_2) inversion problem for Fourier series serves as an excellent illustration of the necessity for using a more sophisticated notion of integration.

A proof of the Weierstrass polynomial approximation theorem. Fourier series of continuous functions, uniqueness, C-1 summability. Statement and proof of convergence theorems for Fourier series of piece-wise smooth functions. Gibb's phenomenon. Response of a linear system to a periodic input. Mean-square approximation of the partial sums of a Fourier series to the function; the Parseval relations, convolutions. An introductory excursion into a simple prediction or time series problem may prove interesting.

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INTRODUCTION TO APPLIED MATHEMATICS
Optimization Option

The purpose of the course is the presentation of important and representative illustrations of the work of applied mathematicians. This work has three major aspects: (a) the formulation of scientific problems in mathematical terms, (b) the solution of the mathematical problems that result, and (c) the interpretation and evaluation of the solution. The primary emphasis of this course will be on the phases of formulation and interpretation. This means that careful attention must be given to the prior training of the students and that the level of the mathematics used must be consistent with their preparation. Mathematical methods, such as Lagrange multipliers or linear inequalities, which the student may not have encountered in his previous courses, are to be developed only to the degree needed for the application. Although this may limit the discussion to special cases of more general models, it seems clear that the central objective of teaching the nature of formulation and interpretation is better served by this restriction. The consideration of mathematical methods in depth is left to a series of courses which can parallel and follow this introduction. It is intended that this course be given in the junior year.

Deterministic Models.

a. Inventory problems. Sources: Churchman, Ackoff, and Arnoff [6] give the most thorough treatment of elementary problems (Chapters 8-10, pp. 199-274). The mathematics used can follow any traditional calculus sequence; an extensive bibliography follows Chapter 8, pp. 232-234. The basic reference on applications is Whitin [34], which summarizes most of the developments up to about 1952. An excellent expository account of more modern techniques of inventory control has been given by Ladermann, Littauer, and Weiss [20]. There are numerous sources of examples and exercises such as Saaty [28] (see Example 2, p. 159 ff.).

b. Growth and survival models. Sources: Kemeny and Snell [19] give a very successful exposition of two dynamic models derived from ecology in Chapter III, pp. 24-34, complete with 13 exercises. The material is drawn largely from Lotka [21] (Chapter VIII). Another source of applications which are based similarly on systems of linear differential equations is given by Rappaport [26], who draws in turn on the original work on arms races by Richardson.

c. Scheduling problems. Sources: Churchman, Ackoff, and Arnoff [6] give an introduction at the proper level (Chapter 16, pp. 450-476) and include a bibliography of 19 items. A special result of importance is contained in Johnson, "Optimal two and three-stage production schedules with set-up times included," [17] and is treated quite well in Bellman and Dreyfus [4], pp. 142-145, with further bibliography on pp. 150-151.

d. Dynamic programming. Sources: This subject could be introduced via the Johnson scheduling result cited above or via the extensive example developed by Kemeny and Snell [19], Chapter IX, pp. 109-119, which is accompanied by 15 exercises. After this the problem is an excess of sources. The basic references are Bellman [3] and Bellman and Dreyfus [4]. The latter is more in the spirit of this course, as are Howard [15] and Hadley [13].

e. Linear programming. Sources: The problem here is again one of too many sources. A rather novel instance of a formulation is the problem of optimization of structural design treated by Prager [25]. The basic reference is Dantzig [7], which emphasizes formulation in Chapter 3, pp. 32-68. Economic formulations and interpretations are well treated in Baumol [2]. At a minimum, the theory and interpretation of duality should be included in this unit. The mathematical treatments of Dantzig [7] or of Gale [11] could be used here. As sources of additional applications, exercises and interpretations, Hadley [12] and Dorfman, Samuelson, and Solow [8] can be recommended.

f. The theory of games. Sources: If we remember that the objective of this course is to introduce the student to the problems of formulation and interpretation encountered in applied mathematics, the selection of articles collected by Shubik [29] seems ideal. This contains such classic papers as Milnor's work on "Games against nature" and McDonald and Tukey on "Colonel Blotto game," as well as significant excerpts from Luce and Raiffa [22] and von Neumann and Morgenstern [33].

g. Nonlinear programming. Sources: Examples abound to introduce this subject. However, a particularly satisfactory example may be found in "Optimal horse race bets" by Isaacs [16]. The textbook situation has been much improved by the appearance of Had..., [13]. The minimum content of the theory to be introduced in this unit is the use of Lagrange multiplier techniques for nonlinear problems constrained by inequalities (e.g., Chapter 6, "Kuhn-Tucker theory," in Hadley). A particularly fine exposition is provided by Tucker [31]. Economic applications are treated by Baumol [2] and in an excellent exposition done by Enthoven as an appendix to Hitch and McKean [14], pp. 361-405. Another source of example and exercise material is Vajda [32].

Stochastic Models.

a. Inventory problems. Sources: Dvoretzky, Kiefer, and Wolfowitz [9], [10]. See also the article by Ladermann, *et al.* [20]. An excellent stochastic dynamic inventory model is developed in Hadley [13], pp. 402-409. Reference should also be made to H. Scarf's elegant paper "The optimality of (S, s) policies in the dynamic inventory problem" in Arrow, Karlin, and Suppes [1], pp. 196-202. There are many other sources for applications to simple stochastic demand, both discrete and continuous.

b. Queueing problems. Sources: A satisfactory textbook treatment appears in Churchman, Ackoff, and Arnoff [6], pp. 389-449. An adequate bibliography is given there and the second half of this selection is the "classical" study of traffic delays at toll booths by L. C. Edie. Chapter 11 of Saaty [28], "Resume of Queueing Theory" is more suited to the level of this course than his book on the same subject and contains a useful listing of applications on pp. 364-367.

c. Markov chains. Sources: A typical example with which this subject can be introduced is the model taken from sociology by Kemeny and Snell [19], Chapter V, pp. 54-65. A more complete treatment of the theory is given by the same authors in [18]. Other applications are discussed by Howard [15] and an important class of problems are drawn from learning theory (see, for example, Bush and Mosteller [5]).

d. Simulation. Sources: Most of the sources here are in the nature of individual applications. However, reference may be made to Rich [27] and Thomas and Deemer [30]. On the "Monte Carlo method" (as opposed to simple simulation) see Metropolis and Ulam [23] and Meyer [24].

e. Utility theory. Sources: For a self-contained account of a major portion of modern utility theory, it is hard to improve on Chapter 2 of Luce and Raiffa [22]. This chapter will lead to much of the relevant literature, both theoretical and experimental (see pp. 34-37).

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BIBLIOGRAPHY

The following booklets contain the recommendations of CUPM for several programs related to the one discussed here. Familiarity with item A is in fact necessary for an understanding of this report; C and E are also cited. These publications may be obtained free of charge by writing CUPM, Post Office Box 1024, Berkeley, California 94701.

A. A General Curriculum in Mathematics for Colleges (GCMC)

B. Pregraduate Preparation of Research Mathematicians

C. Preparation for Graduate Study in Mathematics

D. Recommendations on the Undergraduate Mathematics Program for Engineers and Physicists

E. Recommendations on the Undergraduate Mathematics Program for Work in Computing

F. Tentative Recommendations for the Undergraduate Mathematics Program for Students in the Biological, Management, and Social Sciences