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TENTATIVE RECOMMENDATIONS FOR THE UNDERGRADUATE MATHEMATICS PROGRAM OF STUDENTS IN THE BIOLOGICAL MANAGEMENT AND SOCIAL SCIENCES.

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COMMITTEE ON THE UNDERGRADUATE PROGRAM IN MATH.

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THIS REPORT DESCRIBES A PROGRAM FOR THE UNDERGRADUATE MATHEMATICAL PREPARATION OF STUDENTS IN THE BIOLOGICAL, MANAGEMENT, AND SOCIAL SCIENCES (BMSS). THE COMMITTEE RECOMMENDS A SEQUENCE OF COURSES WHICH IS DESIGNED TO PROVIDE VARIED TRAINING IN MATHEMATICS IN THE LIMITED TIME BMSS STUDENTS HAVE AVAILABLE. OF SPECIAL IMPORTANCE ARE ELEMENTARY ANALYSIS, PROBABILITY AND STATISTICS, AND LINEAR ALGEBRA. THE PRINCIPAL IDEAS UNDERLYING THE SEQUENCE FOR BMSS STUDENTS ARE (1) MATHEMATICS IS NEEDED PRIMARILY AS A LANGUAGE FOR SCIENTIFIC REASONING, (2) BMSS STUDENTS DO NOT NEED AS MUCH TRAINING IN DETAILED TECHNIQUES AS DO MATHEMATICS AND PHYSICAL SCIENCE STUDENTS, (3) IT IS UNREASONABLE TO EXPEND AS MUCH TIME ON RIGOROUS PROOFS FOR BMSS STUDENTS AS FOR MATHEMATICS MAJORS, AND (4) MORE STRESS SHOULD BE PLACED ON APPLICATIONS WHICH ARE OF SPECIAL INTEREST IN THE BIOLOGICAL AND SOCIAL SCIENCES. THE RECOMMENDATIONS OF THE COMMITTEE FALL INTO THREE PARTS--(1) A BASIC FOUR-COURSE, TWO-YEAR SEQUENCE FOR BMSS STUDENTS, (2) A TWO-COURSE PROBABILITY AND STATISTICS SEQUENCE, AND (3) SOME ADVANCED COURSES THAT SERVE AS ELECTIVES. THE COMMITTEE ON THE UNDERGRADUATE PROGRAM IN MATHEMATICS IS CONVINCED THAT THE REQUIREMENT OF SIX COURSES--THE BASIC SEQUENCE PLUS THE PROBABILITY AND STATISTICS SEQUENCE--IS MINIMAL FOR FUTURE GRADUATE STUDENTS IN THE BMSS AREAS. THIS DOCUMENT IS ALSO AVAILABLE WITHOUT CHARGE FROM CUPM CENTRAL OFFICE, P. O. BOX 1024, BERKELEY, CALIFORNIA 94701. (RP)

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TENTATIVE RECOMMENDATIONS FOR THE UNDERGRADUATE MATHEMATICS PROGRAM OF STUDENTS IN THE BIOLOGICAL, MANAGEMENT AND SOCIAL SCIENCES

FIRST DRAFT
Prepared to serve as a basis for discussion. Comments are invited.

COMMITTEE ON THE UNDERGRADUATE PROGRAM IN MATHEMATICS

JANUARY, 1964

The Committee on the Undergraduate Program in Mathematics, supported by the National Science Foundation, is a committee of the Mathematical Association of America, charged with making recommendations for the overall improvement of college and university mathematics curricula at all levels and in all educational areas. Moreover, the Committee devotes its energies to reasonable efforts in realizing these recommendations.

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The CUPM Panel on Mathematics for the Biological, Management, and Social Sciences has been primarily concerned with the mathematics curriculum as it relates to the education of students in these areas. In this report we present our recommendations on required and optional mathematics courses for prospective graduate students in these fields.

In order to encourage the implementation of the curriculum improvements inherent in these Recommendations, the Panel encourages appropriate conferences, experimental programs, and textbook writing.

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NOTE

These recommendations are tentative. This document has been prepared and is being distributed to serve as a basis for discussion by mathematicians and specialists in the biological, management, and social sciences. Comments from readers are invited and will help the Panel in its subsequent work. Please send comments to any Panel member or to the CUPM Central Office, Post Office Box 1024, Berkeley, California 94701.

INTRODUCTION

The Committee on the Undergraduate Program in Mathematics recommends that departments of mathematics offer courses designed for the needs of students in the biological, management, and social sciences (BMSS).

There is an increasing need for improved undergraduate mathematical training for future graduate students in the BMSS areas. The Social Science Research Council years ago recommended substantial undergraduate mathematical training for future social scientists. The following recommendations are in the same spirit as those made by the SSRC. There is also increasing use of advanced mathematics in many branches of biology and medicine. More recently, some graduate schools of business have been requiring more mathematical preparation of entering students, and have been introducing sophisticated mathematical courses as part of their own programs.

A survey of this Panel has shown that graduate schools in all BMSS areas would like to have more mathematical training for their entering students, but that very few of these students today have enough preparation, or preparation of the type needed. Since the exact need of BMSS students is hard to predict, they benefit most from preparation in considerable breadth in both classical and modern topics in mathematics. Of special importance are elementary analysis, probability and statistics, and linear algebra.

Since high speed computers are sure to play an increasingly important role in all BMSS areas, prospective graduate students should have the mathematical training necessary for data processing, and should have some experience with computers.

The problem is to provide this varied training in the limited time BMSS students have available for mathematics during their undergraduate years. It is, therefore, important to recognize that they need mathematics primarily as a language for scientific reasoning, and that they do not need as much training in detailed techniques as mathematics and physical science students. Nor is it reasonable to expend as much time on rigorous proofs for BMSS students as for mathematics majors. Also, more stress should be placed on applications which are of special interest in the biological and social sciences.

These reasons have convinced CUPM that special courses must be developed for BMSS students.

The following comments are offered in order to clarify basic assumptions and other features of the recommendations.

1. Background of the students. In recent years much effort has been expended to improve mathematics education in the elementary and secondary schools. Several programs of improvement in secondary schools have already had considerable effect and can be expected to have a great deal more. The recommendations described herein are not based upon the assumption that students have studied a particular set of materials in the secondary school, but we do assume that the underlying theme of these new programs will become increasingly prevalent. In particular, we assume that mathematics courses in the secondary school will contain a judicious mixture of motivation, theory, and applications, and that students will be accustomed to care and precision of mathematical thought and statement.

2. Prerequisites. We assume that students in the BMSS areas are prepared to begin their collegiate mathematics with a calculus level course, and that all prerequisites for calculus will be taught in high school; though at present it may be necessary to include some analytic geometry in the first-year course. The Panel recognizes that some departments of mathematics offer remedial courses, and in some cases on a credit basis, in order to correct other deficiencies. However, the Panel cannot consider remedial courses as part of a valid college program. In any case, the final level of attainment, rather than the number of courses, should serve as an indication of proficiency.

Specifically, it is assumed that students have at least some acquaintance with sets, functions, trigonometric functions, mathematical induction, binomial coefficients, and the summation notation.

3. Counselling. Historically, mathematics has been closely allied to the physical sciences, especially to physics. In secondary schools and in elementary undergraduate courses, applications of mathematics have usually been limited to the physical sciences. Therefore, it is not uncommon for students whose interests lie in other fields to enroll in a bare minimum of mathematics courses. If students are to possess the prerequisites stated above, then proper counselling both at the high school and collegiate level is imperative. Students must be made aware of the doors that are closed to them in BMSS fields as well as in physical sciences and engineering when they terminate their study of mathematics prematurely. We intend to transmit this message to guidance and counselling personnel, and we urge all concerned to give attention to ways by which counselling of potential BMSS students can be improved in their locality.

4. Time. These recommendations are intended as a program for today, not for several years in the future. Large parts of the recommendations are very similar to emerging programs at various places, and portions of the sample courses we outline are patterned after existing ones.

Five or ten years from now the situation will undoubtedly be different -- in high schools, in research, in applications, and in such adjacent areas as computation. Such differences will necessitate changes in the mathematics curriculum. A good curriculum can never be static, and it is our belief that the present proposal should be continually modified to keep up with developments. However, the Panel envisions that the materials encompassed here will certainly continue to be an important part of the mathematical education needed by students in the BMSS areas. It should be emphasized that the recommendations are minimal in nature.

5. Flexibility. It must be pointed out that the outlines presented in this booklet are course guides. The Panel is convinced that courses embodying the spirit and general nature of the outlines are appropriate for students in the BMSS areas. In order to make its recommendations more meaningful and better understood, the Panel has presented suitable outlines for the suggested courses. The phrase "a course" refers to a three semester-hour presentation of the subject matter described, but appropriate adjustments can easily be made for other curricular arrangements. Although we suggest a definite number of lectures for each topic in our course outlines, these are intended only to serve as approximate guides and to indicate the relative weights we would assign to different topics in a course.

The Panel does not intend to convey the notion that only those topics included in the outlines are appropriate, or that the order of presentation must be rigidly fixed to conform to the outlines. Everyone recognizes that certain aspects of mathematics are of a sequential nature and do not permit an interchange in the order of presentation of topics. However, there must be available considerable flexibility to allow for variations in the quality of students and in the taste of instructors.

6. Audience. These recommendations are intended as a means of communication from CUPM to departments of mathematics. Despite the fact that many of the disciplines in the BMSS area are represented on the Panel, and that leaders in these fields are being consulted, the Panel does not believe that it is the function of a committee of the Mathematical Association of America to exert pressure on other departments to adopt requirements for their majors. We hope that these disciplines give serious consideration to the recommendations. CUPM, through members of its Panels,

members of the CUPM Consultants Bureau, and officers of the Central Office, stands ready to cooperate and consult with faculties that are attempting to improve the mathematical competence of their students. Cooperation and coordination among mathematics, biology, management, and social science faculties is necessary, and may lead to course changes in all fields.

7. Clientele. The suggested courses are recommended for prospective graduate students in the BMSS area. In addition, students who will terminate their education with the bachelor's degree, but who will have completed a strong major and intend to do serious work in their area of interest, should be encouraged to complete the recommended program of study.

The Panel recognizes that, at the present time, very few graduate students in the BMSS area possess a background in mathematics equivalent to the material recommended here. The responsibility for this deficiency must be shared by those departments of mathematics which have failed to make such a program available and by subject matter departments in the BMSS area which have not required or encouraged their students to complete such a program. Thus, for the present, the Panel suggests that graduate students be permitted to enroll in the recommended courses for graduate credit.

8. Motivation. Beginning students in the BMSS areas may be unaware of the spectrum of application of mathematics in their areas of study. Therefore, proper motivation is crucial. Applications should be introduced early and often. Whether topics in mathematics are fully developed and immediately followed by applications or, alternatively, possible applications are introduced initially in order to motivate the development of the

mathematics, is a matter of taste. The Panel recognizes the pedagogical value of both methods and prefers a mixture of both approaches. Some work with models should be woven into each of the courses.

9. Rigor and Precision. It is necessary that BMSS students learn correct and precise definitions of concepts and statements of theorems. Such precise knowledge is far more important for these students than techniques, since techniques cannot be used to substitute for understanding. Thus, a student cannot understand the relation between the economic concepts of cost and marginal cost unless he knows the precise definitions of basic calculus. On the other hand, rigorous proofs are not crucial for BMSS students. Full details are usually easy to give in the finite case, but plausibility arguments are often sufficient for the continuous case. Rigor for the continuous case can come, if ever, in advanced calculus.

* * *

The recommendations fall into three parts: (1) a basic four-course, two-year sequence for BMSS students; (2) a two-course probability and statistics sequence; (3) comments on more advanced courses that serve as electives. CUPM is convinced that the requirement of six courses (the basic sequence plus the probability and statistics sequence) is minimal for future graduate students in the BMSS areas.

The Panel recognizes that subject-matter requirements for premedical students may not allow sufficient time for the six-course sequence. It also recognizes that some students planning to do graduate work terminating at the Master's degree (e.g., the terminal M.B.A. degree) may not need all six courses. In the near future, the Panel hopes to consult with

BMSS area specialists to determine which and how many of these prescribed courses can reasonably become a basic required sequence for such students. We hasten to add, however, that for those premedical students planning a career in medical research, or for those BMSS students planning a Master's degree program in certain special areas (as, for example, in an M.B.A. program with emphasis on computers, statistics, operations research, and management science), serious consideration should be given to requiring the full six courses.

THE BASIC SEQUENCE

We list below the subdivisions of the basic sequence of four courses. It is assumed that there is time for approximately 39 lectures in each course. A list of topics, with a suggested number of lectures for each topic, is given under each subdivision. Naturally, these recommendations merely outline one possible arrangement for such courses. The same goals can be achieved in many other ways.

Each major subdivision is followed by a list of comments. These are designed to bring out the philosophy we recommend to course-planners.

Students in BMSS areas clearly need a selection of topics quite different from the traditional analytic geometry and calculus. To highlight this need, an oversimplified view of problems attacked by mathematical methods is presented in the accompanying figure. Most of the mathematics traditionally taught deals with problems that fall into one cell of this table, whereas problems in the other cells are also of current and evergrowing importance in all applications of mathematics, including those of the BMSS areas.

	Deterministic	Stochastic
Few Variables	I Organized Simplicity	II Disorganized Simplicity
Many Variables	III Organized Complexity	IV Disorganized Complexity

Area I includes the traditional undergraduate analytic geometry-calculus sequence, along with difference and differential equations. Area II includes elementary probability and statistics. Area III includes linear algebra and many variable advanced calculus. Area IV includes complex stochastic models. High speed computers play an important role in all areas, especially in areas III and IV.

The two-year sequence proposed here allocates, in order, about 45%, 25%, 25%, 5% of its time to these four areas. The recommended unit on computing is in addition to the two-year sequence, and is not included in this allocation.

The basic sequence is designed to cover the ideas most important to BMSS students from probability theory, linear algebra, and analysis. The division between the first and second year was made with two major considerations in mind: First of all, students should be enabled to take a more significant statistics course with the first year of the sequence as prerequisite. Second, since many students may fail to elect the second year, the first year must be a meaningful unit in itself. For this reason, the statistics course is designed so that the first semester can be taught after one year of the basic sequence.

A comment is in order to explain the inclusion of a course in probability theory in the basic sequence in addition to the third year ccurse in probability and statistics. Whereas the importance of statistics in the BMSS areas has long been recognized, BMSS students generally have to settle for an oversimplified statistics course. The early introduction of probability theory together with calculus should make it possible to teach a more sophisticated statistics course.

Probability theory, on its own right, has also acquired an important place in the BMSS fields. Stochastic models are proving useful in the biological and social sciences. The theory of games, queuing theory, stochastic programming, and other scheduling techniques are important planning devices in the management sciences. Simulation of various probabilistic processes, by means of computers, promises to play a key role in all BMSS fields. Since all problems concerning human beings involve a strong element of uncertainty, probability theory has been assigned a basic place in the course sequence.

We feel it is reasonable to expect many mathematics departments to offer such a basic sequence for BMSS students. But it would not be reasonable to expect several specialized sequences within the BMSS area. Nor would such specialized sequences be desirable, since students freely switch from one specialty to another during their undergraduate years.

THE FIRST YEAR

The first year course sequence serves as an introduction to the basic ideas of calculus and probability theory, two branches of mathematics most useful in the BMSS areas. The courses outlined below combine the teaching of calculus with probability theory. Considerable economy of effort can be achieved in this manner, and probability theory, instead of physical science applications, can be used to motivate the calculus.

In the outline a full course in calculus is sandwiched between two half courses in probability. However, a closer integration of these topics may prove pedagogically sounder. (See 3(ii).)

It is recognized that many colleges will find it easier to teach a semester of calculus followed by a semester of probability. If this is done, it is important to observe that the semester of calculus here proposed differs significantly from the first semester of a traditional calculus sequence. Care should also be taken to advise students to follow the calculus course with a semester course in probability.

We list the topics of the first year under three major subdivisions.

1. Probability (Finite sample spaces)

Recommendation: We recommend approximately 20 lectures in finite probability to cover the following topics:

(a) Sample space of an experiment, events--treatment using set notation and operations, providing an intuitive understanding of randomness and uncertainty. (4)

(b) Definition of probability of an event via assignment of weights to unit subsets of a sample space, probability as a set function, simple theorems on probability, the special case of equiprobable measure, and counting problems. Both the relative frequency and subjective interpretations of probability should be discussed early. (3)

(c) Conditional probability, composite experiments, Bayes' formula, independent events and independent trials. (3)

(d) Hypergeometric and binomial probabilities, sampling with and without replacement. (3)

(e) Random variables as functions on sample spaces, probability distribution, mathematical expectation, mean, variance and standard deviation, standardized variable, law of large numbers. (5)

(f) Independence of random variables, sums of random variables, mean and variance of sums, the sample mean. (2)

Discussion

(i) We assume that it will be both necessary and desirable to review the elements of sets when treating topic (a), and to review the notion of a function when treating topic (e).

(ii) We believe it is important to develop the notion of probability as a nonnegative, additive set function following the definition based on the assignment of weights to simple subsets of the sample space. This is especially important to pave the way for the later introduction of probability on nonfinite sample spaces.

(iii) In topics (d) and (f) one should take the opportunity to introduce informally some examples of statistical inference.

(iv) The use of flow diagrams, as in Kemeny et al, Finite Mathematics With Business Applications (Prentice Hall, Inc., 1962) is strongly recommended to clarify points for the student and to introduce him to this important device. For example, a student who understands the flow diagrams for the computation of $E(X^2)$ and for $[E(X)]^2$ is not likely to confuse the mean square and the square mean.

(v) Throughout, the student should perform and analyze simple random physical experiments designed to illustrate the material and to give personal experience with variability. Coins, cards, dice, telephone books, almanacs, and random numbers offer readily available equipment.

2. Differential and integral calculus (Functions of one variable)

Recommendation: We recommend approximately 39 lectures in calculus to cover the following topics:

(a) Limits and derivatives, differentiation of algebraic functions (power function, sum, product and quotient formulas, chain rule, implicit differentiation), higher derivatives, Taylor's formula with remainder, analogies with the difference calculus. (12)

(b) Increasing and decreasing functions, extrema of a function, graphs of equations and inequalities, emphasis on end-point extrema. (6)

(c) Definite integral, fundamental theorem of the calculus, areas by integration, Simpson's rule. (6)

(d) Introduction to power functions, exponential and logarithmic functions, their derivatives and integrals. (6)

(e) Integration by parts and by change of variable. (2)

(f) Infinite sequences and series, improper integrals, power series, approximations by convergence. (7)

Discussion

(i) Functions should be defined and illustrated in a number of different ways--as a mapping, as a set of ordered pairs, etc. One common use that these students will have for functions will be for those defined by computer programs. Here the illustration is particularly good, since the function is the program deck, the argument(s) of the function the data card(s), and the value(s) of the function the output card(s). Both continuous and discontinuous and differentiable and nondifferentiable functions should be illustrated. Various range and domain sets should be considered, including some that are nonnumerical.

(ii) We believe it important to emphasize that the finite problems and formulations of the difference calculus are fundamental and closer to reality than the continuous models of the differential calculus. The student should understand that one often introduces a continuous model in an attempt to make more mathematically tractable a problem that is more naturally formulated in discrete form.

(iii) The fundamental theorem of calculus should be applied in a variety of fields.

(iv) In emphasizing end-point extrema in topic (b) we think it well to introduce a simple linear programming problem in two variables. This supplies an important context for graphing linear inequalities and also reinforces the idea of extrema occurring at the boundary.

(v) We recommend the use of flow diagrams throughout this unit as a pedagogic device.

(vi) The topics we include in the calculus unit are those essential for the introduction to continuous and discrete probability in the unit to follow. It is for this reason that we postpone trigonometric functions to the second year and bring such topics as listed in (e) and (f) forward to the first year.

(vii) In teaching the techniques of integration we recommend that the student be taught to use integral tables rather than to derive parts of such a table. For this reason, only two fundamental techniques are recommended in (e). A computer is much better at techniques of integration than a student can ever be, and time thus saved can be devoted to obtaining a better understanding of the meaning of integration.

3. Probability (Infinite sample spaces. Discrete and continuous)

Recommendation: We recommend approximately 19 lectures on probability to cover the following topics:

(a) The notion of probability of events extended to experiments whose sample spaces are countably infinite or are intervals of real numbers, probability as a set function. (2)

(b) Density and distribution functions of random variables. (3)

(c) Expectations as sums of infinite series or as integrals. (2)

(d) Discrete distributions such as the geometric, Poisson, and negative binomial; continuous distributions such as uniform, exponential, and normal. (8)

(e) Poisson and normal approximations to the binomial distribution, with informal examples of statistical inference. (4)

Discussion

(i) In introducing probability here, one should take advantage of the development of the set function idea in the unit on finite probability. (See discussion item (ii) in unit 1 above.)

(ii) Although our listing of topics separates calculus and probability, we think it wise that these subjects be integrated into a more unified course. Notions of probability can often be used to advantage, not only to supply important examples for calculus techniques, but to motivate the consideration of calculus topics. The relation between density and distribution functions, for example, is a reformulation of the fundamental theorem of calculus. The notion of mean value of a random variable can be used to introduce the definite integral, distributions on countably infinite spaces motivate the development of infinite series, etc.

(iii) Here too, flow diagrams can be used to advantage.

(iv) The idea of a stochastic process is very important, but it is postponed until the second year course when difference and differential equation techniques are available.

(v) The topics listed in sections (b) and (c) (density functions and moments) are fully explored only in the context of section (d) topics, when particular density and distribution functions are discussed. Some of the lecture time now assigned to topics (b) and (c) is reserved for this fuller treatment and can be added to the time allotted to section (d).

THE SECOND YEAR

The first course is devoted to linear algebra and related topics, with applications (topic 4). The second course deals with functions of several variables, complex exponentials, difference and differential equations (topics 5 and 6). It is also recommended that approximately thirteen class periods devoted to computation accompany the first course (topic 7). Preferably the work on computation should be organized as a one-credit-hour course, but it may be included in a four-credit-hour course during the first semester.

4. Linear algebra

Recommendation: We recommend approximately 39 lectures in linear algebra and related topics to cover the following:

(a) Vector and matrix operations with examples from BMSS fields. (6)

(b) Linear systems of equations, dimension, axiomatic definition of vector space, constructive solution of linear equations, computation of inverses. (11)

(c) Linear inequalities, convex sets, introduction to linear programming. (6)

(d) Binary relations: equivalence, order (e.g., weak, partial, total), graphs. Operations on relations. (4)

(e) Invariants, canonical forms, eigenvalues and quadratic forms. (6)

(f) Finite Markov chains. (6)

Discussion

(i) It is suggested that use of an input-output example could motivate all the matrix operations and also emphasize the mathematical importance of the individual elements in a matrix in contrast to the approach exclusively from affine geometry. Linear least squares analysis offers another example.

(ii) Although vectors are to be introduced in component form, the axiomatic approach should not be neglected.

(iii) The constructive approach to linear equations should be carried out to make contact with the work in computation in Section 7 below. At least one example (preferably applied and nonsquare) of a set of 10 or more linear equations should be considered, in order that questions of practical computation with such systems can be raised.

(iv) The time allotted in topic (c) obviously does not permit more than an introduction to linear inequalities and linear programming, but geometric solutions for two and three dimensions might be feasible. The simplex method might well be introduced here.

(v) The basic topic under (d) is the development of equivalence relations in preparation for the equivalence relations on matrices. One should develop other relations by appropriate modification and extension of the axioms for equivalence or order. Simple examples from utility theory, communication networks, and measurement theory can be introduced to illustrate BMSS applications of various types of relations.

(vi) The importance of the flow chart as an aid to teaching the solution of linear equations and linear programming should not be overlooked.

(vii) Markov chain models serve as an example of a stochastic process other than the independent processes. These models have already proved useful in BMSS areas, and some examples should be introduced in this unit. Moreover, vector and matrix operations are natural tools here.

5. Analysis

Recommendation: We recommend approximately 19 lectures in analysis to cover the following topics:

(a) Functions of several variables, partial derivatives, extreme values of functions, Lagrangian multipliers, inequality constraints (e.g., the Kuhn-Tucker theorem). (10)

(b) Complex numbers and exponentials, Euler's formula, calculus of trigonometric functions. (9)

Discussion

(i) We suggest using Euler's formula to define complex exponentials. Rather than proving Euler's formula, we propose motivating it by use of trigonometric identities. Since complex exponentials are introduced for solving differential equations, this treatment is actually theoretically adequate.

(ii) The topics in this analysis unit serve as a review and supplement to the introduction to calculus techniques of the first year course.

(iii) The treatment of the calculus of more than one variable should be motivated by and centered on optimization problems.

(iv) Multiple integrals are omitted from this unit and may be introduced in the statistics course as the need arises.

6. Difference and differential equations

Recommendation: We recommend approximately 20 lectures in difference and differential equations to cover the following topics:

(a) Definition and illustration of difference and differential equations, linear difference and differential equations with constant coefficients, interpolation procedures for difference equations. (13)

(b) Systems of linear equations. Equivalence of n^{th} order equation with systems of equations. (4)

(c) Numerical solution of differential equations, iterative methods. (3)

Discussion

(i) The treatment of difference and differential equations should take advantage of the student's background in linear algebra and should use handbook techniques. The theories for linear difference and differential equations with constant coefficients should be developed simultaneously and in parallel.

(ii) Illustrations from BMSS fields should be emphasized in favor of full mathematical proofs of theorems used.

(iii) The need for numerical solutions is paramount. This implies on the one hand that even when explicit solutions are available, there is still a need for actual function values, and on the other hand that numerical solutions should be sought when explicit solutions are not obtainable. The work on numerical solutions should be based on use of a digital computer.

(iv) Throughout this entire unit, flow charts should enter as a major pedagogic device.

7. Computation

Recommendation: We recommend approximately thirteen periods of laboratory work in computation during the first semester of the second year. This material should be designed to include the rudiments of writing a program in some automatic programming language and the running of some nontrivial programs on a digital computer.

(a) Orientation and use of flow diagrams. (3)

(b) Teaching of a simple language, such as Fortran or Algol. (3)

(c) Examples such as tabulating a function, problems in probability theory, solution of linear equations, and numerical solution of calculus problems, use of random number generators. (7)

Discussion

(i) It is assumed that a high-speed computer with Fortran, Algol, or similar compiler is freely available to students. The aim of this work should be a nodding acquaintance with the use of computers, as demonstrated by the completion of a few nontrivial programs by each student.

(ii) We recognize that of all the topics under consideration this is the one most difficult to staff from regular mathematics departments. However, there are usually persons associated with other departments and with laboratories who would be competent, and might even be eager, to handle the computer instruction. To include such instruction as a part of the regular course is desirable but may not be feasible. It has been decided, therefore, to present the instruction in computation as a separate course for the following reasons: (1) section sizes desirable for computer instruction may be different from those for the regular course; (2) most teachers prefer to have undivided responsibility for their courses; (3) some colleges may not have computers available and may have to omit this part of the program; (4) separation gives greater flexibility in use of other plans for computer instruction, for example, concentrated evening programs over a shorter time period.

(iii) We wish to emphasize the availability of programmed materials for teaching some of the routine details of computer programming.

(iv) Some schools may also wish to have a one-credit hour models seminar during the second semester, in which representatives of the several BMSS fields would present models illustrating basic uses of mathematics in their fields. In addition to illustrating a variety of cases in which mathematical models are applicable, the seminar should give a feeling for approximations. Approximations are sometimes made when constructing a model--for instance the use of axioms that do not exactly fit the real situation. Another type of approximation is made when the first few terms of a Taylor's series are used to approximate a function. BMSS students will need to become acquainted with both kinds of approximations.

THE PROBABILITY AND STATISTICS SEQUENCE

The sequence outlined below is to be covered in two semesters. It offers considerable flexibility in its relations with the two-year BMSS basic mathematics sequence. One possibility is for a student to take the first year of the BMSS mathematics program and follow it with the first semester of this two-semester sequence in Probability and Statistics. The student would then have as much solid grounding in statistical inference as is possible in a three-semester mathematics sequence--indeed, nearly as much as is contained in a full standard calculus sequence plus a semester of probability and statistics.

The two semesters of this sequence can conveniently be taken concurrently with the BMSS second year basic mathematics sequence. The second semester of the Probability and Statistics sequence will add considerable strength to the student's algebra, just as the first semester will reinforce his calculus. Finally, the last topic in the second semester will take advantage of the work on differential equations in the BMSS course; though, if taken concurrently, there may need to be some coordination by the instructors since both topics come at the ends of their respective courses.

Upon completion of the second semester of the Probability and Statistics sequence (that is after the total three-year BMSS sequence), the student's mathematical capabilities should be both stronger and broader than with any of the standard programs now offered.

Recommendation: We recommend approximately 78 lectures to cover the following topics:

(a) Integration. Intuitive notion of measure with volume, probability, mass and other distributions as examples. Riemann-Stieltjes integral and average values over distributions. Distributions with a density leading to multiple integrals; joint density function of several random variables. The density of a conditional distribution and the conditional mean or regression function. Evaluation of multiple integrals as repeated integrals with use of transformations of coordinate system. Examples of volumes, centroids, normalizing constants for probability densities, probabilities, derivation of distribution of the sum of two random variables. The particular case of the bivariate normal density to be extensively studied. (See (i) and (ii).) (13)

(b) Moment generating functions. Definition with examples of binomial, Poisson, normal, and gamma distributions. Moment generating function (m.g.f.) of the sum of two independent random variables. Idea of moment problem and use of tables of m.g.f.'s. Sums of random numbers of random variables. (4)

(c) Sequences of random variables. Mean and variance of a sum of random variables in general case (including correlated variables). Sequences of distributions, m.g.f.'s and limiting distributions. Weak Law of Large Numbers using Chebychev's Inequality and Central Limit Theorem for independent identically distributed random variables, derived from m.g.f. theory. Example--the normal approximation to the binomial. The independence and distribution of the mean and variance of a sample from the normal distribution. A study of orthogonal linear combinations of standard independent normal random variables. (5)

(d) Statistical inference. The notion of a point estimator and its reliability. Intuitively desirable properties of point estimators: closeness, consistency, efficiency, sufficiency, unbiasedness. Notion of a loss function and of estimators to minimize the expected loss. Incorporation of prior knowledge through Bayes' formula leading to use of posterior distribution, with and without loss function. The method of maximum likelihood.

Deciding between hypotheses. "Tea-tasting" example of Fisher and randomization. Neyman-Pearson Lemma. Tests of hypotheses, the power of a test and its relation to the design of an experiment. The likelihood ratio method with the example of contingency tables. Consideration of prior knowledge and/or the consequence of the decision taken. (See (ii) and (iii).) (20)

(e) Relationships in a set of random variables. Generalize by matrix algebra to n-variate normal; notions of independence, correlation and regression and prediction in general and for n-variable normal. (4)

(f) Linear Models. A study of the above estimation methods, mainly, with only minor reference to testing, in the models

(I) $y_i = E(y) + u_i$, $E(y_i) = \sum_{i=1}^{\kappa} \beta_i x_{ij}$, u_i independent $N(0, \sigma^2)$, and

(II) $y_i = \sum_{j=1}^{\kappa} \beta_{ij}$ where the β_{ij} are normal random variables with zero means and various correlations.

Model I is the "regression" model where $\beta_1, \ldots, \beta_k$ and σ^2 require estimation, and the matrix $X = (x_{ij})$ is known. Model II is the "components of variance" model and, in the simplest cases (which are all that are treated here), only the variances of the β's require estimation.

As an example of Model I, the fitting of a straight line $E(y) = \beta_0 + \beta X$ is to be treated with comments on how the X's might be chosen (i) if one is sure the line is straight and (ii) if one is not. The rank of X is to be taken to be $k < n$. The simplicity of the case where X has orthogonal columns is to be emphasized. This leads naturally to the analysis of variance and the standard tests. Show that the unweighted least squares estimators lose efficiency when the u_i are not independent and of equal variance. (10)

(g) Design. With Model I, the design problem is that of making a good choice of X. Illustrate by the problem of searching for the optimum of a function of several variables.

The development of Type I models by the intuitive analysis of variability. Notions of treatment effect, block effect, and of interaction effects. Design of experiments so that estimators of effects exist which are uncorrelated and have suitable variances. The analysis of variance for block-designs and factorials.

Design of social experiments in many variables. The design and conduct of sample surveys.

Examples of laboratory experiments and of bioassay. (15)

(h) Stochastic processes and review of Markov Chains. Branching processes (extinction of family names and of genes, for example). Indication of more general Markov processes via Poisson, birth, birth and death processes. Brief discussion of nonstationary processes. Examples from learning theory and biology. (See (iv).) (7)

Discussion

(i) Approximately half of the material in this proposed sequence is general mathematics, rather than probability or statistics. Much of it broadens and extends the student's abilities in calculus and in linear algebra.

(ii) It is intended that the work on formal tests of hypotheses be modest, though the student should be exposed to some of the standard kitchenware such as t-tests, sign test, Wilcoxon two-sample test (sometimes called Mann-Whitney). The student should understand that testing statistical hypotheses does not represent a major portion of a statistician's work, and, indeed, that the method is a rather primitive tool for the analysis of data and decision making.

(iii) Throughout, it is intended to weave a running example to aid the discussion of decision theory for two action decisions. By considering the example with and without loss functions, with and without prior distributions, and with and without the possibility of further sampling, the interconnection between decision theory and notions of testing hypotheses are to be brought out. By leaning more toward a decision-theoretic than a testing-hypotheses attitude, the course more readily prepares students for the theory and philosophy of economics and management.

(iv) For classes containing mostly economics majors, the section on design might be somewhat reduced and the section on stochastic processes omitted so that a section "Economic Time Series" could be inserted. Such a section would have two components: simultaneous regression equations and stationary normal processes. Similar appropriate revisions are possible for classes composed largely of psychology majors, or of biology majors, and so on.

MORE ADVANCED COURSES

The six course sequence described above is minimal and will not completely equip the student to pursue studies in upper level undergraduate mathematics courses without additional work. On the other hand, many individual departments in the BMSS areas encourage students to pursue more advanced work in mathematics. Also, students who develop particular aptitudes in the BMSS areas may wish to extend their mathematical knowledge beyond the minimal recommendations.

It is both unrealistic and unnecessary to supply such students with an additional set of special offerings. The regular advanced undergraduate courses in each mathematics department serve as a natural vehicle for the serious student of this subject.

What is needed is a provision for students who wish to take additional work in mathematics following the BMSS sequence. We encourage departments of mathematics to make provision for such students by planning a transitional course.

Special subject courses in mathematical biology, in areas of business, or in the social sciences, are interdisciplinary efforts which succeed only if and when the necessary talent is available, and will be taught in departments where such talent resides. Also the content of such courses will depend heavily on the particular interdisciplinarian. Therefore, although such courses are considered important and desirable for BMSS students, suggestions for the content of such special courses are not contained in this document.