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INSTRUCTIONAL GUIDE FOR BASIC MATHEMATICS 1, GRADES 10 TO 12.

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LOS ANGELES CITY SCHOOLS, CALIF.

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THIS INSTRUCTIONAL GUIDE FOR MATHEMATICS 1 OUTLINES CONTENT AND PROVIDES TEACHING SUGGESTIONS FOR A FOUNDATION COURSE FOR THE SLOW LEARNER IN THE SENIOR HIGH SCHOOL. CONSIDERATION HAS BEEN GIVEN IN THE PREPARATION OF THIS DOCUMENT TO THE STUDENT'S INTEREST LEVELS AND HIS ABILITY TO LEARN. THE GUIDE'S PURPOSE IS TO ENABLE THE STUDENTS TO UNDERSTAND AND APPLY THE FUNDAMENTAL MATHEMATICAL ALGORITHMS AND TO ACHIEVE SUCCESS AND ENJOYMENT IN WORKING WITH MATHEMATICS. THE CONTENT OF EACH UNIT INCLUDES (1) DEVELOPMENT OF THE UNIT, (2) SUGGESTED TEACHING PROCEDURES, AND (3) STUDENT EVALUATION. THE MAJOR PORTION OF THE MATERIAL IS DEVOTED TO THE FUNDAMENTAL OPERATIONS WITH WHOLE NUMBERS. IDENTIFYING AND CLASSIFYING ELEMENTARY GEOMETRIC FIGURES ARE ALSO INCLUDED. (RP)

INSTRUCTIONAL GUIDE
BASIC MATHEMATICS 1
GRADES 10 to 12

LOS ANGELES CITY SCHOOLS
Division of Instructional Services

Curriculum Branch
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FOREWORD

This Instructional Guide for Basic Mathematics 1 outlines content and provides teaching suggestions for a foundation course for the slow learner in the senior high school. Consideration has been given to the student's interest levels and his ability to learn. The guide's purpose is to enable the students to understand and apply the fundamental mathematical algorithms and to achieve success and enjoyment in working with mathematics. It is anticipated that the teacher, too, will find a challenge in experimenting with a new development of basic topics and that he will experience the reward of observing real progress on the part of class members.

It is suggested that the teacher remind himself of the importance of treating each student as a worthy individual, of providing for frequent changes of pace, and of being generous with praise for accomplishment and for effort. The slower learner needs work which is within his comprehension, yet which encourages exploration and discovery. He needs assignments which are long enough to provide real learning experiences, but brief enough to encourage their successful completion..

Teachers who approach the teaching of Basic Mathematics 1 with enthusiasm and interest will find the experience to be rewarding. The challenge of the general mathematics class calls forth real teaching skills and a thorough understanding of the subject. Most of all, the Basic Mathematics class stimulates the teacher's best efforts in reducing to the simplest of terms the basic ideas and the more sophisticated concepts which are developed.

ACKNOWLEDGMENTS

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Max R. Wainwright, Jr.	Manual Arts High School

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OBJECTIVES OF BASIC MATHEMATICS 1

The course in Basic Mathematics 1 is planned to provide instruction which is suitable for the slow learner in senior high school whose ability in general mathematics is considerabley below expectancy for his grade.

The course is intended to enable the student to achieve:

Vocational competence, as he develops the ability to

--Follow directions accurately.

--Add, subtract, multiply, and divide whole numbers.

--Perform the basic operations of arithmetic with accuracy.

--Judge the reasonableness of his solutions to problems.

--Copy corectly numerals, symbols, and simple geometric figures.

Consumer effectiveness, as he learns to

--Purchase merchandise and other items wisely.

--Plan and manage a budget and handle financial transactions with a bank intelligently.

--Estimate quantities, distances, areas, and weights with reasonable accuracy.

--Read advertisements with understanding to determine the amounts actually required for the purchase of the advertised items.

Responsible community membership, as he grows in ability to

--Understand the importance of integrity and develop a personal commitment to high standards of honesty.

UNIT I. MATHEMATICS IN OUR MODERN WORLD — 7 Teaching Days

A. Development of the Unit

1. Develop an awareness of the importance of mathematics in our daily lives.
   a. Discuss the mathematics which the students need to use as workers and citizens.
   b. Explore the many ways in which mathematics is basic to human activities.
   c. Point out the contribution which mathematics has made in the space program.

2. Discuss the students' need to understand mathematics in order to take a significant place in society.
   a. Point out how mathematics is related to

      Money
      Purchasing and paying bills
      Housekeeping
      Owning a car

3. Use items from newspapers to illustrate the widespread use of mathematics.
   a. Point out the necessity of reading a newspaper discerningly.
   b. Help students to discover articles and ads which involve mathematics and mathematical symbolisms.

4. Introduce the subject of taxes.
   a. Discuss the reasons why taxes are necessary.
   b. Help students to learn the methods for computing and collecting taxes.
   c. Discuss types of taxes collected.

      Local
      State
      Federal

5. Emphasize the importance of learning about measurement.
   a. Time
   b. Temperature
   c. Age
   d. Lengths and distances
   e. Area

B. Suggested Procedures

The teacher may find it useful to:

--Ask students to list ways in which they use mathematics every day.

--Discuss ways in which everyone is called upon to use mathematics.

--Develop recognition of the need for mathematics by studying some of its history.

--Introduce the topic of space exploration, and point out the important role of mathematics.

--Plan with class an imaginary trip to the store to purchase groceries.

--Initiate a discussion about owning a car (purchase price, mileage, gas consumption, insurance, payments, and responsibilities).

--Bring newspapers to school and distribute copies to class members.

--Ask students to cut out articles and ads in which mathematical concepts are used. Discuss their content.

--Point out the need for reading newspapers intelligently.

--Introduce a discussion about the need for taxes.

a. Discuss the establishment of a small western town.

b. Show the need for roads, schools, a fire department.

c. Show the need for a sheriff, and a jail.

d. Discuss possible ways of collecting taxes.

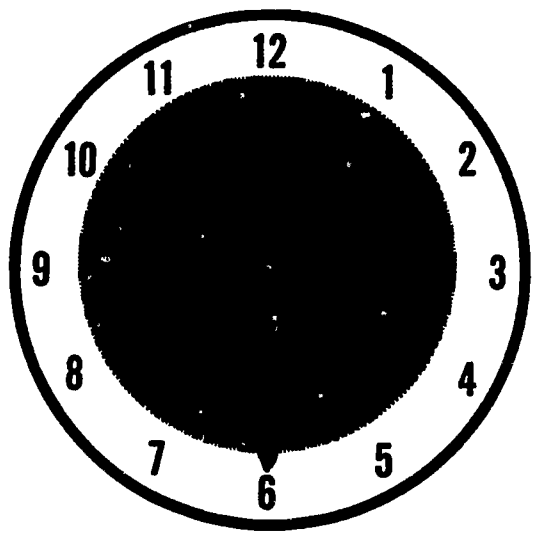
--Ask students to compute their ages in years, months, and days.

--Discuss the day's weather report, and review the use of the thermometer.

B. Suggested Procedures (contd)

--Conduct oral drill in telling time. Ask class members to express time in different ways, such as

9:10	half after 6	half past 5
3:45	quarter to 8	quarter past 1



--Discuss the purposes and uses of measurement.

C. Evaluation

It is expected that the students will be able to demonstrate that they have learned:

1. The need for studying mathematics.

2. To read a newspaper discerningly.

3. To interpret more successfully the world around them and recognize its dependence upon mathematics.

4. To recognize the need for taxes.

5. To appreciate the need for measurement.

UNIT II. THE SET OF WHOLE NUMBERS: ORDER AND RELATIONSHIPS 14 Teaching Days

A. Development of the Unit

1. Introduce the set of whole numbers.
   a. Develop the concept that a whole number can be expressed in many ways.
   b. Show that most fractions and decimals are not whole numbers.
   c. Describe the set of whole numbers as a set which includes 0 and the counting numbers.
   d. Lead students to discover that in the set of whole numbers, there is no largest whole number.

2. Develop the number line as a device to represent number ideas.
   a. Introduce the set of whole numbers on the number line.
   b. Show how to construct a number line.

3. Interpret number as a property of sets or collections.
   a. Relate number property of sets to points on the number line.
   b. Describe equivalent sets as sents which contain the same number of members and which have an identical number property.

4. Present number patterns on the number line.
   a. Develop a variety of number sequences and sets.
   b. Develop the ability to complete open number sequences.

5. Develop the order property of the set of whole numbers.
   a. Relate the order of whole numbers to the number line.
   b. Develop an understanding of related terms, such as:

      "is equal to"

      "is greater than"

      "is less than"

UNIT II. THE SET OF WHOLE NUMBERS: ORDER AND RELATIONSHIPS

A. Development of the Unit (contd)

5. c. Develop an understanding of symbolism.

   = means "equal to"

   > means "is greater than"

   < means "is less than"

6. Introduce the use of whole numbers in problem solving.

   a. Relate an understanding of the order of whole numbers to problem situations.

   b. Develop problem solving skills through the use of problems written by students.

7. Introduce basic geometric concepts.

   a. Develop the concept of a point.

   b. Develop the concept of a line as a set of points.

   c. Present the basic geometric shapes: square, rectangle, triangle, circle, parallelogram, polygon.

   d. Introduce the concept of an angle.

8. Present the fundamentals of measurement.

   a. Develop the concept of linear measurement.

   b. Relate the concepts of "is greater than" and "is less than" to measurement.

B. Suggested Procedures

The teacher may find it useful to:

--Provide students with activities which will illustrate that there are many ways of expressing each whole number

3 1 + 1 + 1 III □□□ 5 - 2 2 + 1 etc.

--Ask students to write the set of whole numbers beginning with 0. Explain what the symbol " . . ." means after the first few numerals.

For example: $\{0, 1, 2, \ldots\}$

--Explain that there is no largest counting number. Demonstrate by taking a very large number and adding 1 to it to produce an even larger number. 5,000,000 + 1 = 5,000,001.

--Show that most fractions and decimal fractions are not whole numbers. $.5 = \frac{1}{2} \neq 1$.

B. Suggested Procedures (contd)

—Illustrate concepts of a number line.

Show students that to make a number line, a unit of length is first established.

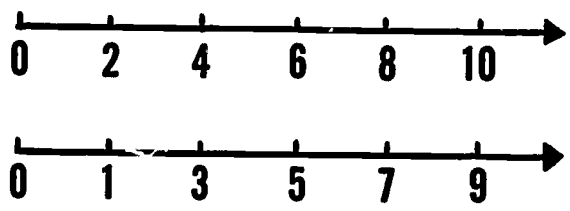
After the unit of length is established, point out that the position of each number is determined in such a way that any two adjacent points representing whole numbers are the same distance apart.

Explain that a number line has no end points. The arrows at the ends of the number line imply this principle.

—Instruct students to relate the number property of a collection of objects in a set to the numerals on the number line.

—Help students to generalize that  and  have the same number property as well as the same number of elements and are therefore called equivalent sets while  and  are not equivalent sets.

—Show by simple illustrations that number lines may have many different patterns.



—Introduce a variety of number patterns for students to explore.

—Ask students to complete many partial number sequences.

{2, 4, ___, 16, ___, ___, 128}

—Use the number line to show that each point represents a number less than any point to its right and greater than any point to its left.

—Conduct drill in the use of "is equal to," "is greater than," and "is less than."

5 is less than 6

3+1 is equal to 2+2

6 is greater than 5

7 _____ 3

4 _____ 0

0 _____ 4

B. Suggested Procedures (contd)

—Introduce symbols > "is greater than" and < "is less than." Help students to generalize that the arrow always points to the smaller number.

$3 > 2$ $2 < 3$

—Provide drill in the use of these symbols.

$3 \bigcirc 4$ $5 \bigcirc 7$

$4 \bigcirc 3$ $7 \bigcirc 5$

—Assign simple word problems that will illustrate each of the concepts developed in this unit.

—Present basic concepts of geometry stressing the concepts developed in this unit.

—Start with a point, and then show that a line is a set of points. Introduce the basic shapes which are present in the physical world. Ask students to answer simple questions about the number of sides and angles in the various shapes. Develop the idea of "greater than" or "less than" as related to sides and angles.

—Introduce students to measurement through the use of a straight edge. (It is recommended that the straight edge be marked only with inch, one-half inch, and one-fourth inch markings.)

—Present many examples to illustrate how measurement is performed. Measure sides of triangles, quadrilaterals, rectangles, squares, parallelograms, and polygons. Show students how the concept of "is greater than" or "is less than" is applied to measurement.

C. Evaluation

It is expected that the student will be able to demonstrate that he has learned to:

1. Distinguish between whole numbers, fractions, and decimal fractions.
2. Make a number line, identify patterns on a number line, and indicate what numbers are necessary to complete a sequence of numbers correctly.
3. Use the concept of a number line to determine which numbers are "greater than" or "less than" other numbers.
4. Solve simple word problems relating to the concept of whole numbers and the order of whole numbers.
5. Use effectively the ideas of point, line, and common geometric shapes.
6. Measure lines and apply the concepts of "greater than" or "less than" to the measured lines.

UNIT III. ADDITION: OPERATION WITH WHOLE NUMBERS — 12 Teaching Days

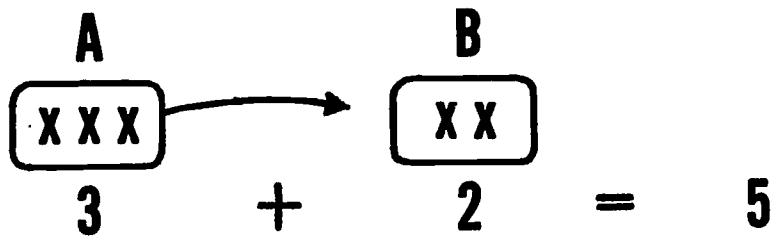
A. Development of the Unit

1. Introduce addition as the joining of sets of objects that have no members in common.
   a. Develop the commutative property for joining sets.
   b. Develop the associative property for joining sets.
2. Introduce addition, using the number line.
   a. Develop an understanding of whole numbers as represented by unit distances from zero on the number line.
   b. Develop addition as the combining of number line distances representing whole numbers.
3. Explore the commutative property for addition of whole numbers.
   a. Develop the concept that the order of addends does not affect the sum.
   b. Emphasize the commutative property in both horizontal and vertical addition.
4. Introduce the concept of zero as applied to addition.
   a. Develop an understanding of zero as the identity element for addition.
   b. Emphasize the use of the identity element in addition problems.
5. Explore the associative property for addition of whole numbers.
   a. Develop the concept that, when three or more numbers are to be added, the sum is independent of the grouping.
   b. Emphasize the associative property by both horizontal and vertical addition.
6. Introduce practical problems which are solved by the use of addition.
7. Help students to increase their understanding of basic geometric concepts.
   a. Develop the concept of line segment.
   b. Develop the concept of perimeter.
8. Help students to understand the fundamentals of measurement.
   a. Develop the addition of measures of lines.
   b. Apply the concepts of addition of line segments to perimeters.
   c. Develop the concept of area.

B. Suggested Procedures (contd)

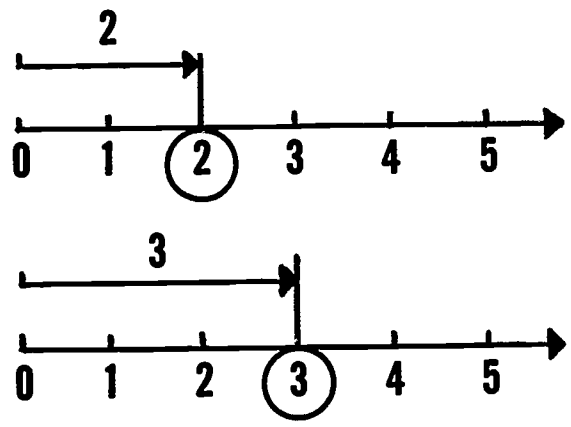
The teacher may find it useful to:

--Provide student activities to help students understand the operation of addition through the joining of two sets.

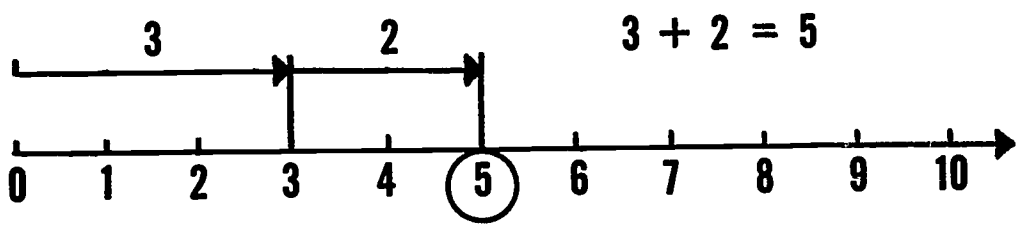


--Develop the understanding of union of disjoint sets to include three or more sets.

--Instruct students to represent whole numbers on the number line as distances from zero.



--Ask students to demonstrate addition facts on the number line.

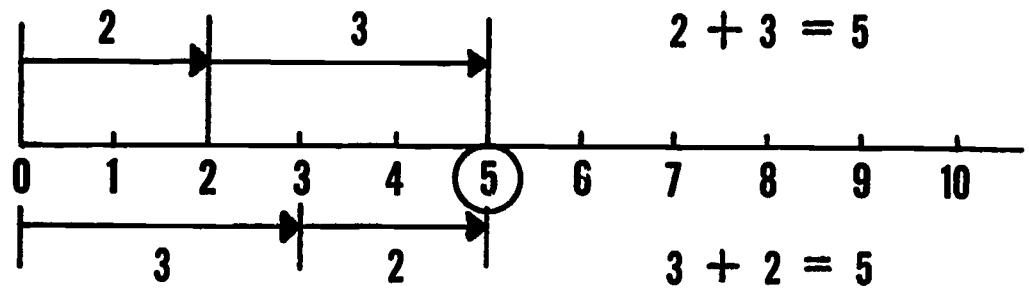


--Provide students with a series of related problems leading to the discovery of the commutative property of addition.

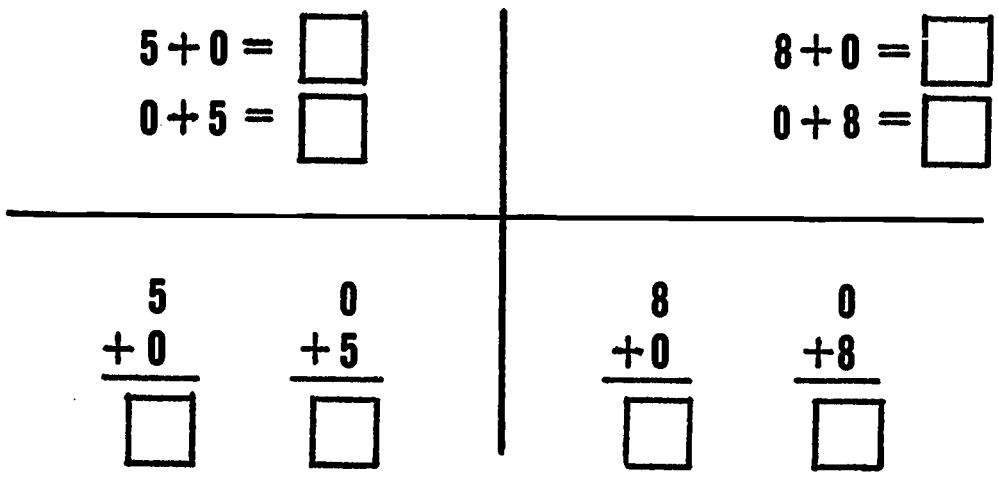
2+3 = ___ 3+2 = ___	5+1 = ___ 1+5 = ___
2 3 +3 +2 □ □	5 1 +1 +5 □ □

B. Suggested Procedures (contd)

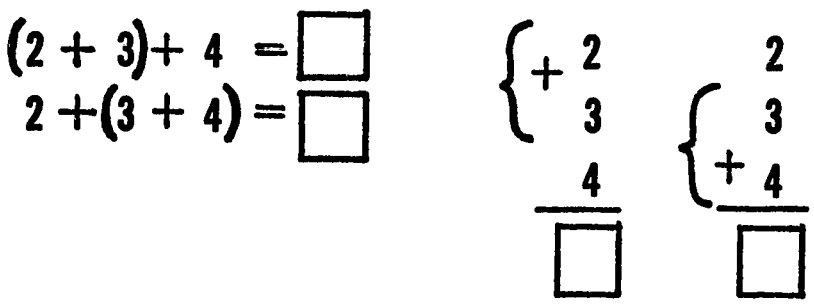
--Ask students to solve related problems on the number line.



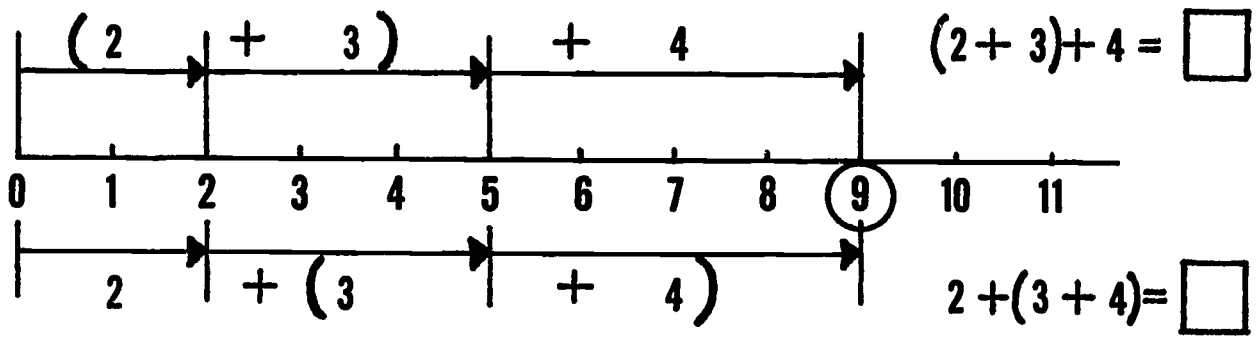
--Supply problems in which zero is introduced as an addend.



--Present problems that will help students to discover the associative property of addition.



--Develop the associative property in the use of the number line.



--Assign a series of simple word problems which require students to employ concepts learned in this unit.

B. Suggested Procedures (contd)

--Encourage students to write and solve their own problems using concepts developed in this unit.

--Instruct students to use a straight edge to draw line segments.

--Ask students to distinguish line segments from lines.

--Help students to discover that the perimeter of any plane figure is the distance around it.

--Help students to understand the addition of denominate numbers and the measures of line segments that make up perimeters or plane shapes.

--Introduce students to the concept of area by counting unit squares.

C. Evaluation

It is expected that the students will be able to demonstrate that they have learned to

1. Recognize the number property from the number of objects in a set and add the elements of two or more sets.

2. Demonstrate the commutative property and the associative property for addition on the number line.

3. Understand that zero is the identity element for addition.

4. Solve simple word problems, using addition of whole numbers in situations.

5. Write and solve simple word problems.

6. Add the measures of line segments and compute the measure of the perimeters of some common figures.

7. Apply concepts of area and its measure to the solution of problems.

UNIT IV. SUBTRACTION: OPERATION WITH WHOLE NUMBERS — 10 Teaching Days

A. Development of the Unit

1. Introduce subtraction as the opposite of addition.
   a. Develop subtraction as a separation of sets.
   b. Show the relationship of addition and subtraction facts on the number line.

2. Present subtraction problems in both horizontal and vertical form.
   a. Relate pairs of equations.
   b. Relate pairs of problems in vertical form.
   c. Interpret subtraction as discovering the missing addend.

3. Introduce the concept of zero in subtraction.
   a. Develop the idea that subtracting 0 from any number results in the same number.
   b. Develop the idea that subtracting any number from itself produces zero.

4. Introduce the vocabulary used in performing subtraction.
   a. Present definitions of subtrahend, minuend, difference, and missing addend.
   b. Illustrate subtrahend, minuend, difference, and missing addend.

5. Introduce problems in which the subtraction process is utilized.

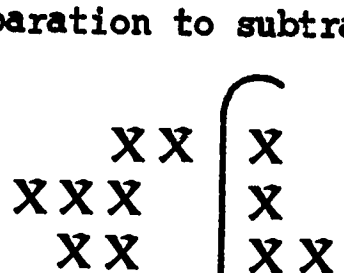
6. Extend understandings of basic concepts of geometry.
   a. Review basic concepts of geometry.
   b. Present examples of a plane, a curve, a closed curve, and interior and exterior regions.

7. Help students to understand measurement.
   a. Redevelop the idea of area by counting and subtracting.
   b. Provide practice in subtraction of the measures of line segments.

B. Suggested Procedures

The teacher may find it useful to:

—Provide activities to show students the relationship of set separation to subtraction.

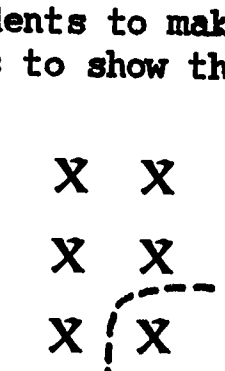


$7 + 4 = \square$

$11 - 4 = \square$

$11 - 7 = \square$

—Ask students to make up their own equations from set illustrations to show this relationship.

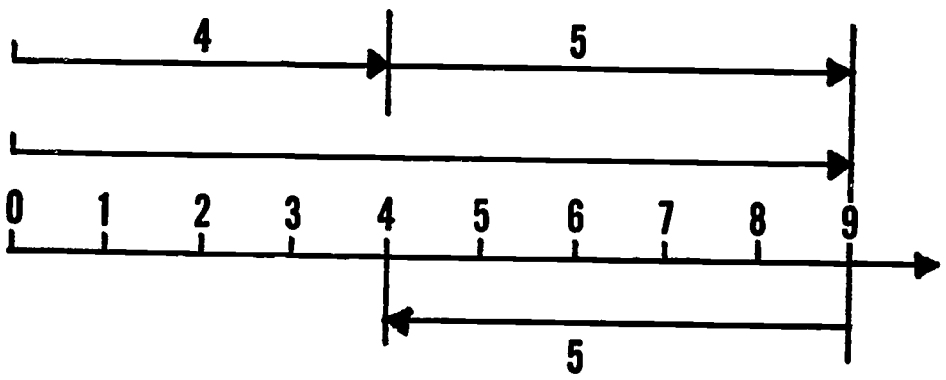


$6 + 2 = \square$

$8 - 2 = \square$

$8 - 6 = \square$

—Instruct students to demonstrate subtraction facts on the number line.



$4 + 5 = 9$

$9 - 5 = 4$

—Ask students to show the relationship between addition and subtraction by writing addition sentences from subtraction equations.

$5 - 3 = \square$

$3 + \square = 5$

B. Suggested Procedures (contd)

--Provide subtraction exercises to be solved in both the vertical and horizontal forms.

$$\begin{array}{ccc} 5 & 7 & 7 \\ +2 & -\square & -\square \\ \hline 7 & 2 & 5 \end{array} \qquad \begin{array}{l} 5 + 2 = 7 \\ 7 - \square = 2 \\ 7 - \square = 5 \end{array}$$

--Introduce zero as the subtrahend in subtraction problems.

$$\begin{array}{cc} 5 & 5 \\ +0 & -0 \\ \hline 5 & 5 \end{array} \qquad \begin{array}{l} 5 + 0 = 5 \\ 5 - 0 = 5 \end{array}$$

> Develop the idea that $n - 0 = n$ and that $n - n = 0$.

--Develop the meanings of terms such as subtrahend, minuend, difference, and addend.

--Assign students a series of word problems, using concepts learned in this unit as well as concepts learned in previous units.

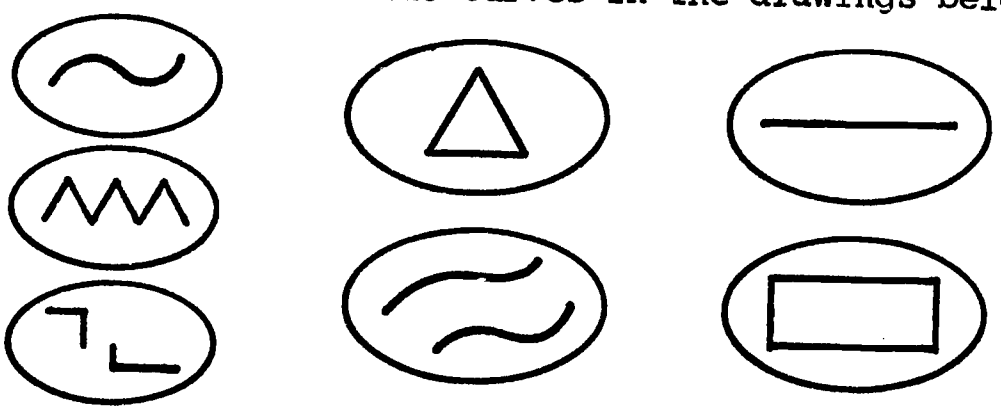
--Encourage students to make up problems of their own involving subtraction.

--Review and redevelop geometric ideas previously presented.

--Develop the concept of a plane by using as examples a desk top, floor, ceiling, and other flat surfaces.

--Help students to interpret a curve as a set of points which can be represented by a pencil drawing made without lifting the pencil from the page.

--Ask them to mark with an X the curves in the drawings below.



--Point out that a closed curve is a figure formed when a person places a pencil point on a paper and moves it around and ends at the point where he began.

B. Suggested Procedures (contd)

--Conduct drill exercises to emphasize the meaning of interior and exterior regions of closed curves.

--Provide activities that will help class members to develop the idea of interior and exterior as applied to regions in geometric figures.

--Assist students in discovering the application of subtraction in the measurement of distances.

C. Evaluation

It is expected that the student will be able to demonstrate that he has learned to

1. Relate addition facts to subtraction facts and use them in solving practical problems.

2. Show subtraction facts on the number line.

3. Complete open number sentences, using addition and subtraction facts.

4. Understand the use of zero as an identity element for subtraction.

5. Apply such terms as subtrahend, minuend, difference, and addend correctly.

6. Interpret applications of plane figures in the real world.

7. Recognize curves and closed curves.

UNIT V. NUMBER CONCEPTS

14 Teaching Days

A. Development of the Unit

1. Introduce number-numeral relationships.
   a. Show that there are many ways of writing a number.
   b. Develop an understanding that numerals are man-made symbols to represent number ideas.
2. Introduce expanded notation.
   a. Develop the concept of numerals having more than one digit.
   b. Relate expanded notation to place value.
   c. Develop the idea of regrouping.
3. Relate ideas of addition and subtraction to expanded notation.
   a. Show additions of whole numbers with and without regrouping.
   b. Show subtractions of whole numbers with and without regrouping.
4. Introduce problems for students to solve in which expanded notation with and without regrouping is used.
5. Extend the understanding of geometric concepts.
   a. Explore the different types of quadrilaterals and triangles.
   b. Develop working definitions of quadrilateral and triangle.
   c. Develop understandings of properties of angles.
6. Continue the study of measurement.
   a. Introduce the use of the protractor for angle measurement.
   b. Use angle measurement to classify angles.

      Straight angle
      Right angle
      Acute angle
      Obtuse angle

B. Suggested procedures

The teacher may find it helpful to:

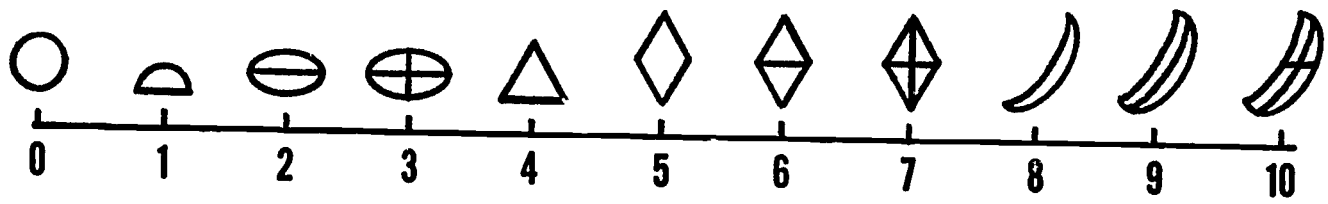
—Assign activities in which students will discover different ways of naming numbers.

7 − 0 　　6 + 1 　　8 − 1 　　SEVEN / VII 　　1 − 1 + 5 　　9 − 2

—Review combinations of numbers that add to 10.

△ + □ = 10

—Develop new symbols to represent numbers from 0 to 10.



—Provide drill exercises in which students will use these new symbols.

—Extend the idea of naming numbers to include numbers larger than 10.

9 + 6 = 15 　　8 + 7 = 15 　　10 + 5 = 15

—Rename numbers to emphasize place value.

17 = 10 + 7
26 = 20 + 6
32 = ___ + 2

135 = 100 + 30 + 5
456 = ___ + 50 +
___ = 300 + 40 + 5

26 = ___ + 6
26 = ___ + TENS + ___ ONES
___ = 2 TENS + ONES

—Introduce regrouping of numbers.

26 = 20 + 6
26 = (10 + 10) + 6
26 = 10 + (10 + 6)
26 = 10 + 16

B. Suggested Procedures (contd)

--Assign practice in regrouping numbers.

$324 = 300 + 20 + 4 = \_\_\_ + \_\_\_ + 4$

$324 = 300 + (10 + 10) + 4 = \_\_\_ + (\_\_\_ + 10) + 4$

$324 = 300 + 10 + (10 + 4) = \_\_\_ + \_\_\_ + (10 + 4)$

$324 = 300 + 10 + 14 = \_\_\_ + 10 + \_\_\_$

Develop an understanding of addition, using the conventional vertical form and using expanded notation.

$$\begin{array}{r} 23 \\ +\ 42 \\ \hline 65 \end{array} \qquad \begin{array}{r} 20 + 3 \\ 40 + 2 \\ \hline 60 + 2 = 65 \end{array}$$

--Extend addition to include regrouping.

$$\begin{array}{r} 28 \\ +\ 45 \\ \hline 73 \end{array} \qquad \begin{array}{r} 20 + 8 \\ 40 + 5 \\ \hline 60 + 13 = 70 + 3 = 73 \end{array}$$

--Develop an understanding of subtraction, using the conventional vertical form and using expanded notation.

$$\begin{array}{r} 35 \\ -\ 23 \\ \hline 12 \end{array} \qquad \begin{array}{r} 30 + 5 \\ -(20 + 3) \\ \hline 10 + 2 = 12 \end{array}$$

--Extend subtraction to include regrouping.

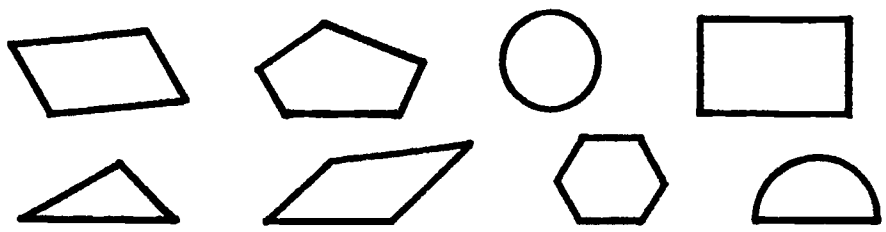
$$\begin{array}{r} 23 \\ -\ 18 \\ \hline 5 \end{array} \qquad \begin{array}{r} 20 + 3 \\ -(10 + 8) \\ \hline \end{array} = \begin{array}{r} 10 + 13 \\ -\ (10 + 8) \\ \hline 5 \end{array}$$

--Provide students with simple word problems using expanded notation and regrouping.

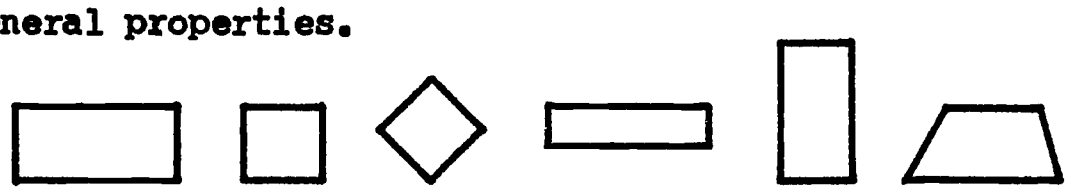
B. Suggested Procedures (contd)

--Provide activities requiring students to identify quadrilaterals.

--Instruct class members to mark with an X the figures that are quadrilaterals:

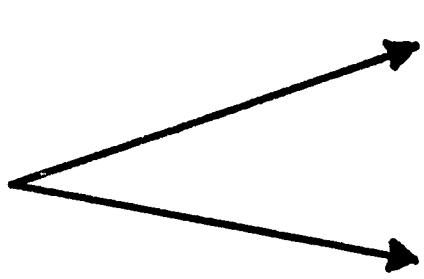


--Discuss quadrilaterals to develop an understanding of their general properties.

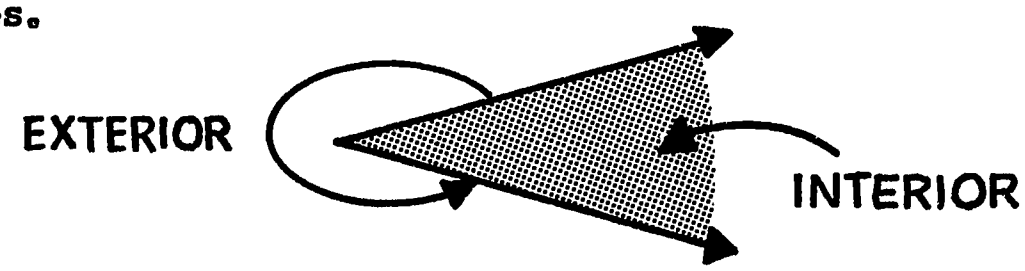


Ask the class, "What do you notice about each?"

--Develop an understanding of an angle as the union of two rays.



--Develop an understanding of interior and exterior regions of angles.



--Relate the concepts of "greater than" and "less than" to angles.

--Discuss the protractor and its composition, and use.

--Provide activities requiring students to use the protractor to measure angles.

--Help students to develop skills in the classification of angles by size.

C. Evaluation

It is expected that the student will be able to demonstrate that he has learned to:

1. Express numbers in many different ways.

2. Express number ideas in the conventional form and in expanded notation.

3. Recognize the positional values of digits in numerals.

4. Apply the concept of expanded notation to solve addition and subtraction problems.

5. Solve problems involving addition and subtraction with and without regrouping.

6. Differentiate between quadrilaterals and other polygons.

7. Differentiate between triangles and other polygons.

8. Classify quadrilaterals, triangles, and angles.

9. Use the protractor to measure angles.

UNIT VI. MULTIPLICATION: OPERATION WITH WHOLE NUMBERS — 16 Teaching Days

A. Development of the Unit

1. Introduce multiplication by using arrays of collections of objects.
   a. Develop the commutative property by using arrays.
   b. Introduce multiplication by using arrays having equal numbers of members.
2. Introduce multiplication by the use of the number line.
   a. Develop an understanding of whole numbers as distances on a number line.
   b. Develop multiplication as repeated marking off of a unit distance along a number line.
3. Introduce the commutative property of multiplication.
   a. Develop the concept that the order of factors does not affect the product.
   b. Demonstrate the commutative property of multiplication on a number line.
4. Introduce the associative property of multiplication.
5. Introduce the concepts of zero and one as applied to multiplication.
   a. Develop the concept of one as the identity element for multiplication.
   b. Develop the concept that the product of zero and any number is zero.
6. Apply concepts of expanded notation and place value in multiplication.
7. Introduce problems for students in which the use of multiplication is involved.
8. Develop further understandings of geometric concepts.

   Ask students to use concepts of properties of angles to construct

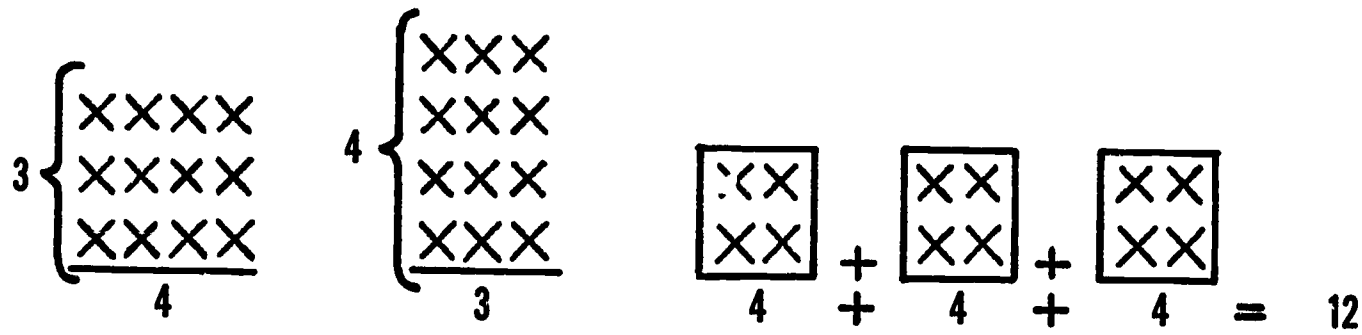
Straight angles	Acute angles
Right angles	Obtuse angles

9. Extend and redevelop the concepts of measurement.
   a. Develop the concept of area.
   b. Develop the concept that the sum of the angles of a quadrilateral is 360 degrees.
   c. Explore the concept that the sum of the angles of a triangle is equal to 180 degrees.
   d. Assist students in developing an intuitive understanding of areas of squares and rectangles.

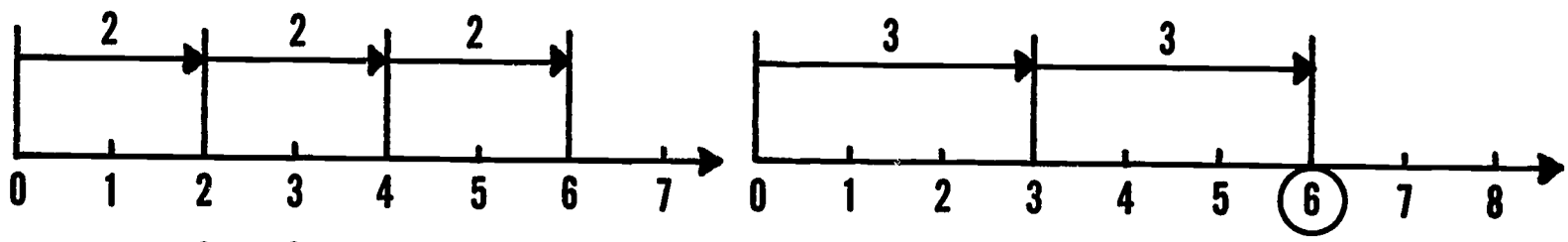
B. Suggested Procedures

The teacher may find it useful to:

--Use rectangular arrays to interpret multiplication.



$3 \times 4 = 12$ $4 \times 3 = 12$ $3 \times 4 = 12$



$2 + 2 + 2 = 6$ $3 + 3 = 6$

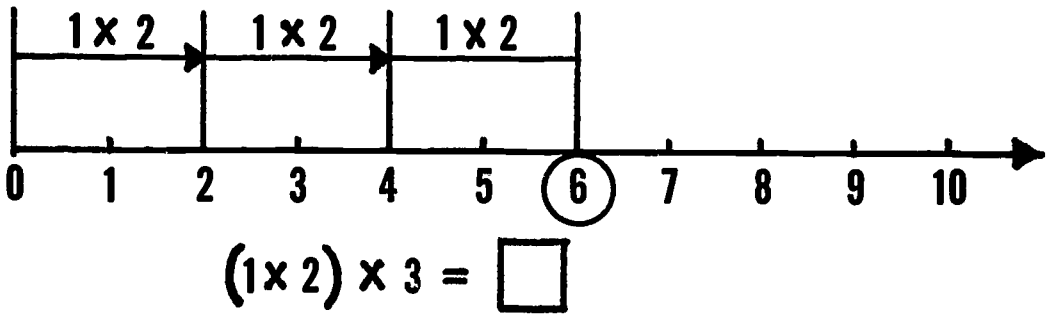
$3 \times 2 = 6$ $2 \times 3 = 6$

--Develop the commutative property by showing pairs of related equations

$1 \times 3 = 3$ $3 \times 1 = 3$	$2 \times 4 = 8$ $4 \times 2 = 8$	$4 \times 1 = 4$ $1 \times 4 = 4$	$18 = 6 \times 3$ $18 = 3 \times 6$
$6 = 2 \times 3$ $6 = 3 \times 2$	$12 = 3 \times 4$ $12 = 4 \times 3$	$3 \times 0 = 0$ $0 \times 3 = 0$	$18 = 9 \times 2$ $18 = 2 \times 9$

B. Suggested Procedures (contd)

--Develop the associative property by showing pairs of related equations on the number line.



(1 x 2) x 3 = □

1 x (2 x 3) = □

(1 x 3) x 0 = □

1 x (3 x 0) = □

--Introduce the multiplication table (matrix) through 9 x 9)

X	0	1	2	3	4	5	6	7	8	9
1										
1										
2			4							
3								21		
4										
5										
6										
7										
8										
9										

--Ask class what pairs of factors give the product in the following

___ x ___ = 21

3 x □ = 21

□ x 7 = 21

FACTOR x FACTOR = 21

FACTOR x □ = PRODUCT

□ x FACTOR = PRODUCT

--Introduce multiplication where zero is a factor

5 x 0 = □

5 x □ = 0

B. Suggested Procedures (contd)

--Combine the commutative and associative properties of multiplication.

--Provide practice writing exercises in numbers in expanded notation.

$$115 = 100 + 10 + 5$$

--Ask students to build multiplication tables (matrices) for multiples of ten.

X	1	10	100	1000
1	1	10	100	1000
2	2	20	200	2000
3	3	30	300	3000
4	4	40	400	4000

X	1	20	30	40
1	1	20	30	40
2	20	40	60	80
3	30	60	90	120

--Help students to relate multiplication facts to expanded notation.

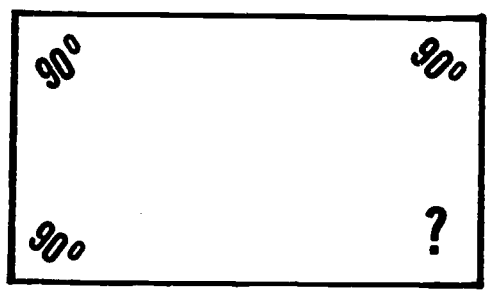
$$23 \times 2 = \begin{array}{r} 20 + 3 \\ \times\ 2 \\ \hline 40 + 6 = 46 \end{array} \qquad \begin{array}{r} 23 \\ \times\ 2 \\ \hline 46 \end{array}$$

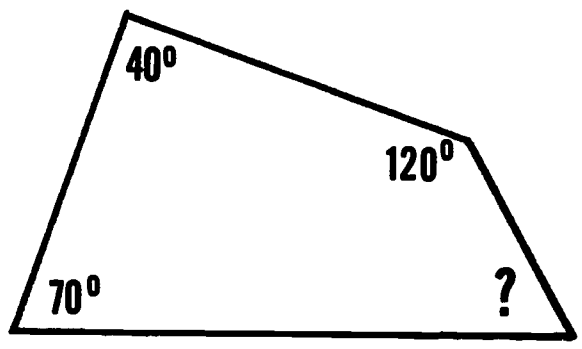
$$132 \times 3 = \begin{array}{r} 100 + 30 + 2 \\ \times\ 3 \\ \hline 300 + 90 + 6 = 396 \end{array} \qquad \begin{array}{r} 132 \\ \times\ 3 \\ \hline 396 \end{array}$$

--Introduce problems for students in which multiplication must be used.

--Explain methods of constructing straight angles, right angles, acute angles, and obtuse angles.

--Ask students to construct quadrilaterals. Instruct them to measure angles and to find the sums.





B. Suggested Procedures (contd)

--Ask students to construct triangles, to measure their angles, and to find the sum of the angles.

--Redevelop the idea of area and correlate the counting of squares to the concept of length and width as it relates to finding areas of rectangles and squares.

C. Evaluation

It is expected that students will be able to demonstrate that they have learned to

1. Use effectively the idea of multiplication as repeated addition.

2. Interpret multiplication by means of arrays and a number line model.

3. Make use of the commutative and associative properties of multiplication.

4. Complete multiplication matrices (tables) for whole numbers through 9 and for decades. (As 20, 30, 40.)

5. Recognize that one and zero have special properties related to multiplication.

6. Solve multiplication problems written in expanded notation and in the conventional vertical form.

7. Solve simple word problems using multiplication.

8. Construct special angles.

9. Measure angles of quadrilaterals and triangles and observe a geometric fact about the sum of the angles of each.

10. Use the formulas for the area of squares and rectangles.

UNIT VII. DIVISION: OPERATION WITH WHOLE NUMBERS — 15 Teaching Days

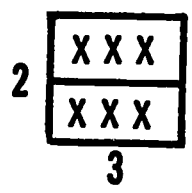
A. Development of the Unit

1. Introduce division as the opposite of multiplication.
   a. Present division as set partitioning.
   b. Present related division and multiplication facts together.
2. Present division as repeated subtraction.
   a. Develop division on the number line.
   b. Show that division can be accomplished by repeated subtraction.
3. Develop division as location of the missing factor.
4. Develop the properties of the operation of division.
   a. Develop the concept that division of any number by one produces that same number.
   b. Develop the concept that dividing zero by any number produces zero.
   c. Develop the concept that division by zero is undefined.
   d. Develop the concept that division is not commutative or associative.
5. Introduce vocabulary used in division.
   a. Present definitions of dividend, divisor, and quotient.
   b. Identify dividend, divisor, and quotient in various types of problems.
6. Introduce geometric concepts that include properties of the circle.
   a. Help students to interpret the relationship between the diameter and the radius of a circle.
   b. Develop the concepts of circumference and arc.
   c. Introduce chord and tangent.
   d. Discuss the concept of the center of a circle.
   e. Develop the idea that a circle is a closed curve dividing a plane with three sets of points.
7. Continue the study of measurement.
   a. Show the area of a circle by counting unit squares.
   b. Assist students in applying the concepts of addition, subtraction, multiplication, and division to denominate numbers.

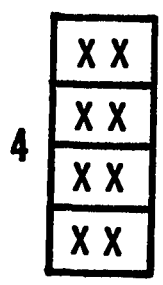
B. Suggested Procedures

The teacher may find it useful to:

--Interpret division as the opposite of multiplication by using sets of physical objects and arrays.



$2 \times 3 = 6$
$6 \div 2 = 3$



$4 \times 2 = 8$
$8 \div 2 = 4$

--Use pairs of related equations to emphasize that division is the opposite of multiplication.

$2 \times \square = 8$
$8 \div 2 = \square$

--Use the concept that Factor x Factor = Product to develop concept that Product - Factor = Factor.

$3 \times 2 = 6$ Factor x Factor = Product

$6 \div 2 = 3$
$6 \div 3 = 2$ } Product ÷ Factor = Factor

--Develop activities to demonstrate that

a. Division of any number by one produces that same number.

$3 \div 1 = 3$
$3 \div \square = 3$

$5 \div 1 = 5$
$\square \div 1 = 5$

b. Division of zero by any number produces zero.

$0 \div 5 = 0$
$0 \div 6 = 0$

c. Division of any number by zero is undefined.

d. Division is neither commutative nor associative.

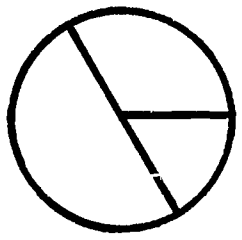
	True or False	
$4 \div 2 = 2 \div 4$	____	____
$6 \div 3 = 3 \div 6$	____	____
$8 \div 4 \div 2 = 8 \div 4 \div 2$	____	____

--Introduce activities to build understanding of terms.

a. Divisor b. Dividend c. Quotient

B. Suggested Procedures (contd)

--Ask students to construct a circle and to draw several diameters.



--Help class members to develop intuitive definitions of diameter and radius. Establish the relationship between radius and diameter.

--Instruct students to use string and tape measures to estimate circumference.

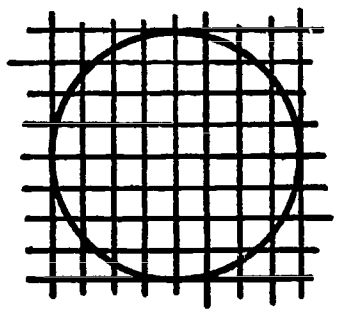
--Assist students in developing an intuitive understanding of the relationship between diameter and circumference.

--Point out to students the names for other parts of a circle.

--Review units of linear measure and area measure.

--Use the counting of unit squares in rectangles and squares to assist students in discovering formulas for area.

--Instruct class members to use squared paper to construct a circle and estimate the area. (They may use inches or any other unit of measure.)



--Provide practice exercises in subtracting, multiplying, and dividing denominate numbers.

3 ft. 2 in.	35° 25′
+ 2 ft. 5 in.	10° 20′
5 ft. 7 in.	47° 45′

C. Evaluation

It is expected that students will be able to demonstrate that they have learned to:

1. Relate division and multiplication.

2. Demonstrate that division of any number by one produces that same number.

3. Demonstrate that dividing zero by any number produces zero, but that division by zero is undefined.

4. Use the idea that Factor x Factor = Product and relate this to the fact that the Product ÷ Factor = Factor.

5. Recognize that division is not commutative nor associative.

6. Know the vocabulary associated with the process of division.

7. Know the vocabulary related to the use of a circle.

8. Develop intuitively formulas for computing the areas of squares and rectangles.