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A TRANSITIONAL CURRICULUM GUIDE FOR MATHEMATICS IN GRADES 7 AND 8.

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THIS TRANSITIONAL CURRICULUM GUIDE WAS DESIGNED TO SERVE THE FOLLOWING PURPOSES--(1) TO POINT OUT THE VARIOUS CONCEPTS, DEFINITIONS, MEANINGS, AND APPLICATIONS RELATED TO CERTAIN AREAS OF MATHEMATICS WHICH SHOULD BE THE CONTENT OF MATHEMATICS IN GRADES SEVEN AND EIGHT, (2) TO BRIDGE THE GAP BETWEEN TRANSITIONAL PROGRAMS AND MORE MODERNIZED COURSES, TO INCORPORATE MODERN TERMINOLOGY WITH THE TRADITIONAL TOPICS, AND TO INTRODUCE NEW CONCEPTS AS APPROPRIATE, AND (3) TO HELP TEACHERS BUILD AN ARITHMETIC BACKGROUND OF THEIR STUDENTS BY PRESENTING NEW IDEAS IN A WAY ACCEPTABLE TO ALL STUDENTS, BY MAINTAINING AND POLISHING COMPUTATIONAL SKILLS, BY INTRODUCING AND USING MODERN TERMINOLOGY AS NEEDED, AND BY DEVELOPING PATTERNS OF THOUGHT NECESSARY TO LATER WORK IN MATHEMATICS. SAMPLE INSTRUCTIONAL UNITS ON A NUMBER OF TOPICS HAVE BEEN INCLUDED. THESE UNITS ARE STRUCTURED TO SHOW HOW THE MATERIAL CAN BE ORGANIZED FOR EFFICIENT TEACHING AND TO PROVIDE SOME HELPFUL IDEAS ABOUT HOW TO PRESENT CERTAIN TOPICS. TOPICS PRESENTED IN THE GUIDE INCLUDE NUMBERS AND OPERATIONS, GEOMETRY, MEASUREMENT, BUSINESS ARITHMETIC, RATIOS, GRAPHS, SETS, MATHEMATICAL SENTENCES, AND STATISTICS. (RP)

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(With Teaching Units)

1966

Prepared by
Mathematics Teachers of New Mexico

Approved by
New Mexico State Board of Education

New Mexico State Department of Education
Santa Fe, New Mexico

Leonard J. De Layo
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Prepared by the

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NEW MEXICO STATE BOARD OF EDUCATION

FOREWORD

This guide has been prepared to serve the needs of New Mexico's teachers of mathematics in grades seven and eight, junior high school principals, and directors of instruction. It is presented for use during a period in which junior high school mathematics, as a specific body of material, is in the process of achieving full stature in its own right. No longer is the role of mathematics in grades seven and eight largely a role of summarizing the learnings of elementary arithmetic. Rather, mathematics study at the junior high school level today is regarded as an integral, and important part of the total mathematics program of the secondary school.

The material within this guide has been carefully prepared and critically evaluated. It is presented with the hope that it may offer maximum assistance in meeting today's need for a program of mathematically-valuable content for study by pupils in the first two years of the secondary school.

To the committee members and staff members who assisted with approximately two years of planning, of research, of writing, and of rewriting which produced this book, are extended sincere appreciation and gratitude.

Leonard J. DeLayo

Superintendent of Public Instruction

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Dr. Merle Mitchell, University of New Mexico, served the Committee as advisor and content-consultant. To her, a special measure of appreciation is extended.

Lura Bennett
Mathematics Specialist
State Department of Education

PREFACE

It is the hope of the planning and writing committee that its work, "A Transitional Curriculum Guide For Grades 7-8 Mathematics," will be found by teachers in the schools of New Mexico to be just what its name implies.

First, it is intended as a guide to the various concepts, definitions, meanings, and applications related to certain areas of mathematics which the committee feels should be the content of mathematics in grades seven and eight. The term "strand" has been used to refer to a whole body of subject matter which should be interspersed throughout the year's program rather than being presented as a single unit. The content has been organized in eight strands, appropriate parts of each being listed for grade seven and for grade eight. It is assumed that the adopted textbook will serve as the key to the sequence of the topics at each level, but teachers are urged to change the textbook order and to supplement the text materials when class needs justify such deviations.

Second, the guide is designated as "transitional." The Committee chose this adjective to describe the guide because it attempts to bridge the gap between traditional programs and more modernized courses. Not based on any one textbook, the guide seeks to incorporate modern terminology with the traditional topics and to introduce new concepts as appropriate. A recognition of the problems of this period of transition brought about this approach.

Third, sample instructional units on a number of topics have been included with the hope that they will show how the material in the strands can be organized for efficient teaching and that they will offer some helpful ideas about how to present certain topics.

The philosophy of the Committee is that junior high school mathematics should be an expansion of previous knowledge and a foundation for the study of advanced mathematics. It should represent a change from the acquisition of mathematical facts and processes needed in daily living to the beginnings of an appreciation of the mathematical way of thinking. Since some students will have had some work in modern arithmetic while others will have only traditional backgrounds, the program of the junior high school must have a flexibility to meet the situation. This "Guide" is designed to help teachers build on whatever arithmetic background their students have by presenting new ideas in a way acceptable to all students, by maintaining and polishing computational skills, by introducing and using modern terminology as needed, and by developing patterns of thought necessary to later work in mathematics.

The junior high school teacher of mathematics has a big and important order to fill. May this "Guide" be of substantial assistance in this task.

Merle Mitchell
Department of Mathematics
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MATHEMATICS FOR THE SEVENTH GRADE

MATHEMATICS FOR THE SEVENTH GRADE

Desired Outcomes and Goals

1. To introduce pupils to a widened world of mathematical ideas, in part by helping them develop understanding of the "whys" of arithmetic processes before the statement of generalizations or rules.

2. To give reasonable practice in several ways of reasoning from the known to logical conclusions about the unknown, and to begin the development of pupil-understanding of the properties of the natural numbers, the integers, the rational and the real numbers at this level.

3. To develop the computational skills needed for applying mathematics concepts and processes to subject matter and practical applications appropriate to this maturity level.

4. To emphasize the language and the use of the basic ideas of sets, in part through developing pupil-understanding of the relationship between points on the number line configuration and the conbined sets of rational and irrational numbers which form the real numbers.

5. To extend the understanding and use of equations and inequations as number sentences, to introduce basic geometric concepts and constructions, to teach the concept of per cents from the ratio-proportion approach, and to assist pupils develop a beginning understanding of statistical methods of describing sets of numerical data.

Terminology

1. "Strand," as used in this Guide, refers to the body of concepts, definitions, meanings, and applications pertaining to an area in mathematics. The material of the seventh grade strand on numbers and operations, as is true of the content of any other strand, should be interspersed throughout the year's program. A mathematics strand, as designated in this Guide, is not intended to delineate a body of subject matter to be taught as a single unit.

Terminology (continued)

2. The word "condition" has been consistently used throughout the Guide because it permits simultaneous reference to equations and inequations.

Symbol Meanings Used in this Guide

$=$	equal	(m)	measurement	$\parallel$	parallel
$\neq$	not equal	$^\circ$	degree	$\perp$	perpendicular
$\approx$	approximately	π	pi	$\widehat{XYZ}$	major arc
$\cong$	congruent	$\bullet$	point	$\widehat{XY}$	minor arc
$\longleftrightarrow$	equivalent	$\leftrightarrow$	line	$\{\ \}$	set
$\sim$	equivalent	$—$	segment	$\{X \mid X < 10\}$	set of all X "such that" X is less than 10
$\sim$	negation, Strand II (some texts)	$\rightarrow$	directed segment	$\{\}$ or ϕ	empty set
$>$	greater than	$\bullet\!\!\longrightarrow$	ray	$\cup$	union
$<$	less than	$\odot$	circle	$\cap$	intersection
$\wedge$	and	Δ	triangle	A X B	A cross B
$\vee$	or	$\angle$	angle	(a,b)	the ordered pair a,b
$\pm$	tolerance	$\square$	parallelogram	$\in$	is a member of

1. Following this, use Strand I - VIII for grade 7
2. Mathematics for Eighth Grade (sheet) and symbol meanings used
3. Strands I - VIII for grade 8
4. Mathematics Terms & Concepts used in this Guide and Sug. Tchg. Equip. for a JHS Math Rm.
5. Appendices - Strand I, III, IV, V (2), VIII (2)

MATHEMATICS CONTENT, EXPERIENCES, AND OUTCOMES
FOR THE SEVENTH GRADE PUPIL
Strand I, Numbers And Operations

Strands And Topics	Content And Competencies To Be Developed	Suggested Background And Resource Material
I. Numbers And Operations		
A. Distinction Between Number And Numeral	1. Pupil learns that: a) A "number" is an idea about specific "numerousness." b) Numerals are written or spoken symbols which represent numbers.	1. If we write 5 and then erase it, do we destroy the number five? We only destroy a symbol for the number five. Some ways of expressing five: V, Roman — (3 + 2) shows five as a sum. IIIII, Egyptian — $\frac{10}{2}$ shows five as a quotient.
B. Positional Numeration Systems	1. Learning that a "base" indicates the pattern of grouping.	1. Some suggestions for work with bases Base five symbols needed: 0, 1, 2, 3, 4 (5 symbols or digits) Base seven symbols needed: 0, 1, 2, 3, 4, 5, 6 (7 symbols or digits)
	2. Recognition that: a) Any number may be represented in different bases. b) The "numerousness" being represented is unchanged, but various symbols are used in different bases.	Base twelve symbols needed: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, T, E (12 symbols or digits) NOTE: Other symbols may be used for T and E.
	3. Recognition of advantage of organizing all addition "facts" in any given base into one table. 4. Construction of addition and multiplication "grids" (square tables) for various bases. NOTE: It is recommended that there shall be no memorizing of such tables.	4. (see Base Six Addition Table and Base Six Multiplication Table below)

+	0	1	2	3	4	5
0	0	1	2	3	4	5
1	1	2	3	4	5	10
2	2	3	4	5	10	11
3	3	4	5	10	11	12
4	4	5	10	11	12	13
5	5	10	11	12	13	14

Base Six Addition Table

X	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	1	2	3	4	5
2	0	2	4	10	12	14
3	0	3	20	13	14	23
4	0	4	12	20	24	32
5	0	5	14	23	32	41

Base Six Multiplication Table

Strands And Topics	Content And Competencies To Be Developed	Suggested Background And Resource Material
B. Positional Numeration Systems (cont.)	5. Translation from base 10 to various bases and vice versa.	5. Example: Changing the decimal numeral 472 to a base five numeral Remainders / Position in Base Five Numeral 5⌊472 5⌊94 -----groups of 5 and ---- 2(ones) ------------ in 1's place 5⌊18 ---groups of 25 and -- 4(fives) ------------ in 5's place 5⌊3 ---groups of 125 and --3(twenty-fives) -- in 25's place 0 ---groups of 625 and --3(125's) ------------in 125's place NOTE: Division ends when quotient is zero. Now we see that $472_{10} = 3342_5$ ← This numeral from this column in reverse order.
C. Cardinal and Ordinal Uses of Numbers	1. Knowledge that cardinal numbers tell "how many."	1. Example of cardinal use of a number: Five cars were needed to transport the class to the pinic.
	2. Knowledge that ordinal numbers tell "which one."	2. Example of ordinal use of a number: Pedro is in the fourth grade.
D. Whole Numbers	1. Knowledge that, for the purposes of this Guide, the set of whole (natural) numbers is composed of zero and the counting numbers.	1. $W = \{0, 1, 2, \ldots\}$ or $N = \{0, 1, 2, \ldots\}$
	2. Knowledge that the set of whole (natural) numbers is endless or unlimited in the number of its elements.	
1. Properties	1. Learning about whole-number properties through experimentation: a) Closure: The sum of any two or more whole numbers is also a whole number. The product of any two whole numbers is also a whole number. ∴ The set of whole numbers is "closed" under both addition and multiplication.	1. a) Demonstrations of the closure property include: $7+8=15$, $9+3+11=23$, $7\times6=42$, $2\times3\times6=36$ Suggestions for pupil demonstration and discovery: Is the set of even numbers closed under addition and multiplication? That is, if a series of even whole numbers are added together, is the sum always an even number? Is the product an even number? Is the set of odd numbers closed under addition? multiplication?

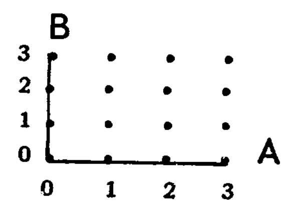
Strands And Topics	Content And Competencies To Be Developed	Suggested Background And Resource Material
1. Properties (cont.)	b) Commutative Property: $a+b = b+a$ and $a \cdot b = b \cdot a$	b) Examples: $3+7 = 7+3$, $10 = 10$; $4 \cdot 12 = 12 \cdot 4$, $48 = 48$
	c) Associative Property: $(a+b)+c = a+(b+c)$ $(a \cdot b) \cdot c = a \cdot (b \cdot c)$	c) Examples: $(7+3)+11 = 7+(3+11)$ $10+11 = 7+14$ $21 = 21$ $(6 \cdot 2) \cdot 5 = 6 \cdot (2 \cdot 5)$ $12 \cdot 5 = 6 \cdot 10$ $60 = 60$
	d) Distributive Property: $a \cdot (b+c) = a \cdot b + a \cdot c$ NOTE: The "distributive" name comes from the fact that the multiplication distributes over the addition	d) Examples: $4 \cdot (5+9) = (4 \cdot 5)+(4 \cdot 9)$ $4 \cdot 14 = 20+36$ $56 = 56$ $5 \cdot (7-3) = (5 \cdot 7)-(5 \cdot 3)$ $5(4) = 35-15$ $20 = 20$
2. Order Relations	1. Pupils demonstrate understanding of meaning and ability to use the symbols: $>$ - - - - - - - is greater than $<$ - - - - - - - is less than $=$ - - - - - - - equals; is equal to $\neq$ - - - - - - - is not equal to $\approx$ - - - - - - - is approximately equal to $\geq$ - - - - - - - is greater than or equal to $\leq$ - - - - - - - is less than or equal to	1. Examples: a) $7>5$ means that the <u>number</u> 7 is greater than the number 5. b) $16<572$ means that the <u>number</u> 16 is less than the number 572. c) $17 = 8+9$ means that here are two names for the same number. d) $\sqrt{2} \approx 1.414$ means that the square root of 2 is approximately equal to but not exactly equal to 1.414. e) $7 \neq 8$ means that 7 is not equal to 8. f) Appropriately used symbols: $7 \geq 7$ $14 \geq 9$ $5 \leq 5$ $17 \leq 248$
	2. Learning that if a and b represent any two whole numbers, one <u>and only one</u> of these statements is true: $a>b$ <u>or</u> $a=b$ <u>or</u> $a<b$	2. Example: Given the whole numbers 3 and 7 we see that: $3=7$ is false, $3>7$ is false, $3<7$ is true
3. Identity Elements	1. Recognition that <u>zero</u> is the identity element in addition.	1. $6+0 = 6$ $0+12 = 12$ $39+0 = 39$ $0+402 = 402$
	2. Recognition that <u>one</u> is the identity element in multiplication.	2. $1 \cdot 7 = 7$ $13 \cdot 1 = 13$ $394 \cdot 1 = 394$ $1 \cdot 89 = 89$

Strands And Topics	Content And Competencies To Be Developed	Suggested Background And Resource Material
D. Whole Numbers (cont.)	1. Discovery that an inverse operation "undoes" what has been done by another operation.	1. Example: The inverse of putting a pencil on the desk would be lifting the pencil from the desk.
4. Inverse Operations	2. Recognition that inverse of addition is subtraction.	2. If 3 is added to 4 the result is 7. Notice we can undo the addition by subtracting either of the addends from the sum. $3 + 4 = 7$ $\quad 7 - 4 = 3$ $\quad 7 - 3 = 4$
	3. Recognition that the inverse of multiplication is division.	3. Suggestion: Provide practice in changing given multiplication problems into related division problems and vice versa.
5. Division by zero	1. Learning that division by zero: a) Is meaningless b) Must be guarded against in the use of general (literal) numbers.	1. If zero is used as a divisor, see what happens: Suppose $24 \div 0 = 24$, then $0 \times 24 = 24$. This is evidently false! So division by zero doesn't show division as the inverse of multiplication. Since division by zero is meaningless (undefined), by mathematical agreement we guard against its use.
6. Order of Operations	1. Discovery that symbols serve as "mathematical punctuation:" a) Common grouping symbols say "Do this part first." ( ) parentheses { } braces [ ] brackets ——— bar b) Without grouping symbols, by agreement: 1) Multiplications and divisions are performed first in the order given, from left to right. 2) Secondly, additions and subtractions are performed in the order given, from left to right.	1. b) Example: $12 \div 2 + 1$ Shall we divide twelve by two, then add one, getting seven as a result? Shall we first add the two and one and then divide twelve by the resulting three, getting four as a result? Examples: $12 \div 2 + 1 = 6 + 1 = 7$ $\quad 16 \div 4 + 3 \times 7 = 4 + 21 = 25$ $120 \div (11 + 9) = 120 \div 20 = 6$ $\quad \frac{12 + 8}{4} = \frac{20}{4} = 5$

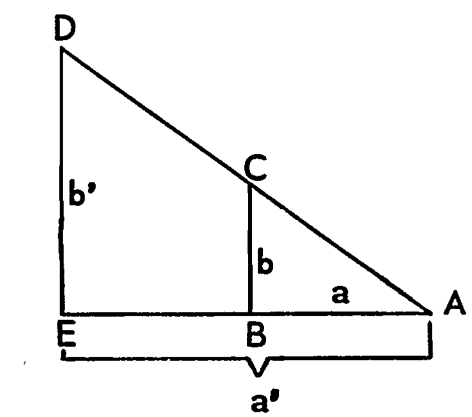
Strands And Topics	Content And Competencies To Be Developed	Suggested Background And Resource Material
E. Factoring and Primes 1. Composite Numbers	1. Mastery of and ability to apply appropriately the following meanings: a) Every whole number except 1 is either prime or composite. b) A composite number is a number which can be expressed as the product of two or more whole numbers (factors) other than 1 and the number itself.	1. NOTE: One, (1), has been excluded as a prime number by mathematicians, in part, to give logical meaning to the unique factorization theorem as applied to any whole number. See following G, 4 section. 2. Examples of composite numbers: $6 = 3 \times 2$, $8 = 4 \times 2$ Notice that the factors of composite number 6 are both less than 6 and greater than 1. Similarly, the factors of the composite number 8 are both less than 8 and greater than 1.
2. Prime Numbers	1. Recognition that a prime number is a number which has only two factors, itself and 1.	1. Examples of prime numbers: $7 = 7 \times 1$, $13 = 13 \times 1$, $5 = 5 \times 1$ NOTE: 5, 7, 13 are primes. In each case the above factorization is the only way that the prime can be expressed as a product unless the factor one (1) is repeated. Because $5 \times 1 = 5$ and $5 \times \underbrace{1 \times 1}_{\text{one}} = 5$ and $5 \times \underbrace{1 \times 1 \times 1 \cdots}_{\text{one}} = 5$, it is apparent that one (1) behaves differently than do primes. Therefore, mathematicians excluded it from the prime numbers.
3. Exponential Notation	1. Learning that exponents, small numerals (superscripts) at the upper right, are a short way of indicating how many times a number is used as a factor. 2. Development of reasonable skill in representing multiplicative situations by means of exponents.	1. Examples: x^3 means $x \cdot x \cdot x$; a^5 means $a \cdot a \cdot a \cdot a \cdot a$ } Exponents 3 and 5 shorten notation. 1. Examples: $10 \cdot 10 \cdot 10$ (or 1000) $= 10^3$; $y \cdot y \cdot y \cdot y \cdot y \cdot y \cdot y = y^7$ } Exponents 3 and 7 shorten notation.
4. Unique Prime Factorization	1. Discovery of the mathematical relationships: a) Every whole number greater than one (1) is either prime or can be factored into a UNIQUE (only one) set of primes. b) Primes are the "building blocks" of the whole-number system.	1. Problem: How many ways can 12 be expressed as product of primes? $12 = 6 \times 2$ (6 is composite so we must factor it) $12 = 2 \times 3 \times 2$ Now all factors are prime. Suppose we approach this problem from another standpoint. $12 = 4 \times 3$ (4 is composite) $12 = 2 \times 2 \times 3$ Now all factors are prime. NOTE: The same set of prime factors results from both approaches.

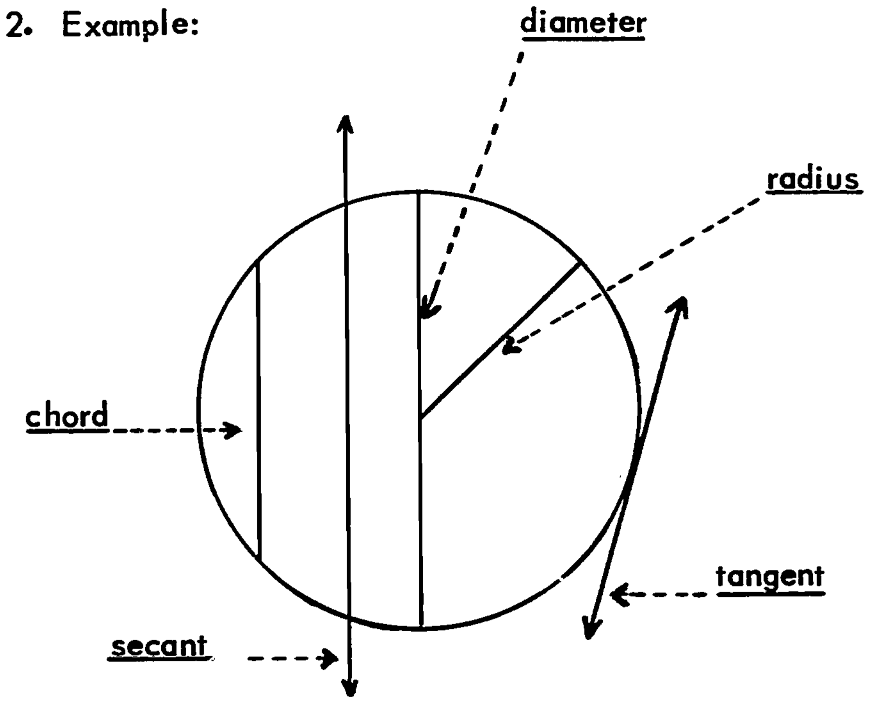
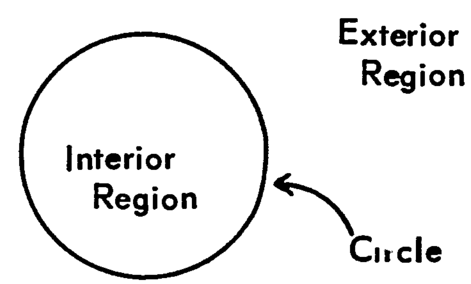
Strands And Topics	Content And Competencies To Be Developed	Suggested Background And Resource Material
F. Rational Numbers 1. Definitions	1. Learning that a rational number is one which may be expressed as the ratio of two whole numbers.	1. Examples: a) $\frac{3}{4}, \frac{1}{2}, \frac{2}{1}, \frac{4}{2}, \frac{16}{1}$ b) .5 is a rational number because it may be expressed as $\frac{5}{10}$ or $\frac{1}{2}$
2. Properties	1. Recognition that properties developed with whole numbers "hold" for rationals.	
3. Decimal Fraction Notation	1. Learning that denominators of decimal fractions are powers of ten.	1. Examples: $\frac{1}{10} = \frac{1}{10^1} = .1$ $\quad \frac{1}{10000} = \frac{1}{10^3} = .001$ $\frac{1}{100} = \frac{1}{10^2} = .01$ $\quad \frac{1}{10000} = \frac{1}{10^4} = .0001$
4. Ordering	1. Student demonstrates ability to put series of common fractions in "order of size" by: a) Finding a common denominator through use of factoring and and LCM (least common multiple). b) Converting to decimal fractions, which means the use of a common denominator that is some power of 10.	1. Example: Arrange $\frac{1}{2}, \frac{1}{3}, \frac{4}{10}$ in order of size. ($10 = 2 \cdot 5$) a) Common denominator is 30, because prime factors 2 x 3 x 5 produce 30. Fractions may be expressed as: $\frac{15}{30}, \frac{10}{30}, \frac{12}{30}$. In order of size: $\frac{10}{30}, \frac{12}{30}, \frac{15}{30}$ or $\frac{1}{3}, \frac{4}{10}, \frac{1}{2}$ b) $\frac{1}{2} = .50$ $\quad \frac{1}{3} = .33\frac{1}{3}$ $\quad \frac{4}{10} = .40$ In order of size: $.33\frac{1}{3}$, .40, .50 or $\frac{1}{3}, \frac{4}{10}, \frac{1}{2}$

Strands And Topics	Content And Competencies To Be Developed	Suggested Background And Resource Material
5. Reciprocals	1. Development of knowledge that: a) Reciprocals are rational numbers whose terms are in inverse order as: $\frac{a}{b}, \frac{b}{a}, \frac{2}{3}, \frac{3}{2}, \frac{29}{1}, \frac{1}{29}$ b) The product of a number and its reciprocal is 1. $\frac{a}{b} \cdot \frac{b}{a} = \frac{ab}{ba} = \frac{ab}{ab} = 1$ c) Every rational number except zero has a reciprocal.	1. Examples: a) The number 2 and the number $\frac{1}{2}$ are reciprocals of each other because the rational number 2 means $\frac{2}{1}$ which is the inverse order of $\frac{1}{2}$. b) $2 \times \frac{1}{2} = 1$ because $\frac{2}{1} \times \frac{1}{2}$ means $\frac{2 \times 1}{1 \times 2}$ or $\frac{2}{2}$ which is another name for 1. c) Zero has no reciprocal because there is no number which, when multiplied by 0, results in 1. $0 \times N = 0$
6. Summary: Operations on Rationals	1. Pupil demonstrates ability to add, subtract, multiply and divide rational numbers according to these mathematical "agreements": a) $\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$, $(c \neq 0)$ b) $\frac{a}{c} - \frac{b}{c} = \frac{a-b}{c}$, $(c \neq 0)$ c) $\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d}$, $(b \neq 0,\ d \neq 0)$ d) $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$, $(b \neq 0,\ c \neq 0,\ d \neq 0)$	1. Examples: a) $\frac{2}{13} + \frac{5}{13} = \frac{2+5}{13} = \frac{7}{13}$ b) $\frac{7}{16} - \frac{4}{16} = \frac{7-4}{16} = \frac{3}{16}$ c) $\frac{3}{8} \times \frac{7}{11} = \frac{3 \times 7}{8 \times 11} = \frac{21}{88}$ d) $\frac{\frac{3}{4}}{\frac{1}{2}} = \frac{\frac{3}{4}}{\frac{1}{2}} \times \boxed{\frac{\frac{2}{1}}{\frac{2}{1}}} = \frac{\frac{3}{4} \times \frac{2}{1}}{\frac{1}{2} \times \frac{2}{1}} = \frac{\frac{6}{4}}{\frac{2}{2}} = \frac{\frac{6}{4}}{1} = \frac{6}{4} = 1\frac{1}{2}$ Property of one or one is identity number of multiplication.

Strands And Topics	Content And Competencies To Be Developed	Suggested Background And Resource Material
II. Geometry A. General Meaning	1. Pupil learns that geometry, in updated math courses, is any logical system developed from the idea of sets of points.	1. Hint: Help pupils accept geometry as a man-created tool of great mathematical usefulness. Individual may choose and state own set of postulates and work within this framework. Not depth work at this stage. 2. The concept that any plane and/or space figure is a set of points, make total study of geometry more understandable.
B. Undefined Terms	1. Recognition that any system of ideas must begin somewhere and develop in orderly manner. 2. Pupil learns that points, lines, planes, and space are undefined because they are the beginning ideas of geometry, but may be described as "sets of points."	1. Example: The system of whole numbers begins with "not any" (zero) digit and increases in size in orderly way. 2. Suggestion: Explain that points, lines, planes, and space may be described and that we accept them as described. These terms could be defined only by use of other terms which would then require defining. This becomes a never-ending process.
C. Symbolism	1. Learning to use geometric symbols as aid in writing clear meanings in brief form. 2. Developing facility in use of symbols to express geometric ideas.	1. Segment AB may be written as $\overline{AB}$; line AB may be written as $\overleftrightarrow{AB}$; ray AB may be written as $\overrightarrow{AB}$; Δ ABC may be used to refer to triangle ABC, etc. NOTE: Some authors reserve "block" capital letters for naming sets and points.
D. Relating Points, Lines, Planes, Space	1. Learning characteristics or properties of points, lines, planes and space. Demonstrating ability to use these ideas correctly. 2. Study and utilization of meaning of: a) Collinear points as points included in same line. b) Coplanar points as points that belong to same plane. c) Graph of Cartesian set A X B (read "A cross B") as part of Cartesian plane.	1. Hint: For the most part use properties as given by the authors of your text. This is one of the few areas in which memorizing is recommended. 2.  Given: A = { 0, 1, 2, 3 } and B = { 0,1,2,3 } Cartesian set, A X B = { (0,0), (0,1), (0,2), (0,3), (1,0), (1,1), (1,2), (1,3), (2,0), (2,1), (2,2), (2,3), (3,0), (3,1), (3,2), (3,3) } NOTE: See Strand V Appendix on Cartesian products.

Strand II, Geometry - - - - Grade 7

Strands And Topics	Content And Competencies To Be Developed	Suggested Background And Resource Material
E. Congruency and Similarity	1. Pupil learns that "congruent" geometric figures are exactly alike in every respect. (Definition)	1. Δ ABC "is congruent to" Δ A'B'C' or $\Delta ABC \cong \Delta A'B'C'$
	2. Pupil learns that "similar" geometric figures have the same shape but not necessarily the same size.	2. Suggestion: The idea of similarity is an excellent area in which to use ratio.  $\frac{a}{a'} = \frac{b}{b'}$
F. Separation	1. Pupil uses the idea of betweenness to demonstrate the meaning of separation.	1. Suggestion: Show that if a point in the boundary of 2 half-figures exists between 2 given points, these points are in 2 different half-figures. NOTE: The term half-figure refers to half-line; half-plane; half-space.
	2. Pupil learns the meaning of the definitions: a - A print separates a line into two half-lines and is the boundary of each half-line. b - A line separates a plane into two half-planes and is the boundary of each half-plane. c - A plane separates space into two half-spaces and is the boundary of each half-space.	2. Warning: Be sure that the students understand that points in the boundary are not included in the half-figures.

<table>
<tr><th>Strands And Topics</th><th>Content And Competencies To Be Developed</th><th>Suggested Background And Resource Material</th></tr>
<tr><td>G. The Circle</td><td>1. Pupil recognizes the circle as a geometric figure and describes it in the terminology of sets.</td><td>1. Example: Given circle A with a radius AB.
We can describe the circle as:
$\{ X \mid \overline{AX} \cong \overline{AB} \}$ which is read, "The set all points X such that segment AX is congruent to segment AB.</td></tr>
<tr><td></td><td>2. Pupil demonstrates, by correct usage, the vocabulary of line segments related to a circle:
a) Basic use: radius, diameter, chord
b) Enrichment: secant, tangent</td><td>2. Example:

</td></tr>
<tr><td></td><td>3. Knowledge that regions of the plane related to a circle constitute 3 distinct sets of points known as:
a) The interior region
b) The exterior region
c) The set of points constituting the circle itself, which is called the boundary of the two regions.</td><td>3. Example:

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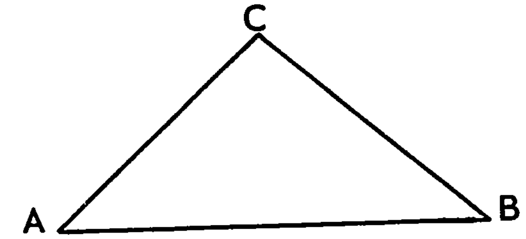
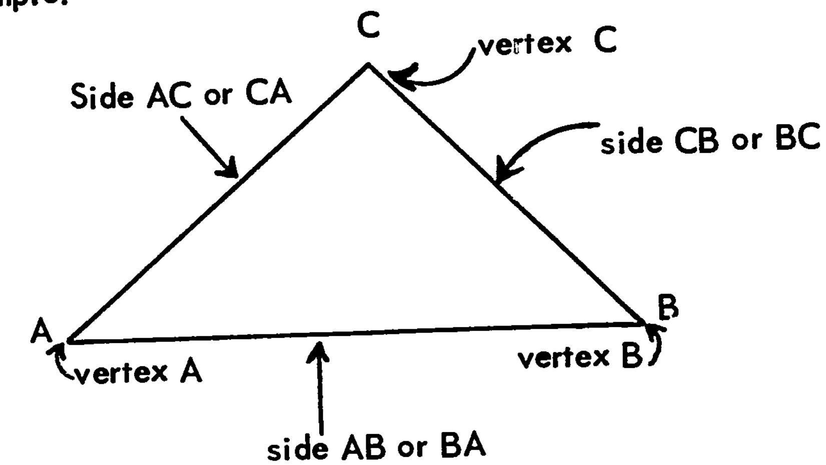
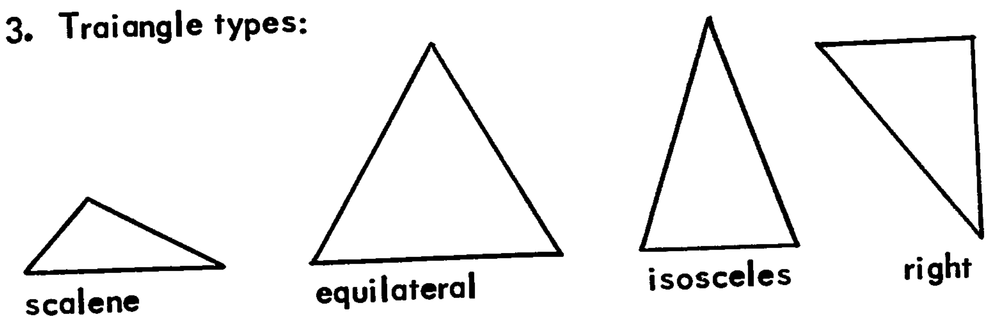
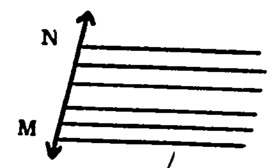
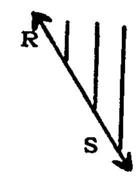
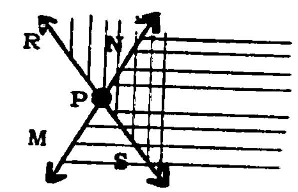
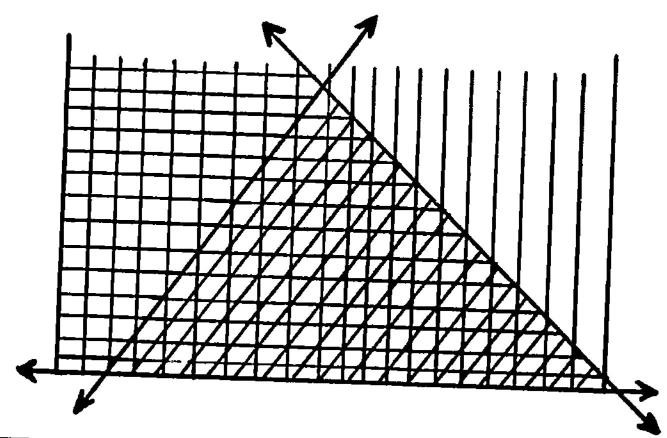
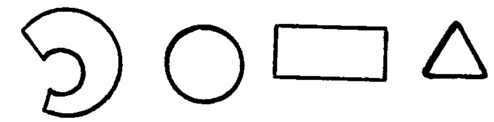
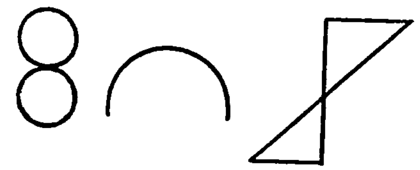
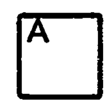
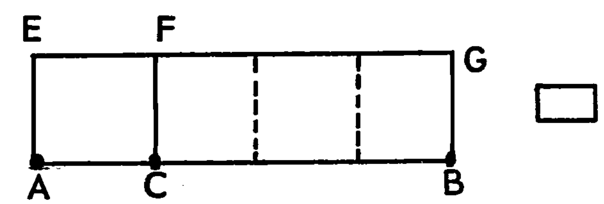
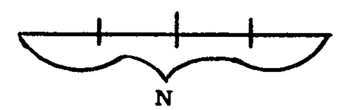
<table>
<tr><th>Strands And Topics</th><th>Content And Competencies To Be Developed</th><th>Suggested Background and Resource Material</th></tr>
<tr><td>J. <u>The Triangle</u></td><td>1. Learning to describe a triangle as the union of 3 segments determined by 3 non-collinear points.</td><td>1.


The union of segments AB, AC and BC would be the triangle ABC.</td></tr>
<tr><td></td><td>2. Learning the standard designation for triangles as:
a) Each side of a triangle is named by the endpoints of the segment forming that side.
b) Each endpoint of a side is a vertex of the triangle.</td><td>2. Example:

</td></tr>
<tr><td></td><td>3. Extending the use of descriptions of types of triangles as:
a) Scalene - <u>no</u> two sides congruent
b) Equilateral - <u>all</u> sides congruent
c) Isosceles - any <u>two</u> sides congruent
d) Right - any triangle having a right angle</td><td>3. Traiangle types:

</td></tr>
</table>

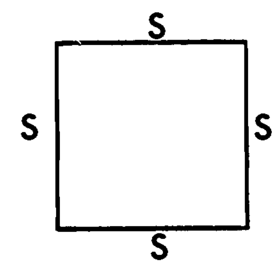
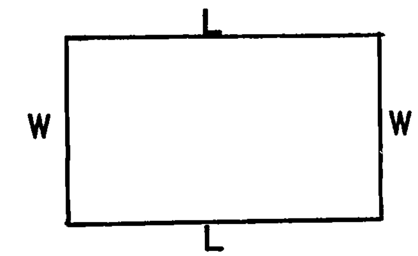
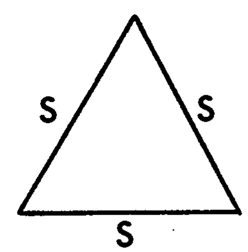
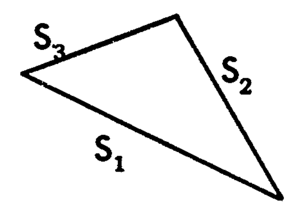
Strands And Topics	Content And Competencies To Be Developed	Suggested Background and Resource Material
K. Regions of a Plane	1. Pupil learns that: a) The intersection of two distinct half-planes is the interior of an angle. NOTE: "Distinct" is used here as synonymous with "different". b) Those points of a plane not in the interior of the angle nor in the exterior of the angle, are included in the angle.	1. Example: Two half-planes shown by use of horizontal and vertical lines  Fig. A  Fig. B  Fig. C By superimposing figure A onto figure B so that the boundary MN intersects boundary RS at point P, we see in figure C that SPN is the angle and that the cross-hatched portion of the plane is the interior of ∠ SPN NOTE: Making use of transparent overlays on an overhead projector is an excellent aid in developing this concept.
	2. Learning that the intersection of the interior of three distinct angles, or of three distinct half-planes, is the interior of a triangle.	2. Example: 

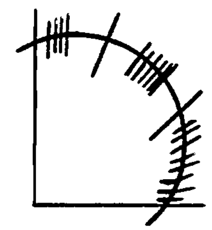
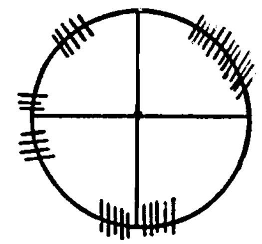
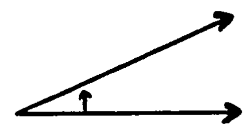
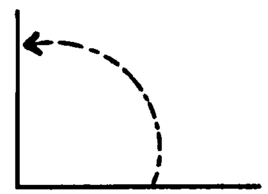
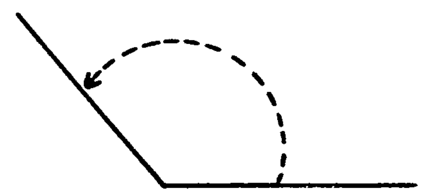
Strands And Topics	Content And Competencies To Be Developed	Suggested Background and Resource Material
K. Regions of a Plane (cont.)	3. Learning the concept that any geometric figure in a plane, which ends where it begins, and which separates the plane into only two regions, is a simple closed curve.	3. Examples: Some simple closed curves are:  These figures are not simple closed curves: 

Strands And Topics	Content And Competencies To Be Developed	Suggested Background And Resource Material
III. Measurement A. Introduction to Measurement	1. Some awareness of the centuries of "idea development" and use on which today's measurements are based	
	2. Learning the difference between: a) "Discrete set" as composed of members which may be counted separately and b) "Continuous set" as a set of elements which are all in one piece without any breaks.	2. a) A discrete set answers the question, "How many?" Arrange for periodic use of discrete sets such as finding the number of marbles in a bag, etc. b) Arrange experiences with "continuous sets" such as measurement of a length of rope, or the length of a football field.
	3. Pupil demonstrates by appropriate action a satisfactory comprehension of: a) Motion Property: A geometric figure may be moved without changing its size or shape.	3. a)  Fig. 1 Cut out Fig. 1, "flip over" into various positions, note "motion" of referent angle A.
	b) Comparison Property: Two continuous geometric figures or sets of the same kind, may be compared to determine if they have the same size or which one is larger.	b)  Fig. 2 Place Fig. 1 over Fig. 2 for comparison of size and shape
	c) Matching Property: If two continuous geometric figures or sets are both made up of parts such that every part of one can be matched with a part of the same size in the other, then the two continuous figures or sets have the same size.	c) Try "matching" a copy of Fig. 1 with Fig. 2
	d) Subdivision Property: A geometric continuous figure or set may be subdivided.	d)  ABGE is subdivided by $\overline{CF}$
	4. a) Discovery that measurement is approximate rather than being exact. b) Identifying "approximately equal to" symbol as $\approx$.	4. Measure lengths of objects by use of a stick so chosen that lengths are seen to be "so many sticks" and less than half or more than half of another stick-length. Report measure to "nearest stick-length".

Strands And Topics	Content And Competencies To Be Developed	Suggested Background And Resource Material
A. Introduction to Measurement (cont.)	5. Learning that: a) Measurements are reported to the nearest whole unit of measure. b) Greatest possible error (g.p.e.) is the greatest possible difference between the real length of a segment and the measurement that is stated. c) Because of the practice in a) above, greatest possible error is limited to $\frac{1}{2}$ of the measuring unit. d) Tolerance is a term used in science and industry in in referring to greatest possible error.	5. Examples: a) Three sticks and a little more are reported as "three sticks" Three sticks and a little more than half another stick are reported as "four sticks." b) and c) If we judge, by using an accurate ruler, that a length is $\frac{7}{8}''$ then we have named the length to the nearest $\frac{1}{8}''$ The real measure may be $\frac{1}{2}$ of $\frac{1}{8}''$ less than we named or $\frac{1}{2}$ of $\frac{1}{8}''$ more than we named. $\frac{1}{2} \times \frac{1}{8} = \frac{1''}{16}$ greatest possible error $\pm \frac{1''}{16}$ is the "tolerance" in this case.
B. Linear Measurement	1. Satisfactory comprehension of: a) Characteristics of a unit of measure as: 1) Unit of measure must be of same nature as object or quantity to be measured. 2) Unit of measure must be capable of being moved or copied. b) The meaning of linear measure as the measurement of line segments. c) The idea that any unit can be subdivided. d) A standard unit of measure as any unit agreed upon and used by a large number of people. e) The most commonly used units of measure in the English and metric systems.	1. Examples: a) The unit used in measuring a pupil's height can't be used for measuring his weight because of the difference in the nature of the two properties being measured. A foot ruler and a pound weight can be moved for use in measuring and can be reproduced quite simply. c) Let N be a unit of linear measure. Any unit of measurement can be subdivided into sub-units of measurement.  d) English Measure: 12 in. = 1 ft.; 3 ft. = 1 yd.; $16\frac{1}{2}$ ft. = 1 rd.; 320 rd. = 1 mi.; 5280 ft. = 1 mi. Metric Measure: 10 millimeters (mm) = 1 centimeter (cm); 100 cm = 1 meter (m); 1000 m = 1 Kilometer (km)

Strands And Topics	Content And Competencies To Be Developed	Suggested Background And Resource Material
B. Linear Measurement (cont.)	2. Development of an understanding of the basic relationship that for every point on the ruler there is a corresponding point on the number line.	
	3. Use of ruler in making both English and metric measurements.	
	4. Learning that congruent line segments are segments of the same length.	
	5. Development of the knowledge that precision in linear measurement: a) Depends on the size of the smallest subdivision used in taking the measure. b) Increases as size of measuring unit decreases.	5. a) Practice measuring common objects (desk tops, books, etc.) for varying precision as "to fourths or sixteenths of an inch" and/or to "centimeters or millimeters". b) NOTE: Here is an important practical application of "inverse or opposite variation".
	6. Pupil demonstrates understanding that: a) Greatest possible error in measurement of line segments may be indicated as an average measure $\pm \frac{1}{2}$ of the measuring unit. b) Real length lies between the extremes of the reported measure plus or minus g.p.e.	6. Example: Given: measure of line segment is $2\frac{3}{8}''$ The measurement together with the greatest possible error is indicated as: (m) $2\frac{3}{8}'' \pm \frac{1}{16}''$ $\therefore$ Real length, or true measure, lies between $2\frac{5}{16}''$ and $2\frac{7}{16}''$

Strands And Topics	Content And Competencies To Be Developed	Suggested Background And Resource Material
C. Perimeter	1. The study of: a) Perimeter as the total length of a closed curve. b) How to measure the perimeter of polygons such as triangles and quadrilaterals. 2. Learning to develop appropriate mathematical sentences and to demonstrate their use in determining the measure of perimeter for: a) Rectangle: $P = 2L + 2W$ or $P = 2(L + W)$ b) Square: $P = 4S$ c) Triangles: Equilateral: $P = 3S$ Scalene: $P = S_1 + S_2 + S_3$	1. Example of perimeter: The distance an ant would walk if it started at one corner of a rectangle and kept walking along the side till it returned to its starting point. 2. Applying the above "ant explanation" to the following pictures, lead children to develop for themselves the correct arithmetic sentence to fit each picture.  $S + S + S + S = P$ or $P = 4S$, $4S = P$  $L + W + L + W = P$ or $2L + 2W = P$, $P = 2(L + W)$  (equilateral) $S + S + S = P$ or $3S = P$, $P = 3S$  (Scalene) $S_1 + S_2 + S_3 = P$ or $P = S_1 + S_2 + S_3$ NOTE: Help pupils discover why the sentence describing the perimeter of a square cannot be sensibly applied to the triangle, etc.

Strands And Topics	Content And Competencies To Be Developed	Suggested Background And Resource Material
D. Angles	1. a) Pupil studies the concept of the "degree" as a standard unit of measure for an angle. b) Pupil uses the protractor in the measurement of angles.	1. a) The degree is $\frac{1}{90}$ of the measure of a right angle.  The degree is $\frac{1}{360}$ of the measure of the circumference of the circle. 
	2. Knowledge that the: a) Right angle is an angle of 90°. b) Acute angle is an angle whose measure is less than 90°. c) Obtuse angle is an angle whose measure is more than 90°.	2.  Measure of acute angle < 90°  Measure of right angle = 90°  Measure of obtuse angle > 90°

Strands And Topics	Content And Competencies To Be Developed	Suggested Background And Resource Material
D. Angles (cont.)	3. Pupil demonstrates comprehension of: a) The comcept of 90° as the measure of any angle formed by rays or line segments which are perpendicular to each other. b) Perpendicular lines as lines that form a right angle. c) Coplanar lines as lines that are included in the same plane. d) Transversal as a line that intersects two or more coplanar lines in distinct points.	3. a) and b) Examples of perpendicularity:

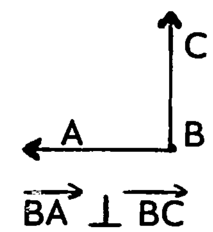


Fig. 1

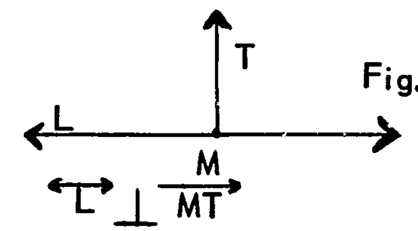


Fig. 2

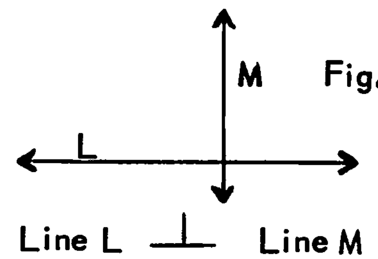


Fig. 3

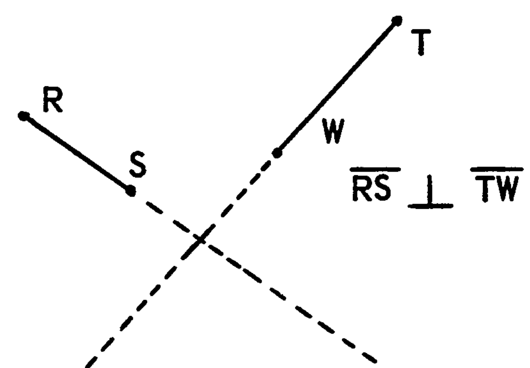


Fig. 4

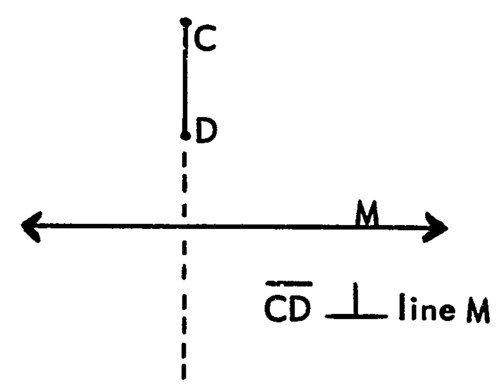
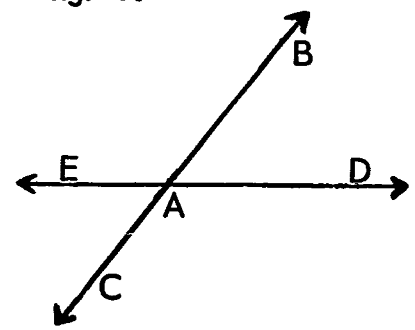
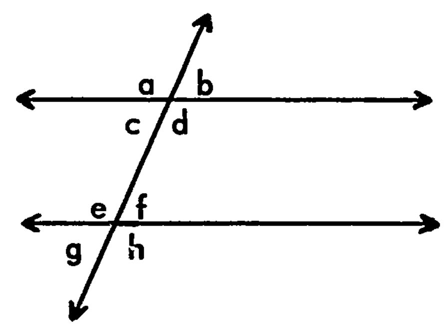
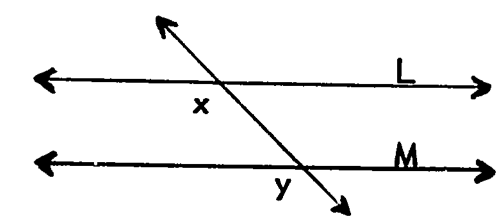
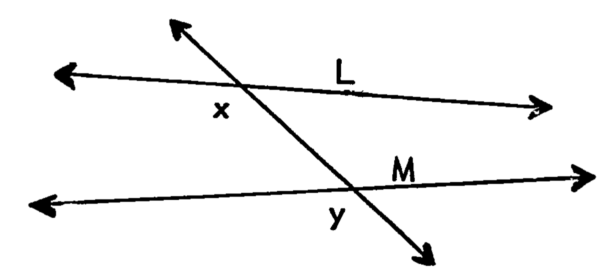
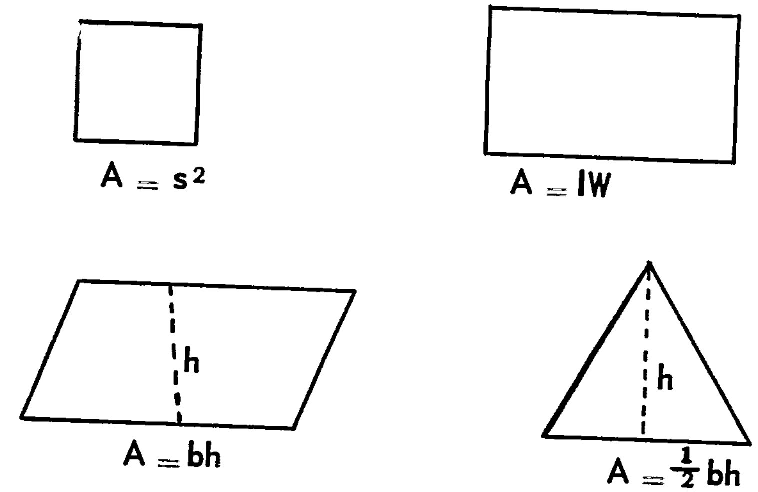
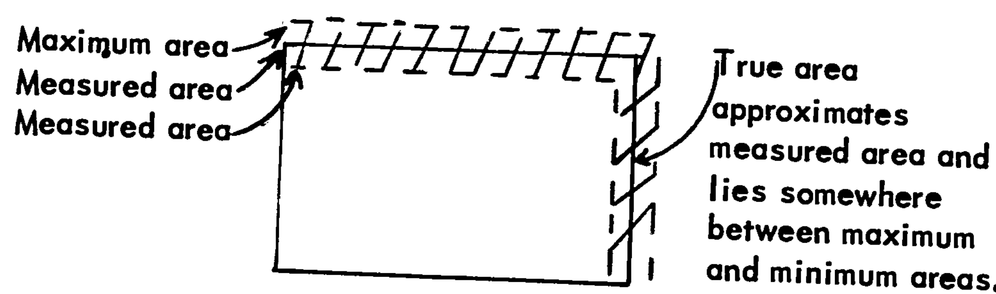
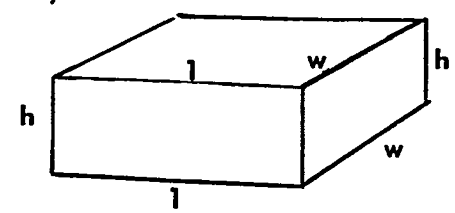
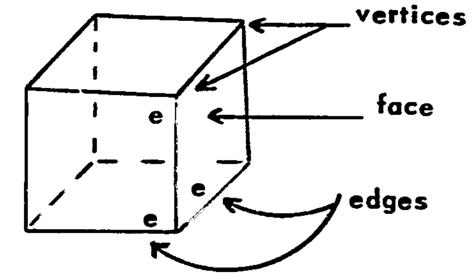
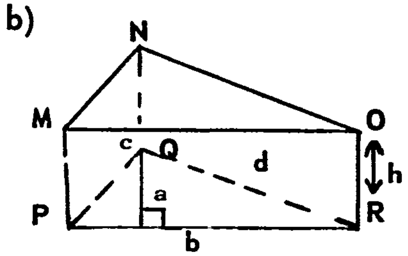


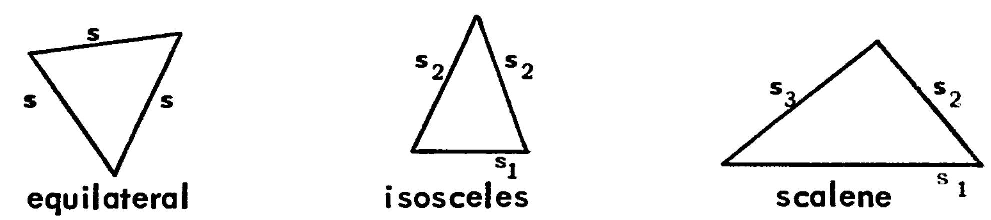
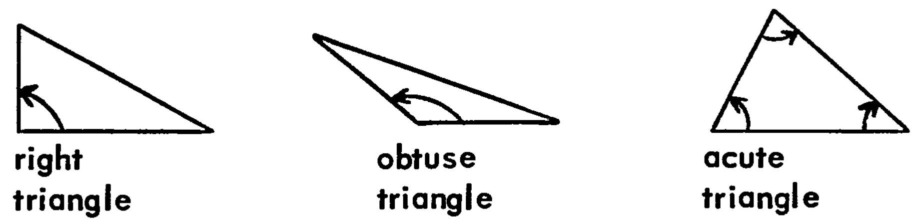
Fig. 5

Strands And Topics	Content And Competencies To Be Developed	Suggested Background And Resource Material
D. Angles (cont.)	e) The relationship and measurement of specific angles: 1) Adjacent angles are any two angles which have a common ray, a common vertex, and whose interiors have no point in common. 2) Vertical (or opposite) angles are two non-adjacent angles formed when two lines intersect. 3) Complementary angles are two angles such that the sum of their measures is 90°. 4) Supplementary angles are two angles such that the sum of their measures in degrees is 180 degrees. 5) Corresponding angles are two angles formed by two coplanar lines and a transversal in such a way that the interior of both angles are subsets of the same half-plane bounded by the transversal, and a side of one angle is a proper subset of a side of the other angle.	e) Examples of angles:  ∠ BAE and ∠ BAD are adjacent angles. ∠ EAC and ∠ BAD are vertical angles. ∠ EAC and ∠ DAC are supplementary angles. Of course ∠ EAC and ∠ DAC are also adjacent!  ∠ b and ∠ f are corresponding angles. ∠ d and ∠ h are corresponding angles.

Strands And Topics	Content And Competencies To Be Developed	Suggested Background And Resource Material
D. Angles (cont.)	4. Pupil illustrates with sketches the basic reason for corresponding angles having equal or unequal measures as: a) When in the same plane a transversal intersects two lines and the corresponding angles have equal measures, then the two lines do not intersect and are parallel. b) When in the same plane a transversal intersects two lines and the corresponding angles have unequal measures, then the two lines will intersect.	4. Examples:  ∠ x and ∠ y are equal, corresponding angles. Line L ‖ line M Read "parallel to".  ∠ x and ∠y are unequal, corresponding angles. Line L is not parallel to line M.
	5. Pupil uses angle measure notation as: a) M means measure b) m (∠ABD) is read, "The measure of angle ABD"	
	6. Pupil demonstrates how to measure angles of simple geometric figures such as rectangles, triangles, and parallelograms.	
E. Area	1. Learning that: a) The term "area" refers to the number of square units measured in the closed region of a simple closed curve. b) The square unit is a convenient measure for use in covering regularly-shaped surfaces without waste space.	

Strands And Topics	Content And Competencies To Be Developed	Suggested Background And Resource Material
E. <u>Area</u> (cont.)	2. Demonstration of pupil ability to compute area of regular quadrilaterals and triangles, making use of: a) Standard units of measure, English <u>and</u> metric English-sq. in., sq. ft., sq. yd., sq. mi. metric-sq. mm., sq. cm., sq. meter b) The mathematical sentences: $A = lw$ (rectangle) $A = s^2$ (square) $A = bh$ (parallelogram) $A = \frac{1}{2}bh$ (triangle)	2. 
	3. Pupil notices that idea of greatest possible error can be applied to area of rectangle.	3. Greatest possible error (g.p.e.) may be defined as the difference between calculated maximum or minimum area and calculated measured area.
	4. Acquiring basic understanding of meanings: a) Calculated or measured area of a regular quadrilateral is the measured length times the measured width. b) True area lies between that of the smallest area and the largest area of a simple closed curve. c) Maximum area is the largest area and minimum area is the smallest area.	4. 
F. <u>Surface Area</u>	1. Extending the understanding that surface area means the sum of the area of all of the faces.	
	2. Pupil demonstrates understanding of <u>parts of space figures</u>: a) Face is a flat side. b) Edge is a line segment which is the intersection of two faces. c) Vertex of a space figure is the intersection of three or more faces at a point.	2. Suggestion: Repeated and continuing use of models and sketches is essential to demonstrations of real understanding. NOTE: Right rectangular prisms have 6 faces, 12 edges, 8 vertices.

Strands And Topics	Content And Competencies To Be Developed	Suggested Background And Resource Material
F. Surface Area (cont.)	3. Use of mathematical sentences to find the surface area of rectangular and triangular right prisms: a) rectangular: $A = 2\ (lw + wh + lh)$ b) triangular: $A = ab + h\ (b + c + d)$	3. a)  $A = 2\ (lw + wh + lh)$  $A = 6e^2$ b)  The area of Δ MNO + area of Δ PQR is the area, ab, of a parallelogram. The areas hb, hc, and hd are the areas of the three side faces of the prism. The total surface area of the prism is represented by: $A = ab + hb + hc + hd$ $\therefore A = ab + h\ (b + c + d)$
G. Volume	1. Study of the cube as a standard unit for measuring volume because it "fills up" 1 unit of space. 2. Pupil demonstrates ability to compute volume of right prisms making use of: a) Standard units of measure, English and metric English: cu. in., cu. ft., cu. yd. metric: cu. mm., cu. cm., cu. meter	2. Examples: See figures in No. 3 above. See figures in Grade 8, Strand III, Section G.

Strands And Topics	Content And Competencies To Be Developed	Suggested Background And Resource Material
G. Volume (cont.)	b) The mathematical sentences: 1) $V = e^3$ (cube) 2) $V = lwh$ (rectangular right prism) 3) $V = \frac{1}{2}abh$ (triangular right prism)	
	3. Pupil recognizes that: a) The idea of greatest possible error can be applied to volumes of solids. b) "True volume" lies between maximum and minimum volume.	3. a) Greatest possible error (g.p.e.) in volume may be defined as the difference between calculated maximum or minimum volume and calculated measured volume.
H. Triangles	1. Learning the definition of triangle as "the union of three segments determined by three non-colinear points".	
	2. Learning the two structural classifications of triangles to be: a) According to measure of sides: 1) Equilateral triangle has three equal sides. 2) Isosceles triangle has two equal sides. 3) Scalene triangle has no two sides equal in measure.	2. Examples: a) Triangles named according to the measure of their sides 
	b) According to measure of angles: 1) Right triangle contains one angle with a measure of 90°. 2) Obtuse triangle contains one angle with a measure of more than 90°. 3) Acute triangle has three angles, each with a measure of less than 90°.	b) Triangles named according to the measure of their angles 

Strands And Topics	Content And Competencies To Be Developed	Suggested Background And Resource Material
H. Triangles (cont.)	3. Recognition of the basic relationships of sides and angles of of triangles as: a) Longest side is opposite largest angle. Shortest side is opposite smallest angle. b) If two sides are equal in length, then the angles opposite those side have equal measure. c) If two angles are equal in measure, then the sides opposite these angles will be equal in length.	
	4. Discovery by experimentation that the sum of the degree measure of the angles of a triangle is 180 degrees.	4. NOTE: This "discovery" results from the investment of sufficient pupil time to allow for measuring the angles of many differently-shaped triangles and finding the sum of those measures.
	5. Pupil demonstrates ability to find measures of area and perimeter of triangles while making use of: a) Standard units of measure, English and metric b) The mathematical sentences: equilateral: ⟶ $A = \frac{1}{2}bh$, $P = 3S$ isosceles: ⟶ $A = \frac{1}{2}bh$, $P = s_1 + s_2 + s_3$ scalene: ⟶ $A = \frac{1}{2}bh$, $P = s_1 + s_2 + s_3$ c) Same math sentence for area of all triangles regardless of shape.	
I. Other Measures	1. Pupil demonstrates reasonable computational proficiency with commonly-used units of measure and their subdivisions. NOTE: Subdividions are not limited to geometric figures.	1. Add: a) 3 hr. 18 min. + 6 hr. 52 min. = 10 hr. 10 min. Subtract: b) 6 yd. 1 ft. 4 in. − 2 yd. 2 ft. 9 in. = 3 yd. 1 ft. 7 in. Multiply: c) 1 gal. 3 qt. 1 pt. x 2 = 3 gal. 3 qt. 0 pt. Divide: d) 2) 5 wks. 3 days = 2 wks. 5 days

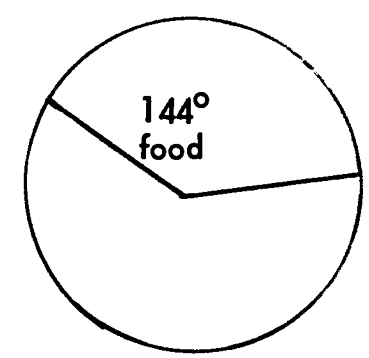
Strand IV, Business Arithmetic - - - - Grade 7

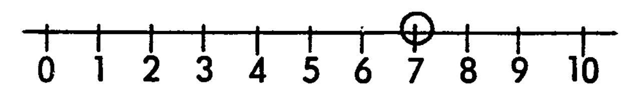
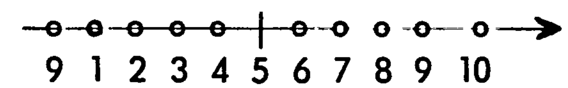
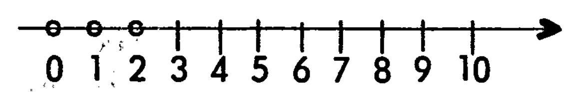
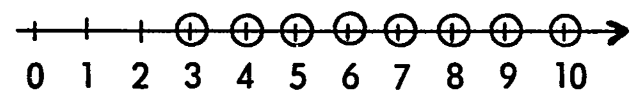
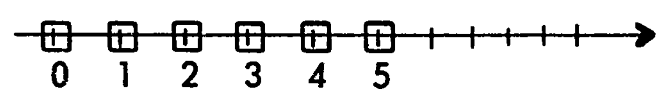
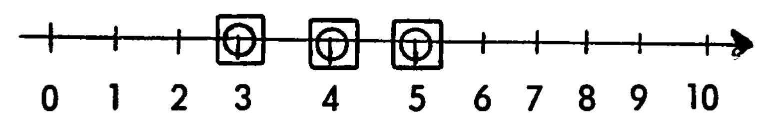
Strands And Topics	Content And Competencies To Be Developed	Suggested Background And Resource Material
IV. Business Arithmetic A. Per Cent in Business	RECOMMENDED: Intuitive development of meanings of business arithmetic terms. OPPOSED TO: Requiring mastery of such terms. RECOMMENDED: Use of business terms as tools for problem solving. NOTE: See Strand 5 for development of concept of per cent as a ratio expressing a given number per 100.	
1. Interest	1. Pupil accepts the meaning of simple interest as rent paid for the use of borrowed money, etc.	1. Common - 4%, 5½ % or 6%
	2. Pupil solves problems involving simple interest.	2. Mrs. Smith borrows $7,500. The bank charges 6% interest. How much interest will she pay in one year? $\frac{x}{7,500} \sim \frac{6}{100}$
	3. Recognition of applications of simple interest in life situations.	
2. Discounts	1. Pupil accepts the meaning of discount as a reduction from original price.	1. La Chez dress shop advertises all dress 33 $\frac{1}{3}$ % off. If a dinner dress originally cost $30, what is its sale price? $\frac{x}{30} \sim \frac{2}{3}$
	2. Reasonable practice in the solution of problems involving simple discount.	
	3. Application of the idea of discount to practical problems.	3. Use daily newspaper advertisements.
3. Profit and Loss	1. Development, intuitively, of meanings of "profit" and "loss."	
	2. Pupil accepts simple general meanings of the terms "gross income" and "net profit."	
	3. Pupil can compute: a) Profits in dollars and as % increase on cost. b) Loss in dollars and as % decrease from original.	

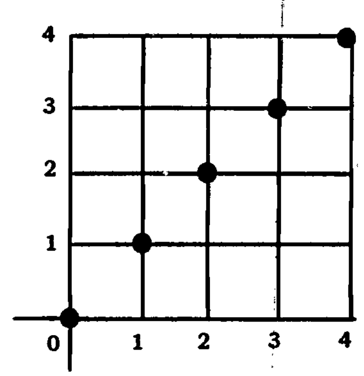
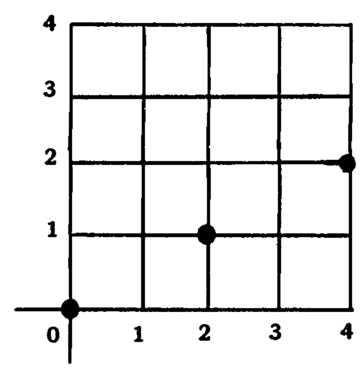
Strands And Topics	Content And Competencies To Be Developed	Suggested Background And Resource Material
B. Banking 1. Types of Banks 2. Functions of Banks	1. Class explores the basic differences between multiple-purpose and commercial banks.	1. This is an excellent point at which to bring in qualified and appropriate resource people. A carefully-prepared-for field trip is recommended.
	1. Learning about the "clearing-house" service of banks to depositors and demonstrating ability to: a) Write and properly endorse checks. b) Properly make out deposit and withdrawal slips.	
	2. Class considers the values of bank savings accounts and demonstrates ability to compute simple interest on deposits.	
	3. Learning that bank safety deposit boxes a) Are valuable for certain uses. b) Are governed by specific restrictions.	
	4. Learning something of a bank's service through the lending of money. Explores meaning of : a) Long term vs short term loans. b) Secured vs personal loans.	4. Study briefly the practical family needs and common business needs for short-term and long-term loans and compute the comparative costs of both types.
	5. Pupil demonstrates reasonable proficiency in using percent to compute the "cost" of a loan.	
C. Investments	1. Class discussion of need for sound investments as opposed to "idle" money.	
	2. Class informally evaluates the advantages, disadvantages and suitability of simple types of investments as: a) Bank savings accounts b) Insurance c) Credit unions d) Government savings stamps and bonds e) "Shark" loans	2. With an appropriate resource person addressing them, class considers the most favorable way of providing money for: a) An unforeseen expense as the complete failure of the family refreigerator. b) Hospital bills and living expenses during father's illness. c) Serious injury to the family car. d) Legal action against the family as owner of a car involved in collision. e) Securing money to purchase a new car. f) Providing for distant goal: college for 6-year-old, or trip to Europe <u>after</u> college as planned for an early teen-ager.

Strand V, Ratios, Relations, And Graphs - - - - Grade 7

<table>
<tr><th>Strands And Topics</th><th>Content And Competencies To Be Developed</th><th>Suggested Background And Resource Material</th></tr>
<tr><td>V. Ratios, Relations, Graphs
A. Graphs</td><td>1. The simple procedures of gathering data and tabulating information
2. The making and labeling of suitable vertical and horizontal scales
3. The assigning of informative titles to graphs
4. Pupil applies rounding off of tabulated figures.</td><td>1. Suggestion: Investigate relation between circumference and diameter of circles of different sizes. Tabulate information so comparisons may be studied.
<table>
<tr><th>size of diameter</th><th>size of circumference</th><th>comparison of circumference to diameter</th></tr>
<tr><td>3"</td><td>9 +"</td><td>3^{+} times as large</td></tr>
<tr><td>$4\frac{1}{2}$"</td><td>14 +"</td><td>3^{+} times as large</td></tr>
<tr><td>____</td><td>____</td><td>____</td></tr>
</table></td></tr>
<tr><td>1. Broken-line Graphs</td><td>1. Learning to represent tabulated data by line graphs</td><td></td></tr>
<tr><td>2. Bar Graphs</td><td>1. Learning to represent tabulated data by both vertical and horizontal bar graphs</td><td>1. Examples:

Two ways to represent same data</td></tr>
<tr><td>3. Circle Graphs</td><td>1. Pupil measures and draws the central angles of circle graphs in relationship to the quantities indicated in tabulated information.</td><td>1. Example:
40 % of budget for food means
40 % of 360° for food.
The measure of the central angle will be 144°.

</td></tr>
</table>

Strands And Topics	Content And Competencies To Be Developed	Suggested Background And Resource Material
A. Graphs (cont.) 4. Picture Graphs	1. Pupil interprets information given in picture graphs.	1. Suggestion: Select simple, clear diagrams from newspapers and magazines.
5. Number Line	1. Graphing the truth set of equations, inequations and inequalities on number line when the universe is all natural numbers. NOTE: In this guide 0 is considered to be a whole number (natural number). N = {0, 1, 2, . . .}	1. NOTE: Truth set is the solution set of a condition (or equation). Examples: X + 4 = 11 (equation)  N ≠ 5 (inequation)  N < 3 (inequality) 
	2. Pupil demonstrates reasonable skill in graphing compound conditions using natural numbers.	2. Given: x + 3 > 5 and 4 + x < 10 x + 3 > 5  U = natural numbers (with 0) 4 + x 10   Solution set = {3, 4, 5}

Strands And Topics	Content And Competencies To Be Developed	Suggested Background And Resource Material
A. Graphs (cont.) 6. Coordinate Plane Graphs	1. Learning the meaning and use of ordered pairs of natural numbers.	1. (1, 2) is an ordered pair which, by mathematical agreement, may represent one space to the right and two spaces up from some reference point on a coordinate plane.
	2. Development of some idea of the usefulness of coordinate diagrams or graphs.	2. NOTE: The terms diagram, picture, graph, and illustration are interchangeable.
	3. Pupil demonstrates reasonable grasp of the processes of: a) Graphing Cartesian products using natural numbers (counting numbers and zero) b) Given: Set A and Set B Find A X B (Read: A cross B) and locate corresponding points on coordinate graph.	3. See Strand V Appendix.
	4. Given a universe of ordered pairs, graphing solution sets for problems of the types: a) $x = y$ b) $2y = x$	4. Given: $A = \{0, 1, 2, 3, 4\} = B$ and universe of ordered pairs is A X B a) $x = y$ Ordered pairs: (0, 0), (1, 1), (2, 2), (3, 3), (4, 4)  b) $2y = x$ Ordered pairs: (0, 0), (2, 1) (4, 2) 
	5. Graphing such inequalities as $x < y$, $x > y$, and graphing inequations as $x + y \neq 6$	

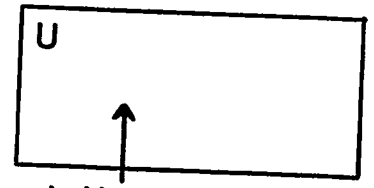
Strands And Topics	Content And Competencies To Be Developed	Suggested Background And Resource Material
B. Relations 1. Ratio	1. Setting up and solving problems by use of ratio as $\frac{1}{2}$, 1:2, (1, 2), 1/2	1. NOTE: A relation can be described as "a set of ordered pairs." Problems involving ratio and percent can be solved using ordered pair notation. A ratio such as 1 : 2 can be thought of as any element of the set of ordered pairs, $\{(1, 2), (2, 4), (3, 6) \ldots\}$. In this sence, a ratio is a relation. Similarly, 80 % can be thought of as any element of the set of ordered pairs. $\{(4, 5), (8, 10), (12, 15) \ldots\}$.
	2. Learning how to determine whether two ratios are equal or equivalent and how to solve for any missing part of a proportion.	2. Examples: $4 : 3 = 8 : 6$ (times 2, times 2) $2 : 5 = \square : 15$ (times 3) $\therefore 2 \times 3 = \square$
	3. Developing recognition of the relationship of decimal and common fractions and rational numbers, as expressed by their "ratio form." 4. Solving "rate pair" problems by use of proportions.	3. Examples: a) $\frac{3}{4}$ means $\frac{30}{40}$ or $\frac{300}{400}$ .75 means $\frac{75}{100}$ or $\frac{300}{400}$ $\therefore$ $\frac{3}{4}$ and .75 name the same quantity. b) The fraction $\frac{12}{6}$ names the same quantity as rational no., $\frac{2}{1}$ $\therefore \frac{12}{6} = \frac{2}{1}$ Ratio forms of given fraction and rational number are equal.
2. Percents	1. Development of concepts a) Symbol "%" means "per 100." b) Any percent means ratio whose second term is 100.	1. Examples: 18% means $\frac{18}{100}$ $9\frac{1}{4}$ % means $\frac{\left(\frac{37}{4}\right)}{100}$ or $\frac{37}{400}$
	2. Considerable practice in expressing and solving by use of ratios; the three customary conditions of simple percent problems. 3. Developing recognition of the relationship between the "part of" quantity and the "all" quantity in percent problems.	2. See Strand V Appendix.

Strands And Topics	Content And Competencies To Be Developed	Suggested Background And Resource Material
VI. Sets A. Meaning of Set	1. Developing the meaning of a set as a definite (well-defined) collection of objects or ideas	1. Examples: A collection of stamps, that flock of birds, his herd of cattle, the junior class, may be designated as sets.
B. Elements (members) of Sets	1. Learning that members or elements of a given set are those items which belong to the set.	1. Example: If we speak of "the set of boys in this class," then each boy who is a member of this class is an element of or a member of the set to which we referred.
C. Set Symbolization	1. Introduction of the sign language of sets with attention to its usefulness, as: a) Braces, { }, are used to denote membership in a set. b) Capital letters are a helpful way of designating sets.	1. Given: The set of odd counting numbers from 1 through 9 a) If braces are used to group the members of the set, the notation becomes: {1, 3, 5, 7, 9 } b) N = { 1, 3, 5, 7, 9}
	2. Learning ways of indicating elements or members of a set by: a) Description b) Tabulation	2. Given: The set of numbers used in counting from 2 through 7 a) The given statement is a description of a set of numbers. The description might be written: N = {The numbers used in counting from 2 through 7} b) Tabulating a set: N = {2, 3, 4, 5, 6, 7 } Tabulation may be spoken or written but it means listing the names of members. Suggestion: Provide for pupil-practice in changing the tabulation of a set into the description of the same set.
D. Set Comparison	1. Learning ways to compare sets: a) Definition: Two sets are equal if the sets have exactly the same members	1. a) The sets P = {a, b, c } and Q = {c, a, b } are equal because they have the same members. (Tabulating order makes no difference.) ∴ P = Q By definition, any two sets with same members are equal.

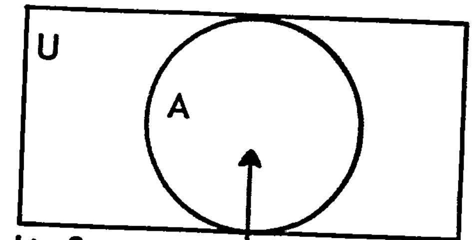
Strands And Topics	Content And Competencies To Be Developed	Suggested Background And Resource Material
D. Set Comparison (cont.)	b) Definition: Two sets are equivalent if their members can be put in one-to-one correspondence.	b) Example: Set A = { Tom, Bill, Joe } ↕ ↕ ↕ Set B = { bicycle, ball, skates } ∴ A ⟷ B or A ~ B Set M = c, d, e, f, g ↕ ↕ ↕ ↕ ↕ Set N = x, y, z, w, v ∴ M ⟷ N or M ~ N One-to-one correspondence between equivalent sets.
	2. Use of counting to determine set equivalence.	1. Suggestion: Counting the members of both sets is a quick way of determining the equivalence of two sets, each having few members. Given: A = {x, y, z} The number of members of set A is written n (A) = 3 Hence, if n(A) = 3 and n(B) = 3, A is equivalent to B or B is equivalent to A.
E. Kinds of Sets	1. Learning to distinguish between types of sets as: a) Definition: A finite set is a set for which the members can be counted. b) Definition: An infinit set is a set for which the members cannot be counted.	1. a) and b) Hint: If a set is not finite, then it is infinite. Example: A = {1, 2, 3 ··· 1000} (Dots indicate omitted members) Finite set because members can be counted. B = {1, 2, 3 ···} Infinite set because members continue endlessly.
	c) Definition: The empty (null) set is a set that has no members. NOTE: Authors use both { } and ϕ to represent the empty set.	c) Example of empty set: The set of even numbers that ate divisors of 13. Suggestion: In grade 7, it is urged that the empty set be usually denoted by { }.

Strands And Topics	Content And Competencies To Be Developed	Suggested Background And Resource Material
F. Subsets	1. Learning that sets within sets are defined as subsets: a) Definition: If each member of one set is also a member of a second set, then the first set is a subset of the second set. b) Definition: A proper subset is a subset such that the subset contains some but not all members of the original set. c) Definition: Improper subsets of any set are the empty set and the given set.	1. a) Ex: A set of cattle is a subset of the set of all animals. Ex: Set K = {a, b, c, d, e} If we consider the vowles a and e, we say that a and e are members of a subset of set K. ∴ { a, e } is a subset of K. b) In example a) above, the set whose members are a and e is proper subset of set K. c) Set K = {a, b, c, d, e} Improper subsets are { } and { a, b, c, d, e} NOTE: The original set and the empty set, in all instances, are considered to be subsets of the original set.
G. Universe or Domain of Sets	1. Developing the meaning of universe or domain as: Definition: Universe is the set of objects from which answers, or members of the solution set, must be chosen. 2. Learning to designate a stated universe as: U = set of natural numbers less than 20 or U = {0, 1, 2, 3••• 19 }	1. Hint: Numbers, symbols, and ideas are among the "objects" which may be included in the "universe," designated for various problems. 2. Hint: "Universal set," "universe," and "domain" are three different ways of referring to the set of objects from which answers, or members of the solution set must be chosen.

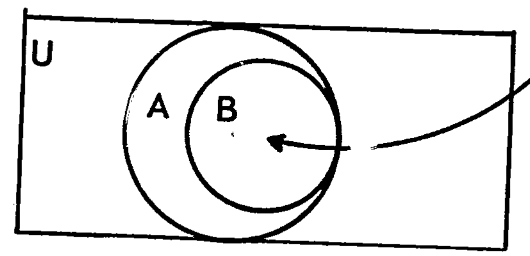
Strands And Topics	Content And Competencies To Be Developed	Suggested Background And Resource Material
H. Set Diagrams	1. Exploration of the Venn diagram technique for showing meaning of: a) Universe; b) subset; c) disjoint sets; d) intersecting sets; e) identical or equal sets	1. Suggestions: Diagrams used to picture relations between sets are called Venn diagrams. Mathematicians us·ally use a rectangle to represent a Universal set and various patterns of circles to represent the subsets within the Universe.



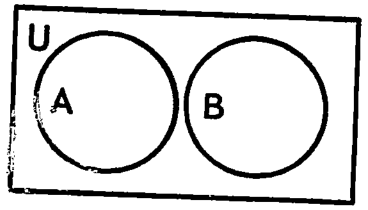
a) Universe = all people in N. M.



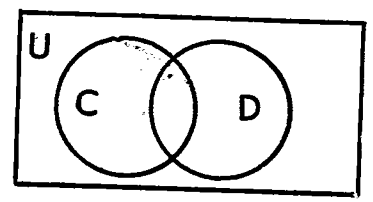
b) Set A = all men in N. M.



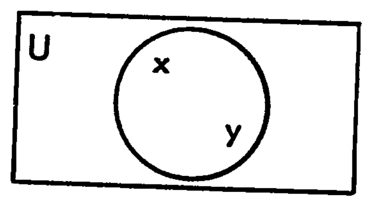
c) Set B = all men in N. M. more than 30 years old. All members of Set B are also members of Set A



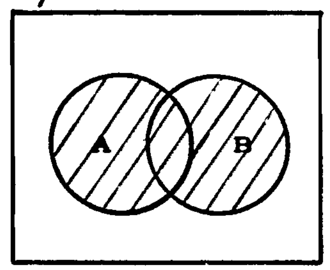
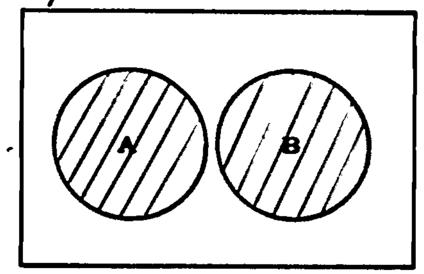
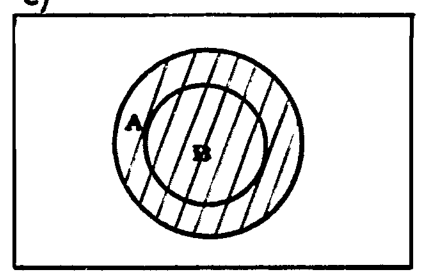
d) A = set of boys in class
B = set of girls in class
Here A doesn't intersect B.
The 2 sets have no members in common.
They are disjoint sets.

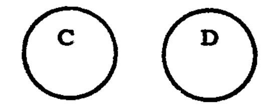
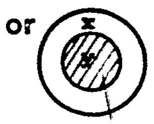


e) C = set of boys in grade 7
D = set of boys in JHS math club
Set C intersects Set D.
Some members of C are also members of D.

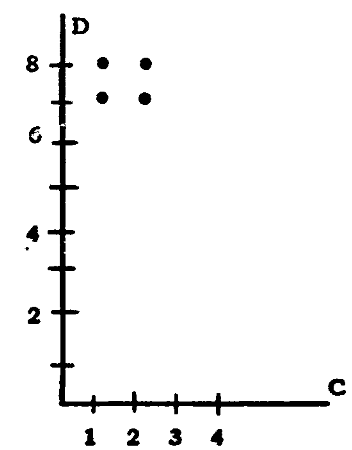


f) Identical or equal sets
x = set of even natural numbers less than 20.
y = set of natural numbers less than 20 divisible by 2.

Strands And Topics	Content And Competencies To Be Developed	Suggested Background And Resource Material
I. Set Operations	1. Learning the meaning of the union operation of two sets as being: a) All members of either of 2 sets, or of both of the sets if the given sets intersect. b) All members which belong to either of the sets, if the given sets are disjoint. c) All members of the larger set if one set is a proper subset of the other set.	1. NOTE: The symbol U represents the union operation of sets. C U B is read, "The union of Set C and Set B. a)  A U B for intersecting sets b)  A U B for disjoint sets c)  A U B when Set B is a subset of Set A a) If A = {1,2,3} and B = {3,4,5} then A U B = {1,2,3,4,5} b) If A = {1,2,3} and B = {4,5,6} then A U B = {1,2,3,4,5,6} c) If A = {1,2,3,4} and B = {1,2} then A U B = {1,2,3,4}

Strands And Topics	Content And Competencies To Be Developed	Suggested Background And Resource Material
I. Set Operations (cont.)	2. Learning the meaning of the intersection operation for two sets as the finding of a set which includes only those members common to both sets. c) Intersection set is composed of only those members common to both sets, if the original sets intersect. b) Intersection set is an empty set if original sets are disjoint. c) Intersection set is composed of all members of smaller set if one set is a proper subset of the other set.	2. NOTE: The symbol $\cap$ represents the intersection operation of sets. $A \cap D$ is read, "The intersection of Set A and Set D." a) If A = {1,2,3} and B = {3,4,5} then $A \cap B$ = {3} or  b) If C = {1,2,3} and D = {4,5,6} then $C \cap D$ = { } or  c) If x = {1,2,3,4} and y = { 2} then $x \cap y$ = {1.2} or  Examples: See Strand I 1) To find G.C.F. of 9 and 15. 9 = 3 x 3 and 15 = 3 x 5 Intersection of factors of 9 and factors of 15 = {3} or G.C.F. 2) To find L.C.M. of 9 and 15. Multiples of 9 = {9,18,27,36,45 · · ·} Multiples of 15 = {15,30,45 · · ·} Intersection of multiples of 9 and multiples of 15 = {45} or L.C.M.
J. Sets In Space	1. Beginning recognition of geometry as sets of points in space based on: a) A "point" as a mathematical idea. b) A line as a set of points. c) A geometric figure as set of points.	1. Suggestions: See Strand II. The "points" idea is worthy of emphasis through all phases of geometry.

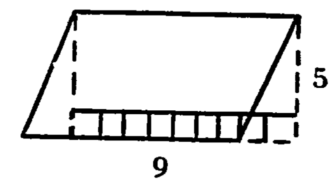
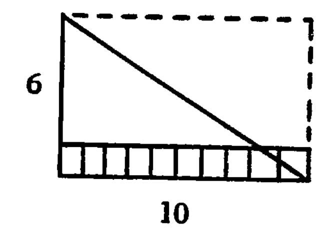
Strands And Topics	Content And Competencies To Be Developed	Suggested Background And Resource Material
K. Sets And Sentences	1. Pupil demonstrates reasonable facility in using: a) Placeholder as a symbol that holds the place for numerals or the names of things being talked about.	1. Examples: a) Placeholders: $\square + 7 = 10$, $x + 9 = 16$. $\square$ and x are placeholders for some numerals
	b) A variable as a single term used in referring to quantities of various sizes.	b) Variable: $U = \{0, 1, 2, 3 \cdots\}$ If $x > 7$, then $x = \{8, 9, 10 \cdots\}$ The variable x is of various sizes.
	c) Universe, replacement set, or domain as the set of objects from which answers must be chosen.	c) The given set of objects from which answers must be chosen must be specified for a problem. Sets of numbers are used very frequently.
	d) Solution set as the set consisting of all members of the universe that make an open sentence into a true statement.	d) See b) above. $x = \{8, 9, 10 \cdots\}$ is solution set because any of these values make a true statement of the open sentence, $x > 7$.
	e) A condition as the requirement expressed by an open sentence.	e) See b) above. $x > 7$ is an open sentence. To make it a true statement, the variable x must be replaced by numerals which represent natural numbers greater than 7.
	f) An equation is a true statement that includes the idea of equality.	f) $7 = 5 + 2$ is an equation. $7 = 5 \div 3$ is <u>not</u> an equation because it is not a <u>true</u> statement of equality.
	g) An inequality (or inequation) is a true statement that includes the idea of "greater than," "less than," or "not equal to."	g) $9 > 6$; $6 < 9$; $6 \neq 9$ are inequalities. $7 \neq 7$ is <u>not</u> an inequality because it is not a <u>true</u> statement of inequality.
	h) A "standard description" as braces and an open mathematical sentence used to describe a solution set.	h) $\{y \mid y > 7\}$ is a "standard description" and is read, "The set whose members are all y that satisfy the condition that y is greater than 7."

Strands And Topics	Content And Competencies To Be Developed	Suggested Background And Resource Material
L. Sets of Ordered Pairs	1. Learning the meaning of an ordered pair as being a pair of objects in which the objects appear in a specific order. See Grade 7, Strand V.	1. (1,2) is an ordered pair which, by mathematical agreement, may represent 1 space to right and 2 spaces up from some reference point. NOTE: The first and second components of an ordered pair are the objects which are listed in first and second places, respectively, in the pair.
	2. Pupil demonstrates reasonable facility in the use of ordered pairs in mathematical sentences with two variables. See Grade 7, Strand VII.	2. Given: $U = \{0,1,2 \cdots\}$ To find $\{(x,y) \mid 2x + y = 7\}$ ← Here we are to solve for the ordered pair, (x,y) Solution set $= \{(0,7), (2,3), (3,1)\}$
	3. Development of the meaning of: a) A set of ordered pairs as being the set formed by matching each object in a given set with each object in a second given set.	3. a) Given: A = {Mike, Joe} and B = {skates, bat} In this case the set of ordered pairs is {(Mike, skates), (Mike, bat), (Joe, skates), (Joe, bat)}
	b) Cartesian set, such as C X D, as being the set of all ordered pairs that can be formed by matching, in turn, each member of set C as the first component of the pairs, with each member of set D as the second component of the pairs.	b) If $C = \{1,2\}$ and $D = \{7,8\}$, then the Cartesian set C X D (read "C cross D") is $\{(1,7), (1,8), (2,7), (2,8)\}$
	c) The Cartesian plane as being the plane which contains all points determined by the ordered pairs of a Cartesian set.	c) The set above in b) could be represented as: 

Strand VII, Mathematical Sentences: Equations and Inequations or Conditions of Equality and Inequality - - - - Grade 7

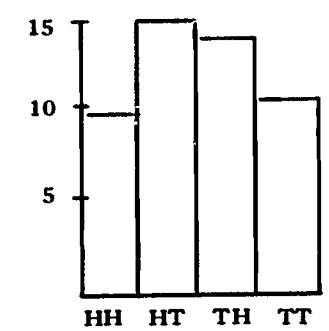
Strands And Topics	Content And Competencies To Be Developed	Suggested Background And Resource Material
VII. Mathematical Sentences A. Basic Meanings	1. Learning to write mathematical sentences which are open or closed, true or false, based on the definitions: a) Open sentences express requirements about objects or situations. b) Closed sentences may be true or false in their expression of ideas about objects or situations.	1. Examples: a) $\square + 3 < 11$ is an open sentence. b) $7 + 3 < 11$ is a closed, true sentence. $9 + 3 < 11$ is a closed, false sentence.
	2. Reasonable skill in writing simple conditions (or equations) in one variable	2. a) John has \$10.50. He bought a gift for his mother and then had \$6.20 left. How much did he spend for the gift? $\$10.50 - x = \$6.20$ b) $x + 3 = 6$ is a simple (or single) condition in one variable.
	3. Reasonable facility in determining solution sets for simple (or single) conditions of equality	3. Example: U = N means Universe = $\{0, 1, 2 \cdots\}$ Given: If a number is added to 20, the sum is 35. Find the number. $20 + x = 35$ ⟵ simple condition of equality $s = \{15\}$ ⟵ solution set
	4. Learning to determine solution sets for inequalities	4. Example: U = C means Universe = $\{1, 2, 3 \cdots\}$ Given: What numbers can be added to 10 so that each sum is less than 21? $10 + x < 21$ ⟵ simple condition of inequality $S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ ⟵ solution set
B. Simple Conditions	1. Reasonable skill in writing simple conditions with two variables when: a) $U = N \times N$ and $N = \{0, 1, 2 \cdots\}$ b) $U = C \times C$ and $C = \{1, 2, 3 \cdots\}$	1. Examples: $x + y < 8$ ⟵ simple (single) condition a) N X N means matching of each appropriate member of the natural numbers, in turn, with each appropriate member of the natural numbers. $N \times N = \{(0,0), (0,1), (0,2) \cdots (1,0), (1,1), (1,2) \cdots\}$ b) C X C means the matching of each appropriate member of the counting numbers, in turn, with each appropriate member of the counting numbers. $C \times C = \{(1,1), (1,2), (1,3) \cdots (2,1), (2,2), (2,3) \cdots\}$

Strands And Topics	Content And Competencies To Be Developed	Suggested Background And Resource Material
C. Compound Conditions	1. Learning to write mathematical sentences using the connective "and" to form compound conditions.	1, 2. Examples: $x + 2 > 6 \wedge 4 + x < 12$ (↑ and) $U = C$
	2. Learning to tabulate solution sets of compound conditions as being the intersection of the sets described by the simple conditions, if "and" is used as the connective.	Solution set is intersection of two sets described by the simple conditions as: $x + 2 > 6$ Soln. Set = $\{5, 6, 7 \cdots\}$ $4 + x < 12$ Soln. Set = $\{7, 6, 5, 4, 3, 2, 1\}$ Intersection of sets described by simple conditions is the solution set of given compound condition, $S = \{5, 6, 7\}$
	3. Learning to write compound conditions with two variables when the universe is either natural or counting numbers.	3. Examples: a) $x + y < 15 \wedge y > 10$ Remember: $N = \{0, 1, 2 \cdots\}$ and $C = \{1, 2, 3 \cdots\}$ $\therefore$ The ordered pairs that are members of C X C will not have 0 as either a first or a second component. b) $x > \$8 \wedge x + \$3 < \$19$ (↑ and) NOTE: Each member of solution set must satisfy both conditions.
D. Conditions For Equivalence	1. Learning that a proportional relation is a set of ordered pairs in which each member is equivalent to each of the other members.	1. Example: $\{(3, 1), (6, 2), (9, 3)\}$
	2. Practice in the use of proportional relations to write sentences that state rate and comparison conditions	2. What is the cost of 5 cans of milk if 3 cans of this milk cost 54¢ $3:54 \sim 5:x \rightarrow 3x = 270$, $x = 90 \rightarrow S = \{90¢\}$
	3. Some practice in tabulating solution sets of equivalence conditions.	3. $x:16 \sim 7:4$ $4x = 112 \longrightarrow x = 28$ or $S = \{28\}$

Strands And Topics	Content And Competencies To Be Developed	Suggested Background And Resource Material
D. Conditions For Equivalence (cont.) 1. Standard units of measure	1. Reasonable skill in using simple conditions of equivalence to convert standard units of measure	1. Example: a) How many inches in 5 ft.? $\frac{12}{1} \sim \frac{x}{5}$ b) $6\frac{1}{2}$ hrs. = how many minutes? $\frac{\frac{13}{2}}{N} \sim \frac{1}{60} \rightarrow N \cdot 1 = \frac{13}{2} \cdot 60$ $N = 390 \rightarrow S = \{390\}$
2. Area	1. Reasonable skill in using simple conditions of equivalence to compute area of square, parallelogram and triangle	1. Example: a) Find area of given figure  Since there are 9 square units in each row and 5 rows in the total surface, we see that: $\frac{9}{1} \sim \frac{N}{5}$ No. of units in 1 row is same proportional relation, or ratio, as no. of units in all rows. b) Find area of given figure 2x = A of rectangle x = A of triangle  $\frac{10}{1} \sim \frac{2x}{6}$ Units in 1 row are in same proportional relation, or ratio, as no. of units in all rows.

Strands And Topics	Content And Competencies To Be Developed	Suggested Background And Resource Material
E. Compound Conditions for Equivalence	1. Pupil recognition of equivalent rate pairs and per cents as members of proportional relations NOTE: See Grade 7, Strand V	1. Examples: a) Joe's walking rate of 105 steps in $1\frac{1}{2}$ minutes is equivalent to how many steps per minute? $\frac{105}{\frac{3}{2}} \sim \frac{x}{1}$ b) Five is what per cent of 25? $\frac{5}{25} \sim \frac{x}{100}$
	2. Reasonable facility in writing and solving mathematical sentences which involve compound conditions of equivalence	2. Example: A store gave a 20% discount on a pair of shoes regularly priced at \$15.00. The net price of the shoes amounted to how many dollars? Let x = number of dollars discount and y = number of dollars net price. Then compound condition is: $\frac{20}{100} \sim \frac{x}{15} \wedge 15 - x = y$ Solution: $\frac{20}{100} \sim \frac{x}{15}$ $100 \cdot x = 300$ $x = 3$ or \$3 discount Then: $15 - 3 = y$ $12 = y$ or net price Solution set $= \{(3, 12)\}$

Strands And Topics	Content And Competencies To Be Developed	Suggested Background And Resource Material
VIII. Statistics A. Meaning of Statistics	1. Learning that the content of statistics is the collecting, arranging, and describing of sets of numerical data (facts).	1. Discussion: What sets of data (facts) could we record for this class? Observation: Do magazines and newspapers have clear ways of "picturing" sets of facts?
B. Interpreting Numerical Data	1. Pupils demonstrate reasonable facility in: a) Obtaining data by counting and measuring. b) Arranging collected data in tables.	1. a), b) Suggestion: Have pupils collect and tabulate data about heights and weights of groups of classmates, ways of transportation to school, scores on math tests, absence and attendance figures, etc.
	c) Organizing numerical data and presenting it in graphical form. See Strand V on graphs.	c) Suggestion: Class discussion of the types of graphs best suited to a clear display of growth in height, comparative costs of 3 items, parts of a total income designated for certain types of expense.
	d) Interpreting numerical data by: 1) Reading the "stories" presented in line, bar, circle, and picture graphs. 2) Making generalizations from the tabulations of results of simple experiments.	d) 1) Have pupils tell the stories presented by graphs which they collect from magazines and newspapers. 2) Following tabulation of birthdays of 1 specific month in which several birthdays of class members fall, encourage pupil-made generalizations concerning: a) Most common birth date? b) More birthdays in first or last part of month? c) Are birthdays "clustered" about certain dates or "spread" through the month?

Strands And Topics	Content And Competencies To Be Developed	Suggested Background And Resource Material
B. Interpreting Numerical Data (cont.)	2. Learning the meaning and importance of: a) "Frequency tally" or "array" of outcomes of experiment b) Histogram as a graph which indicates frequency of occurrence of pictured elements.	2. Suggestion: Have each pupil conduct his own experiment and record the heads-and-tails results of tossing 2 pennies fifty times. One possible record is: a) Frequency Tally H H ~~||||~~ ~~||||~~ H T ~~||||~~ ~~||||~~ ~~||||~~ T H ~~||||~~ ~~||||~~ |||| TT ~~||||~~ ~~||||~~ | b) Frequency array Coin Faces / No. of Times H H - - - - - 10 H T - - - - - 15 T H - - - - - 14 T T - - - - - 11 c) Histogram Freq. 

Strands And Topics	Content And Competencies To Be Developed	Suggested Background And Resource Material
C. Measures of Central Tendency	1. Learning that a "measure of central tendency" is a measure (or number) about which data (facts) tend to center or cluster.	1. Suggestions: a) Have pupils discuss which student should be chosen to represent three students who are respectively short, medium and tall in height. b) If the average of 5 different numbers is 13, can we be sure that some of the numbers are greater than 13 and that some of the numbers are less than 13?
	2. Exploration of important measures of central tendency: a) Definition: The mode is the number (or element) which occurs most frequently in a set of data. b) Definition: The median is the middle element when an odd number of elements are arranged in order of size. The median is the average of the two middle elements when an even number of elements are arranged in order of size. c) Definition: The arithmetic mean (or average) is a single representative of a set of elements found by dividing the sum of the elements by the number of elements.	2. Example: In a math test a group of nine pupils made scores of 70, 75, 90, 65, 70, 80, 85, 80, 95. a) 70 and 80 are the modes or most-frequently-occurring numbers. b) Arranged in order of size the numbers are: 65, 70, 70, 75, 80, 80, 85, 90, 95 ↖ M (pointing to the first 80) In this group of nine elements, the middle element or number, is 80. This is the median. c) Arithmetic mean in b above is found by: $(65 + 70 + 70 + 75 + 80 + 80 + 85 + 90 + 95) \div 9 = \frac{710}{9}$ or $78\frac{8}{9}$ NOTE: Here the teacher is urged to help pupils understand that computing an average is a "cutting down" of the large numbers, 95, 90, 85, 80, and a "building up" of the smaller numbers, 65, 70, 75, until the middle figure is reached at $78\frac{8}{9}$.
	3. Discussion of the usefulness of the arithmetic mean, the median and the mode as single elements, each of which serves as a way to summarize a large group of data.	

2

Strands And Topics	Content And Competencies To Be Developed	Suggested Background And Resource Material
D. Measures of Spread or Dispersion	1. Study of meaning and ways of measuring "spread" or "scatter" of a set of elements a) Definition: The range is the difference between the largest and the smallest elements of a set. b) Definition: The average deviation is the average of the individual differences of each element from the arithmetic mean.	1. a) $C = \{20, 30, 40\}$ $D = \{29, 30, 31\}$ The mean and the median of both set C and set D are 30, but the members of set C are spread farther apart than the members of set D. The range of set C is 20, but the range of set D is only 2. The description, "Set C has a median of 30 and a range of 20" shows much greater scatter or spread of elements than is indicated in the description, "Set D has a median of 30 and a range of 2." b) Given: $N = \{6, 2, 5, 10, 6, 2, 4\}$ Arithmetic mean = 5 as seen in $\frac{6+2+5+10+6+2+4}{7} = \frac{35}{7}$ or 5 Tabulating deviations (or differences) of nos. from 5 (arith. mean) we have: Nos. — Diff. From Mean of 5 6 - - - - - - - - - - - - 1 2 - - - - - - - - - - - - 3 5 - - - - - - - - - - - - 0 10 - - - - - - - - - - - - 5 6 - - - - - - - - - - - - 1 2 - - - - - - - - - - - - 3 4 - - - - - - - - - - - - 1 Total: 14 Average deviation of nos. from arithmetic mean = 2 because $\frac{1+3+0+5+1+3+1}{7} = \frac{14}{7}$ or 2

Strands And Topics	Content And Competencies To Be Developed	Suggested Background And Resource Material
D. Measures of Spread or Dispersion (cont.)	2. Enrichment: Pupil discover that in a set of numbers the sum of the signed differences of the elements from the arithmetic mean is zero.	2. Given: N = 6, 2, 5, 10, 6, 2, 4 and mean $= \frac{35}{7}$ or 5 We rearrange numbers in order of size and use positive and negative numbers (signed differences) and zero to represent the differences from the mean. Nos. — Difference From Mean 10 - - - - - - - - + 5 6 - - - - - - - - + 1 6 - - - - - - - - + 1 Mean 5 - - - - - - - - 0 4 - - - - - - - - $^-1$ 2 - - - - - - - - $^-3$ 2 - - - - - - - - $^-3$ Sum of differences from mean: $5 + 1 + 1 + 0 + (^-1) + (^-3) + (^-3) = 0$

MATHEMATICS FOR THE EIGHTH GRADE

MATHEMATICS FOR THE EIGHTH GRADE

Desired Outcomes and Goals

1. To develop a foundation for the study of the mathematics which will be required of the pupil as he progresses through high school.
2. To give reasonable practice in several ways of reasoning from the known to logical conclusions about the unknown, and to extend the development of pupil-understanding of the properties of the natural numbers, the integers, the rational and the real numbers.
3. To develop computational skills needed for applying mathematics concepts and processes to subject matter and practical applications appropriate to this maturity level.
4. To extend the understanding and use of the basic concepts of sets and of operation on sets, in part through extending pupil-understanding of the relationship between points on the number line configuration and the combined sets of rational and irrational numbers which form the real numbers.
5. To extend the understanding and use of equations and inequations as number sentences, to extend pupil-knowledge of basic geometric concepts and relationships, to teach the ratio-proportion approach in converting English-system measurements to metric measure, and to extend pupil-understanding of statistical methods of describing sets of numerical data.

Terminology

1. "Strand," as used in this Guide, refers to the body of concepts, definitions, meanings, and applications pertaining to an area in mathematics. The material of the eighth grade strand on numbers and operations, as is true of the content of any other strand. should be interspersed throughout the year's program. A mathematics strand, as designated in this Guide, is not intended to delineate a body of subject matter to be taught as a single unit.

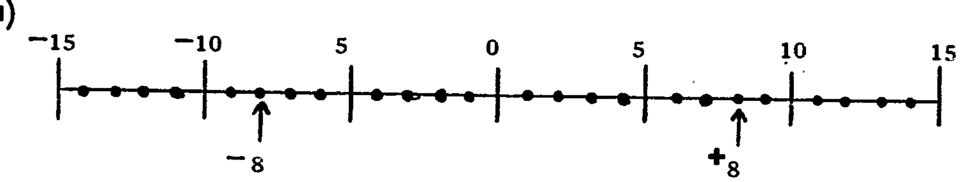
2. The word "condition" has been consistently used throughout the Guide because it permits simultaneous reference to equations and inequations.

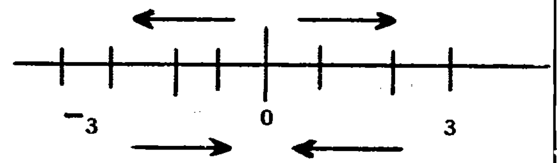
Symbol Meanings Used in this Guide

=	equal	(m)	measurement	$\parallel$	parallel
$\neq$	not equal	°	degree	$\perp$	perpendicular
$\approx$	approximately	π	pi	$\widehat{XYZ}$	major arc
$\cong$	congruent	•	point	$\widehat{XY}$	minor arc
$\longleftrightarrow$	equivalent	$\longleftrightarrow$	line	{ }	set
~	equivalent	—	segment	$\{X \mid X < 10\}$	set of all X "such that" X is less than 10
~	negation, Strand II (some texts)	—>—	directed segment	{} or $\emptyset$	empty set
>	greater than	•—>	ray	$\cup$	union
<	less than	$\odot$	circle	$\cap$	intersection
$\wedge$	and	Δ	triangle	A X B	A cross B
$\vee$	or	$\angle$	angle	(a,b)	the ordered pair a,b
$\pm$	tolerance	▱	parallelogram	$\in$	is a member of

MATHEMATICS CONTENT, EXPERIENCES, AND OUTCOMES
FOR THE EIGHTH GRADE PUPIL

Strand I, Numbers And Operations

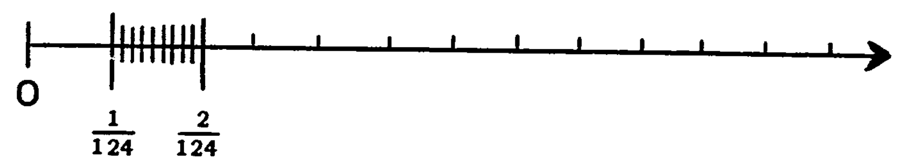
Strands And Topics	Content And Competencies To Be Developed	Suggested Background And Resource Material
I. Numbers And Operations A. The System of Integers 1. Positive and Negative Integers	1. Review grade seven unit on whole numbers. 2. Pupil recognizes oppositeness of directions of travel paths, thermometer readings, profit and loss, etc., as showing the need of positive and negative integers.	2. Teaching suggestions: a) If $^{+}40°$ represents a temperature of 40° above zero then a temperature of 10° below zero is written____. (answer: $^{-}10°$) b) If $^{+}1000$ represents a profit of \$1000, then a loss of \$550 is written____. (answer: $^{-}550$) c) If $^{+}15$ represents 15 steps upstairs, then 15 steps downstairs is written____. (answer: −15) Generalization: Positive (⁺) and negative (⁻) signs are used with numerals to represent oppositeness of direction, action, situation, relationship, etc.
2. Absolute Value	1. Pupil demonstrates understanding that: a) "absolute value" of integer tells the number of units from zero to the given number.	1. Example: a) 
	b) Sign of number indicates its location left or right of zero on number line:	b) Negative 8, ($^{-}8$), and plus 8, ($^{+}8$), have the same absolute value, 8, because they are the same distance, 8 units, from zero. Symbolically: $\|y\|$ is read "absolute value of y" or "absolute y". $\|^{+}6\| = 6, \quad \|^{-}6\| = 6$
	c) The integer zero, which is neither positive nor negative, separates positive integers from negative integers on the number line.	c) Using zero: If 0 is positive, then $x + 0 = x$. If 0 is negative, then $x + (-0) = x$. ∴ A distinction between $^{+}0$ and -0 is useless.

Strands And Topics	Content And Competencies To Be Developed	Suggested Background And Resource Material
The System of Integers (cont.) 3. Additive Inverse	1. Recognition that: a) Additive inverse of a number, when added to that number, results in zero.	1. Examples: a) By "right trips" and "left trips" on the number line we see intuitively that: $(^{+}3)$ and $(^{-}3)$ result in 0, $(^{-}3)$ and $(^{+}3)$ result in 0. 
	b) Additive inverse of any number may be found by changing its sign.	b) $(^{+}5) + (^{-}5) = 0$, $(^{-}17) + (^{+}17) = 0$, etc.
	c) Zero is its own additive inverse.	c) By use of number line explore possible sizes of, "Suppose we had $^{+}0$ and $^{-}0$" with intuitive recognition that "the right and left trips go nowhere," so the result is zero.
4. Laws of Integers	1. Discovery by experimentation that positive and negative integers behave according to familiar "whole number" laws. a) Closure: sum or product of two integers is an integer.	1. Examples: a) $(^{-}3) + (^{+}9) = (^{+}6)$, $(^{+}3) \cdot (^{-}6) = (^{-}18)$, $(^{-}3) \cdot (^{-}6) = (^{+}18)$
	b) Commutativity: $(^{-}a) + b = b + (^{-}a)$	b) $(^{-}7) + (^{+}4) = (^{+}4) + (^{-}7)$, $(^{-}18) + (^{-}5) = (^{-}5) + (^{-}18)$ $^{-}3 = {}^{-}3$ $\qquad$ $^{-}23 = {}^{-}23$
	c) Associativity: $-a \cdot (b \cdot c) = (^{-}a \cdot b) \cdot c$	c) $(^{-}2) \cdot (3 \cdot 4) = (^{-}2 \cdot 3) \cdot 4$ $(^{-}2) \cdot 12 = (^{-}6) \cdot 4$ $^{-}24 = {}^{-}24$
	d) Distributivity of one factor over sum: $^{-}a \cdot (b + c) = (^{-}a \cdot b) + (^{-}a \cdot c)$	d) $^{-}5 \cdot (4 + 7) = (^{-}5 \cdot 4) + (^{-}5 \cdot 7)$ $(^{-}5) \cdot (11) \qquad (^{-}20) + (^{-}35)$ $^{-}55 = {}^{-}55$

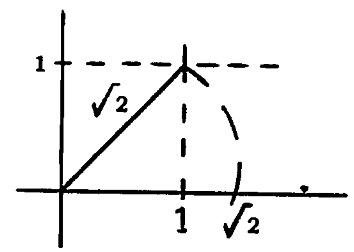
Strands And Topics	Content And Competencies To Be Developed	Suggested Background And Resource Material
A. The System of Integers (cont.) 5. Fundamental Operations With Integers	1. Ability to apply the "action patterns" of trips on a number-line, running a movie film backward, etc. in explaining the processes of addition, subtraction, multiplication and division of integers.	1. The teacher is urged to follow the "action pattern" introduction with a reasonable amount of computational practice in finding the sums, differences, products, and quotients of integers with like and unlike signs.
B. Exponents 1. Notation	1. Pupil demonstrates knowledge of and effective computational use of: a) Complete prime factorization b) G. C. F. (greatest common factor) c) L. C. M. (least common multiple)	1. Examples: a) Unique prime factorization of 12 is $2 \cdot 2 \cdot 3$ b) To find G. C. F. of 12 and 27 Factoring: $12 = 2 \cdot 2 \cdot 3$ and $27 = 3 \cdot 3 \cdot 3$ $\therefore$ G. C. F. is 3. c) To find L. C. M. of 12 and 27 Factoring: $12 = 2^2 \cdot 3$ and $27 = 3^3$ $\therefore$ L. C. M. $= 2^2 \cdot 3^3$ or $4 \cdot 27 = 108$
2. Fundamental Operations with Exponents	1. Pupil demonstrates grasp of meaning and ability to apply basic "laws of exponents" to express repeated factors as: a) $a^n \cdot a^m = a^{n+m}$ b) $a^n \div a^m = a^{n-m}$ c) $(a^n)^m = a^{n \cdot m}$	1. Examples: a) $2^2 \cdot 2^3 = (2 \cdot 2) \cdot (2 \cdot 2 \cdot 2) = 2^5$ or 2^{2+3} b) $2^5 \div 2^3 = \frac{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2}{2 \cdot 2 \cdot 2} = 2 \cdot 2$ or 2^2 or 2^{5-3} c) $(2^4)^3 = 2^4 \cdot 2^4 \cdot 2^4 = (2 \cdot 2 \cdot 2 \cdot 2) \cdot (2 \cdot 2 \cdot 2 \cdot 2) \cdot (2 \cdot 2 \cdot 2 \cdot 2) = 2^{12}$ or $2^{4 \cdot 3}$

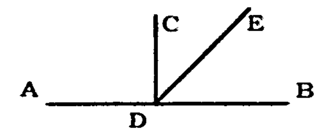
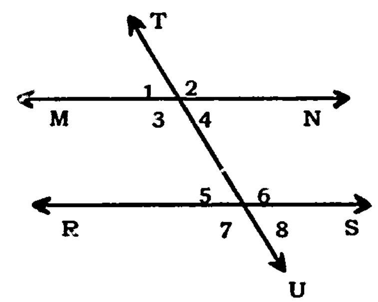
Strands And Topics	Content And Competencies To Be Developed	Suggested Background And Resource Material
B. Exponents (cont.) 3. Negative Exponent s	1. Recognition, following careful observation, that a^{-n} means $\frac{1}{a^n}$	1. Look at this pattern $1000 = 10^3$ (or 10 x 10 x 10) On left of equal sign we divide each no. by 10 → $100 = 10^2$ (or 10 x 10) $10 = 10^1$ (or 10) On right of equal sign we decrease each exponent by one $1 = 10^0$ To keep pattern going we have $\frac{1}{10} = 10^{-1}$ $\frac{1}{100} = 10^{-2}$ $\frac{1}{1000} = 10^{-3}$ NOTE: $\frac{1}{10^n} = 10^{-n}$
	2. Demonstration of computational ability in use of positive and negative integral exponents	2. $\frac{1}{3^2} \cdot \frac{3^3}{1} = \frac{3^3}{3^2} = \frac{27}{9} = 3^1$ $3^{-2} \cdot 3^3 = 3^{-2+3} = 3^1$
4. Expansion of Integers to Powers	1. Recognition that in mathematical language: a) An exponent tells how many times a number is used as a factor. b) Positive exponents mean expansion is in the numerator. c) Negative exponents mean expansion is in the denominator.	1. Examples: a) 3^4 means $3 \cdot 3 \cdot 3 \cdot 3$ b) 4^3 means $4 \cdot 4 \cdot 4$ c) 4^{-3} means $\frac{1}{4^3} = \frac{1}{4 \cdot 4 \cdot 4} = \frac{1}{64}$

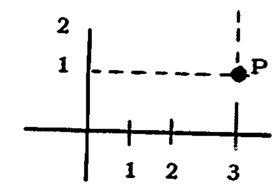
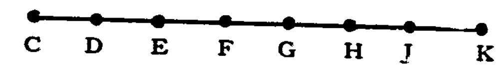
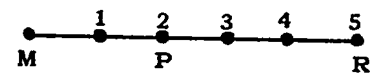
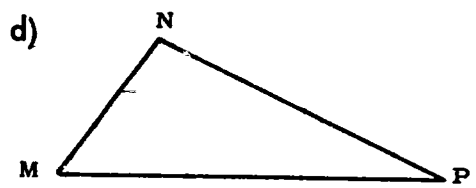
Strands And Topics	Content And Competencies To Be Developed	Suggested Background And Resource Material
4. Expansion of Integers to Powers (cont.) a) Zero and as Exponents	1. Pupil learns that: a) $a^0 = 1$, $(a \neq 0)$ by definition b) $a^1 = a$ by definition	1. Suggestions: a) Make repeated use of "pattern" in B (3) example above, to show that $12^0 = 1$, $Z^0 = 1$, $(Z \neq 0)$, etc., for <u>infinite</u> number of ways of "naming" the numeral one. Also: $5^3 \cdot 5^{-3}$ means $\frac{5^3}{1} \cdot \frac{1}{5^3} = \frac{5^3}{5^3} = 1$ b) NOTE: Since a^1 means that a is to be used as a factor one time, it is customary to omit the exponent.
b) Scientific Notation	1. Pupil demonstrates comprehension of the way scientific notation utilizes powers of 10 and the laws of exponents to simplify computations with very large and/or very small numbers.	1. Examples: a) <u>Multiply</u> Rounding off to 3 significant digits: 14,596,438 by 27,692,179 → 14,600,000 × 27,700,000 $1.46 \cdot 10{,}000{,}000$ times $2.77 \cdot 10{,}000{,}000$ 1.46×10^7 times 2.77×10^7 By Associative Law: $(1.46 \times 2.77) \times (10^7 \times 10^7)$ Rounding to 3 signif. digits: $4.0442 \times 10^{14} = 4.04 \times 10^{14}$ b) Use similar development: $.000012 \times .00000241 = 1.20 \times 10^{-5} \times 2.41 \cdot 10^{-6}$ $= 2.89 \times 10^{-11}$

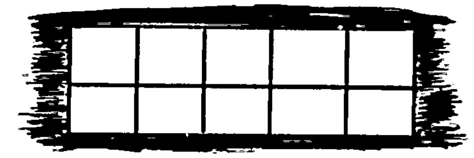
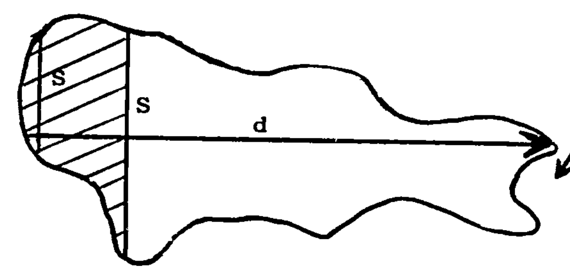
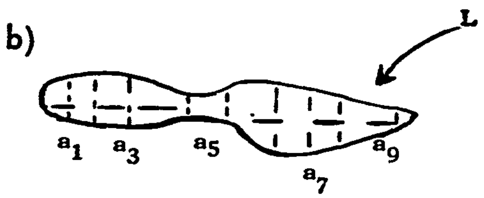
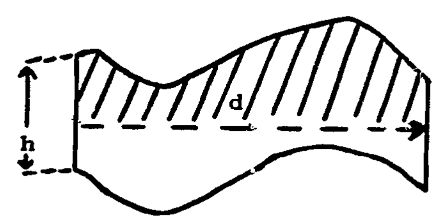
Strands And Topics	Content And Competencies To Be Developed	Suggested Background And Resource Material
C. Rational Number System 1) Positive and Negative Rational Numbers	1. Review grade seven, Strand I, material on rational numbers. 2. Extienson of understanding to include negative rational numbers.	2. Examples: $\left(\frac{3}{8}\right)+\left(\frac{-1}{8}\right) = \frac{3-1}{8} = \frac{2}{8} = \frac{1}{4}$ $\frac{3}{8} \cdot \left(\frac{-1}{8}\right) = \frac{-3 \cdot 1}{8 \cdot 8}$ or $\frac{-3}{64}$
2) Density	1. Pupil learns, by use of number line that: a) Any two rational numbers have other rational numbers between them.	1. Examples: a) Is there a rational number between $\frac{1}{124}$ and $\frac{2}{124}$?  Many tiny divisions of given space are possible. As long as there is any space between nos. on line, we can take half of that space to find another number between the two given numbers. (Theoritically, there is no end to this process.)
	b) Common fractions and decimal fractions can be used to show that there are always other rational numbers between two given rationals.	b) Another way to find one rational number between $\frac{1}{124}$ and $\frac{2}{124}$ is to note that the average of the given rationals is greater than $\frac{1}{124}$ and less than $\frac{2}{124}$, so it must lie on the number line between the given points. (This method is useful in locating a rational number between any two distinct points representing rationals on the number line.)
	c) The set of rationals is "everywhere dense".	c) There are many rational numbers between 2.5173 and 2.5174. A few of them are: 2.51731, 2.51732, 2.51733, etc.

Strand I, Numbers And Operations - - - - Grade 8

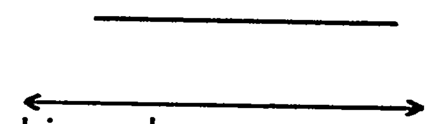
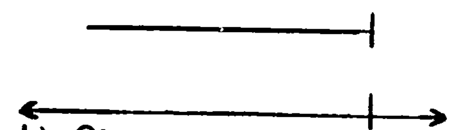
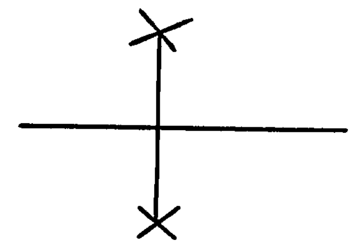
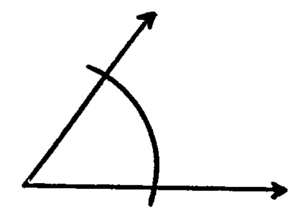
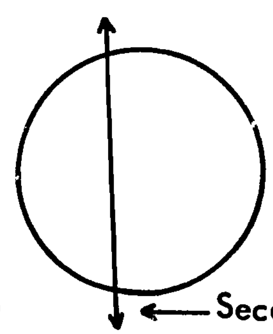
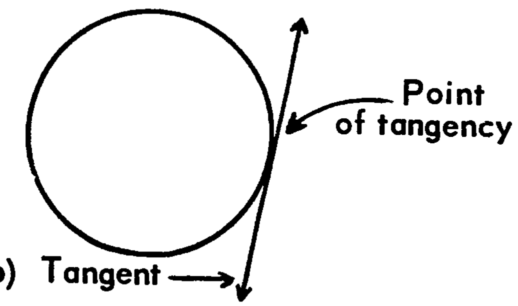
Strands And Topics	Content And Competencies To Be Developed	Suggested Background And Resource Material
D. Irrational Numbers	1. Learning that: a) Irrational numbers are represented by "endless numerals" such as the value of pi. b) All remaining "holes" on number line are "filled in" by irrational numbers. NOTE: Thorough discussion of irrationals not recommended at this time.	2. $\sqrt{2}$, $\sqrt{3}$, $\sqrt{8}$, etc., are irrational numbers whose location can be marked on the number line:  NOTE: Proof of irrationality of $\sqrt{2}$ not recommended. See a modern Algebra I if proof is desired.

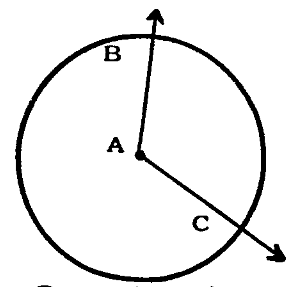
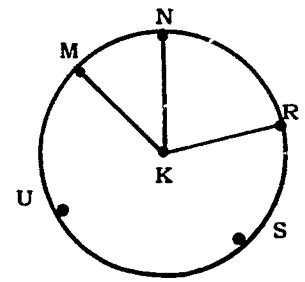
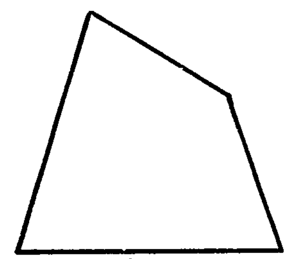
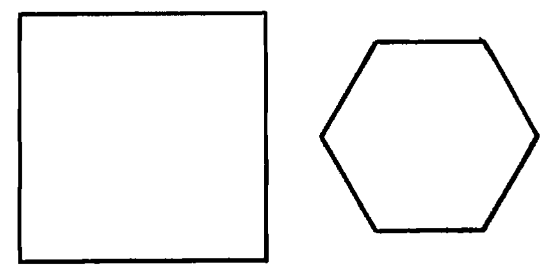
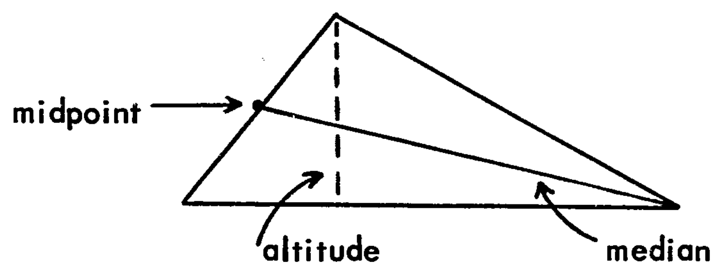
Strands And Topics	Content And Competencies To Be Developed	Suggested Background And Resource Material
II. Geometry A. Angle Relations	1. Learning the meaning and something of the usefulness of a basic description of angles as: a) complementary angles b) corresponding angles c) supplementary angles d) vertical angles	1. & 2. Examples: a)  Fig. (1) If $\overline{CD} \perp \overline{AB}$, then $\angle$ EDC is complementary to $\angle$ EDB and $\angle$ EDB is supplementary to $\angle$ ADE b)  Fig. (2) If $\overleftrightarrow{MN}$ is parallel to $\overleftrightarrow{RS}$, then angles 4 and 8, 6 and 2, 1 and 5, 3 and 7, are examples of pairs of corresponding angles.
	2. Developing definitions for the above angles. See Strand III, Grade 7, Section D.	c) Angles 1 and 2, 2 and 4, 4 and 3, 1 and 3, 5 and 6, 6 and 8, 7 and 8, 5 and 7, are examples of pairs of supplementary angles. Also, angles 1 and 6, 2 and 5, 1 and 7, 2 and 8, 4 and 7, 3 and 8, are supplementary angles. (Use Figure 2 above) d) Angles 1 and 4, 2 and 3, 5 and 8, and 6 and 7 are examples of vertical angles. (Use Figure 2 above)

Strands And Topics	Content And Competencies To Be Developed	Suggested Background And Resource Material
B. Geometry and Measurement in a Plane	1. Exploring the meaning of certain geometric figures in a plane: a) Coordinates of a point b) Subdivisions of a segment c) Measure of a segment d) Measure of an angle e) Angle described by measure as: "a 50° angle" or "This road meets the highway at a 30° angle."	1. Examples: a)  coord. of P = (3,1) b)  Some subdivisions of $\overline{CK}$ are: $\overline{DE}$, $\overline{DG}$, $\overline{JF}$, $\overline{HC}$, etc. c)  M ($\overline{PR}$) = 3 or "measure of segment PR is 3" d)  $\angle$ M° = 50 or "measure in degrees of $\angle$M is 50"
	2. Review of: a) Measures associated with quadrilaterals and their areas and perimeters b) Measures associated with triangles and their areas and perimeters	2. See Strand III.
	3. Learning the differences between: a) Transversal, described as line intersecting 2 or more lines in a plane. b) Diagonal, described as a segment from one vertex of a quadrilateral to the opposite vertex.	3. Suggestions for illustration: a) Give "observation" assignments in which pupils recognize transversals and diagonals in the structural lines of construction and architecture. b) Pupil use overhead projector and transparencies to show "transversal" and "diagonal" forms observed in nature, in design, etc.

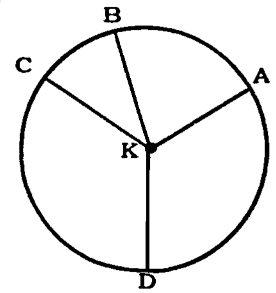
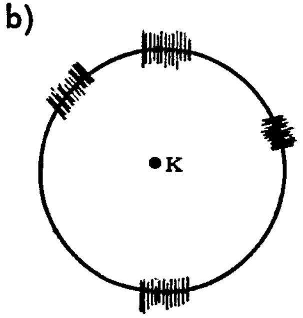
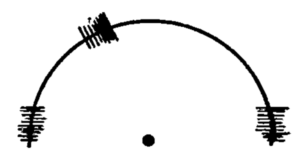
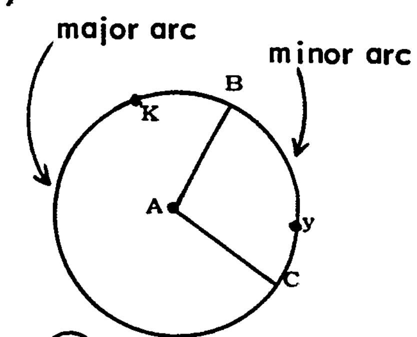
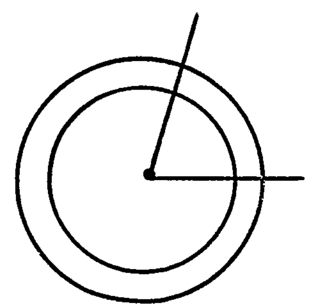
Strands And Topics	Content And Competencies To Be Developed	Suggested Background And Resource Material
B. Geometry and Measurement in a Plane (cont.)	4. Exploring intuitively the meaning of: a) Closed region as the union of a simple closed curve and its interior. b) Area of a closed region as the number of non-overlapping unit regions that can be included in the closed region.	4.  a) Closed Region is union of simple closed curve and interior.  b) The area of this rectangular region (a closed-curve region) is the sum of the areas of the square units.
	5. Exploring intuitively the meaning of "sweeps" as: a) A "sweep" is a line segment moving from left to right and "sweeping" an irregular plane region. b) Distance segment sweeps is measured in direction perpendicular to segment.	5. Example:  6. a) Example: Approximate area of this figure = average length of sweeping segment times distance moved. S = segment or sweep which varies in length. d = distance "swept" by segment
	6. Study of the concepts that: a) Approximate area of irregular region = average length of sweeping segment x distance moved. b) Approximate area of irregular geometric figure = average of several altitudes (including maximum & minimum) x maximum length of figure. c) Finding area of a regular geometric figure of unusual shape makes use of the idea of a "sweep".	6. b)  Area = average of several altitudes (including maximum and minimum) times maximum length of figure. c)  Because shaded region above line can be placed below unshaded region to form a rectangle, then h x d gives us the square measure of this figure.

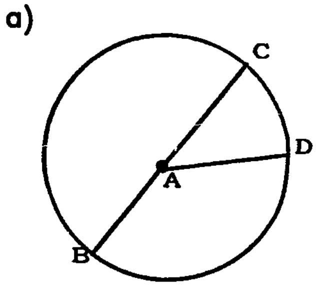
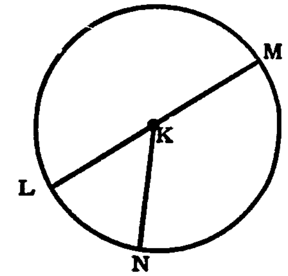
Strands And Topics	Content And Competencies To Be Developed	Suggested Background And Resource Material
B. Geometry and Measurement in a Plane (cont.)	7. Exploring problems involved in finding surface areas of solids (3-space figures) a) Regular prisms and pyramids b) Cylinder c) Cone d) Sphere	7. See Strand III.
	8. Review and extension of pupil understanding of the measure of volume of: a) Regular prisms and pyramids b) Cylinder, cone, sphere	8. See Strand III.
	9. Pupil discovery that the sum of the angles of a regular plane triangle is 180°	9. Suggestion: Laboratory work by students measuring angles in variously-shaped triangles with determination of the approximate sum of the angles is helpful.
	10. Demonstration of congruency of triangles by use of basic properties: a) Angle, side, angle property b) Side, side, side property c) Side, angle, side property	10. Suggestions: a) This development should be intuitive and non-rigorous. b) Making use of earlier definition of congruency, lead pupils to conclude for themselves that it is impossible to show congruency by use of angle, angle, angle property. c) By use of angle, angle, angle property lead pupils through correct comparison of "similarity" of two triangles and "congruency" of two triangles.

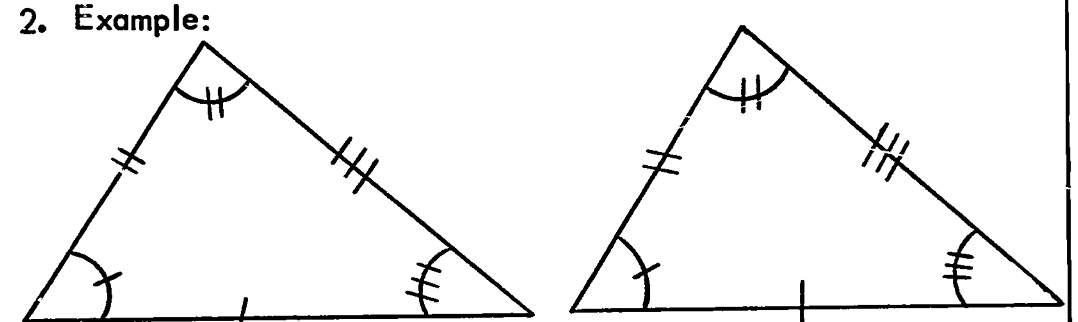
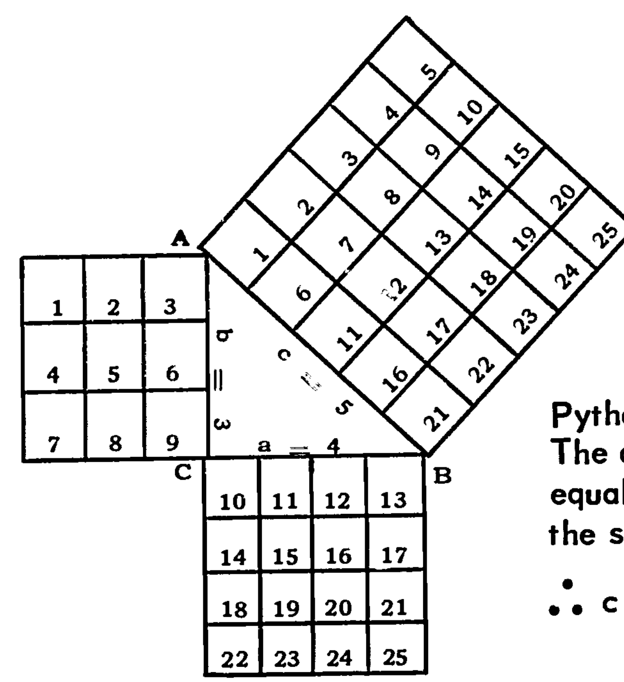
Strands And Topics	Content And Competencies To Be Developed	Suggested Background And Resource Material
C. Constructions and Drawings	1. Pupil demonstrates ability to use protractor, compass and straight edge for construction and drawing of geometric figures such as: a) Line and segment b) A given segment measured on a given line c) Perpendicular bisector of a segment d) An angle e) Angle bisector f) An angle which is congruent to a given angle	1. Hint: The use of an overhead projector and transparent drawing tools is an advantage here.  a) Line and segment  b) Given segment on given line  c) Perpendicular bisector of segment  d) Drawing of an angle  d) Construction of angle bisector
D. Intersections of Lines and Circles	1. Study and use of the concepts: a) Secant of a circle defined as a line whose intersection with a circle contains exactly two points. b) Tangent of a circle as a line included in the plane of a circle in such way that its intersection with the circle contains exactly one point. c) Point of tangency as the point at which the tangent and circle intersect.	1.  a) Secant  b) Tangent

Strands And Topics	Content And Competencies To Be Developed	Suggested Background And Resource Material
E. Central Angles and Arcs	1. Pupil demonstrates ability to: a) Identify a central angle as an angle which has vertex at center of circle and whose sides intersect the circle. b) Distinguish between major and minor arcs of a circle by use of either 2 or 3 points. c) Use major and minor arc symbolism correctly.	1.  a) Central angle  b) Major and minor arcs b) major $\overset{\frown}{RM}$ means $\overset{\frown}{RSM}$ minor $\overset{\frown}{RM}$ means $\overset{\frown}{RNM}$
F. Polygons And Regular Polygons	1. Learning about and demonstrating the difference between a) and b): a) Polygon as any simple closed curve which is the union of segments. b) A regular polygon as a simple closed curve which is the union of segments of the same length that are joined so that equal angles are formed. 2. Enrichment: On the basis of the number of sides, pupils can identify polygons by name through dodecagon.	2. Examples:  Polygon  Regular Polygons
G. Segments Related to the Triangle	1. Recognition of meaning and ability to sketch or construct perpendicular bisector of a side, bisector of an angle, altitude, median of a triangle.	1. 

Strands And Topics	Content And Competencies To Be Developed	Suggested Background And Resource Material
H. Theorems and Converses	1. Learning that a theorem is a statement which is subject to proof.	1. Suggestion: Make use of postulates in proving theorems. No statement should be made unless a reason can be given for making such a statement.
	2. Recognition that a theorem may be true, but its converse may not be true.	2. Example: Statement - If it is snowing, I leave my car in the garage. Converse - If I leave my car in the garage, it is snowing.
I. Finite Geometry Note: This section should be used as enrichment for able students.	1. Learning that finite geometry is a geometry of a limited number of points in space. NOTE: The term "straight line" is not used in finite geometry.	1. For the more able students, the emphasis should be on the existence of geometries other than Euclidian.[1] Finite geometry is one which will serve this purpose. In a finite geometry there are only a countable number of points on a line. Consider an example in which there are three points on each line and three lines through each point. To emphasize that "point" and "line" are undefined and subject to various interpretations, we interpret "point" to mean "student in a class" and "line" to mean "committee." We interpret the property of a point being on a line or of the line being through a point as a student being a member of a committee. Then, for a given class of students, we make these seven assumptions: a) There exists at least one committee; b) Every committee has at least 3 members; c) Any 2 committees have at least 1 member in common; d) If A and B are students, there exists a committee of which both are members; e) If A and B are students, there exists at most one committee of which both are members; f) Not all students are members of the same committee; g) No committee has more than 3 members. The throrems in a finite geometry may be proved as in any other geometry. (1) Meserve and Sobel, Mathematics For Secondary School Teachers, 1962. Prentice-Hall.

Strands And Topics	Content And Competencies To Be Developed	Suggested Background And Resource Material
III. Measurement A. Circles	1. Learning the meaning of specific circle terms and how to measure each: a) Definition - A circle is a set of points in a plane such that all points are an equal distance from a fixed point called the center. b) An arc is a part of a circle. c) A semi-circle is an arc determined by the endpoints of a diameter of the circle. d) A central angle has its vertex at the center of a circle and by its own measure determines the measure of minor and major arcs.	1. Examples: a)  m $(\overline{KA}) = m(\overline{KB}) = m(\overline{KC})$, etc. b)  m (⊙ K)° = 360 c)  m (◠ K)° = 180 d)  ∠ BAC is central angle. Minor arc, $\overset{\frown}{BYC}$, has same measure in degrees as ∠ $\overset{\frown}{BAC}$. Measure of major arc, $\overset{\frown}{BXC}$, in degrees = 360° − m $(\overset{\frown}{BYC})°$.
	2. Pupil learns that: a) Concentric circles are two or more circles having a common center but having different radii. b) By definition of a degree of circular measure, arcs having the same degree measure have different length measures in concentric circles.	2. 

Strands And Topics	Content And Competencies To Be Developed	Suggested Background And Resource Material
A. Circles (cont.)	3. Knowledge that for a circle: a) Radius means the distance from the center to any point on the circle. b) Diameter is the line segment which contains the center of a circle and whose endpoints lie on the circle. A diameter is the longest chord of a circle. c) Circumference means the length of the circle. d) Something of the importance of the symbol pi, π , as an inherited mathematical term representing the ratio of the circumference of any circle to its own diameter. e) The circumference of any circle is exactly π times as large as its own diameter and approximately $3\frac{1}{7}$ times its own diameter. NOTE: See appendix on pi.	3. a)  b) NOTE: Pupils should "discover" the constancy of the comparison of length of diameter and length of circumference by making "string measurements" of many cardboard circles. $m(\overline{BA}) = m(\overline{AC}) = m(\overline{AD})$ is radius, (r) $\overline{BC}$ is diameter (d) $m(\overline{BC}) = m(\overline{BA}) + m(\overline{AC})$ or $m(2r)$ $\therefore d = 2r$ c) Suggestion: Use differing lengths of string to form circles on desk tops as "quickie" class exercise. Does approximate ratio of $3\frac{1}{7}$ to 1 hold for circumference and diameter?
	4. Pupil demonstrates understanding of and ability to apply the mathematical sentences: a) Circumference of any circle: $C = 2\pi r$ or $C = \pi d$ b) Area of any circle: $A = \pi r^2$ or $A = \pi \cdot \left(\frac{d}{2}\right)^2$ 5. Knowledge that congruent circles have the same radius, circumference, and area measurements.	4.  $\overline{LM}$ is diameter. $m(\overline{MK}) = m(\overline{KN}) = m(\overline{KM})$ Each segment measures radius. $C = 2\pi$ $C \approxeq 2 \cdot 3\frac{1}{7} \cdot m(\overline{KM})$ $A = \pi^{2}$ $A = 3\frac{1}{7} \cdot m(\overline{KM})^2$

Strands And Topics	Content And Competencies To Be Developed	Suggested Background And Resource Material
B. Congruent Triangles	1. Learning that when two figures have the same size and shape, they have the same area measure and are called "congruent" (≅)	
	2. Learning that two triangles are congruent if the following properties "hold": a) Angle, side, angle (asa): Two angles and included side of one triangle are congruent respectively to two angles and included side of other triangle. b) Side, side, side (sss): Three sides of one triangle are congruent respectively to three sides of the other triangle. c) Side, angle, side (sas): Two sides and included angle of one triangle are congruent respectively to two sides and the included angle of the other triangle.	2. Example:  Figure A Figure B Figure A is congruent to Figure B by application of any of 3 given properties.
	4. Studying and applying the relationships expressed by the Pythagorean Property as: a) The area of the square on the hypotenuse of a right triangle is equal to the sum of the areas of the squares on the other two sides. b) Demonstrating use of a model of the Pythagorean Property in determining the length of the third side of a right triangle when lengths of the other two sides are given.	4. Example:  Pythagorean Property: The area of the square on $\overline{AB}$ equals the sum of the areas of the squares on $\overline{BC}$ and $\overline{AC}$ $\therefore c^2 = a^2 + b^2$
	5. Learning: a) To use tables of squares and square roots of numbers b) To approximate square roots to two or three decimal places	

Strands And Topics	Content And Competencies To Be Developed	Suggested Background And Resource Material
C. Metric System of Measurement	1. Discusses the usefulness of: a) Relationship of metric units of linear measure b) Abbreviations representing metric measure units c) Equivalent of each metric linear measure in meters d) Meter equivalents expressed in scientific notation	1. Metric Table with equivalents
	2. Pupil demonstrates reasonable proficiency in converting from metric to English units of measure. NOTE: Emphasis on "conversion" work is not recommended.	2. Conversion tables:

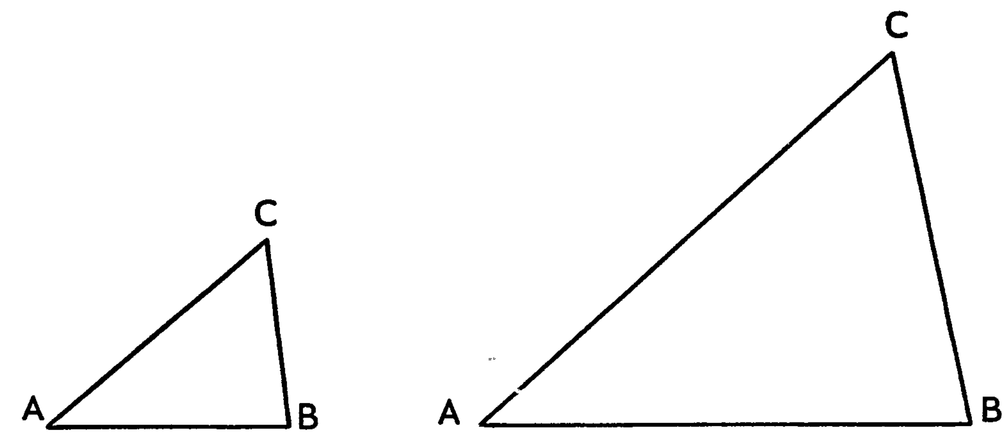
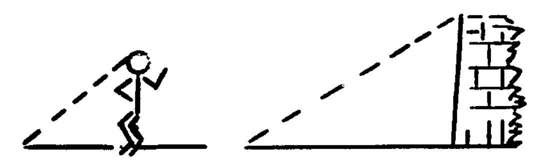
1. Metric Table with equivalents

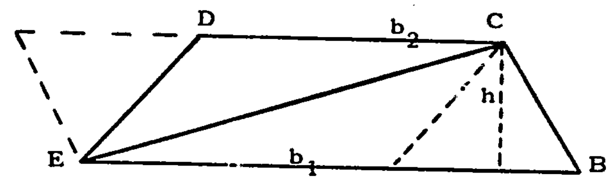
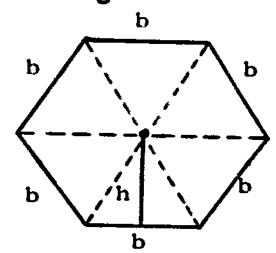
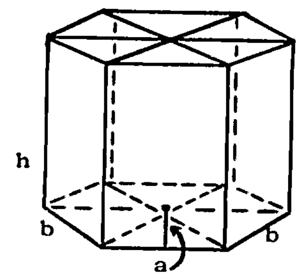
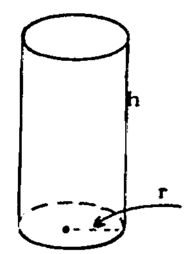
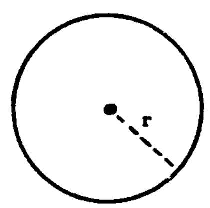
Linear Unit	Abbrev.	Equiv. In Meters	Equiv. In Meters In Sci. Notation
1 millimeter	1 mm.	$\frac{1}{1000}$ m.	10^{-3} m.
1 centimeter	1 cm.	$\frac{1}{100}$ m.	10^{-2} m.
1 decimeter	1 dm.	$\frac{1}{10}$ m.	10^{-1} m.
1 meter	1 m.	1 m.	10^{0} m.
1 dekameter	1 dkm.	10 m.	10^{1} m.
1 hectometer	1 hm.	100 m.	10^{2} m.
1 kilometer	1 km.	1000 m.	10^{3} m.

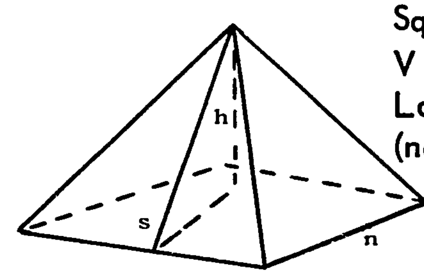
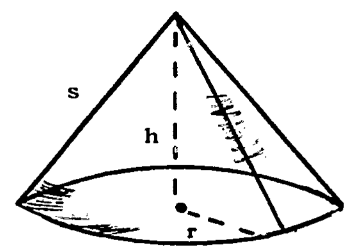
2. Conversion tables:

Length	Weight	Capacity
.039" ≈ 1 mm.	1000 milligrams (mg.) = 1 gram (g.)	1000 milliliters (ml.) = 1 liter (l.)
.394" ≈ 1 cm.	1000 grams = 1 kilogram (kg.)	1000 liters = 1 kiloliter (kl.)
1.094 yds. ≈ 1 m.	.035 oz. ≈ 1 gram	1.06 qts. ≈ 1 liter
.62 mi. ≈ 1 km.	2.2 lbs. ≈ 1 kilogram	1 gal. ≈ 3.785 liters

Strands And Topics	Content And Competencies To Be Developed	Suggested Background And Resource Material
C. Metric System of Measurement (cont.)	3. Developing working knowledge of metric units of mass and volume.	3. Example: If the volume of the interior of a container is 500 cu. cm., then the mass of water it contains is 500 grams. Note that the numerical measures are the same.
	4. Pupil discusses the "useability" of the relationship between metric units of measure for volume, mass, and weight.	4. Volume = Mass = Weight 1 cu. cm. = 1 g. = 1 g. 1 cu. dm. = 1 kg. = 1 kg. 1 cu. m. = 1 metric ton = 1 metric ton
D. Relative Error	1. a) Pupil learns that in a numeral a digit is said to be a "significant digit" if it serves a purpose other than that of locating (or emphasizing) the decimal point. b) Pupil can determine the number of significant digits in a numeral.	1. Examples: Given: 39060 Here, 3, 9, 0, 6 are significant digits. (The last zero is not significant.) Given: 8.0057 Here, 8, 0, 0, 5, 7 are significant digits. Given: 7.0 Here, both 7 and 0 are significant digits.
	2. Pupil learns that: a) "Relative error" is the relationship existing between the g.p.e. and the total measure given. b) "Accuracy" means the relative error, which may be expressed a a "per cent of error."	2. Model: relative error $= \frac{\text{measure of greatest possible error}}{\text{total measure given}}$
	3. Pupil demonstrates reasonable skill in: a) Determining the indicated "precision" of measurement and the "accuracy" of measurement stated in simple problems from science and industry. b) Determining greatest possible error in customary problem situations.	3. Given: 3.5 in. measured length ← (3,5 are significant digits) a) Precision = .1" reported in 3.5" b) Relative error $= \frac{.05}{3.5}$ (.05 ← g.p.e.; 3.5 ← total given measure) $= .01$ ∴ Accuracy ≈ .01 or a 1% error.

Strands And Topics	Content And Competencies To Be Developed	Suggested Background And Resource Material
E. Similar Triangles	1. Pupil learns the properties of measurement related to similar triangles. Definition: Two triangles are said to be similar if there is a one-to-one correspondence between the vertices so that corresponding angles are congruent and the ratios of the measures of corresponding sides are equal, thus making corresponding sides proportional in length.	1. Examples of similar triangles a)  $\angle A \cong \angle A'$ $m(\overline{A'B'}) = 2 \cdot m(\overline{AB})$ $\angle B \cong \angle B'$ $m(\overline{A'C'}) = 2 \cdot m(\overline{AC})$ $\angle C \cong \angle C'$ $m(\overline{B'C'}) = 2 \cdot m(\overline{BC})$
	2. Pupil demonstrates ability to indirectly by use of the properties of similar triangles.	b) An 8 ft. shadow is cast by a man who is 6 ft. tall. At the same time a building casts a shadow 40 ft. long. How high is the building?  Similar triangle problem: By definition ratios of corresponding sides must be the same. Ratio: height to shadow $\frac{6}{8} = \frac{3}{4}$ Ratio: height to shadow $\frac{y}{40} = \frac{3}{4}$ Ratios must be same $\therefore$ We solve: $\frac{y}{40} = \frac{3}{4}$ $4y = 120$ $y = 30$, height of bldg.

Strands And Topics	Content And Competencies To Be Developed	Suggested Background And Resource Material
F. Areas of Certain Polygons	1. Learning the meaning of and utilizing mathematical sentences to compute area of: a) Trapezoid: $A = \frac{1}{2} hb_1 + \frac{1}{2} hb_2$ b) Regular polygon (geometric figure whose sides have equal measure and whose angles have equal measure) as regular pentagon, etc. $A = \frac{1}{2} hp$ (p = perimeter)	1. Examples: a) Area of trapezoid  $A = \frac{1}{2} hb_1 + \frac{1}{2} hb_2$ b) Area of regular hexagon  Hexagon $A = \frac{1}{2} hp$ (p = 6b) Decagon $A = \frac{1}{2} hp$ (p = 10b)
G. Volume	1. Development of mathematical sentences for finding volume of certain "regular" solids, or "closed regions in space" 2. Pupil demonstrates reasonable facility in the use of the specific mathematical sentences: a) Hexagonal right prism: $V = Bh$ or $V = \frac{1}{2} aph$ b) Right circular cylinder: $V = Bh$ or $V = \pi r^2 h$ c) Oblique prisms: $V = Bh$ or $V = \frac{1}{2} aph$ d) Regular pyramids (Square, hexagonal, triangular, etc. bases): $V = \frac{1}{3} Bh$ e) Right circular cone: $V = \frac{1}{3} \pi r^2 h$ or $V = \frac{1}{3} Bh$ f) Sphere: $V = \frac{4}{3} \pi r^3$	1. and 2. Examples:  ← NOTE: ← B = area of base ← S = surface  Hexagonal Right Prism $V = Bh$ or $V = \frac{1}{2} aph$ (p = 6 b) $S = ap + hp$ Right Cylinder $V = Bh$ or $V = \pi r^2 h$ $S = 2 \pi rh + 2 \pi r^2$  Sphere $V = \frac{4}{3} \pi r^3$ $S = 4 \pi r^2$

Strands And Topics	Content And Competencies To Be Developed	Suggested Background And Resource Material
H. Surface Area	1. Development of mathematical sentences for finding surface area of certain "regular" solids 2. Pupil shows reasonable facility in use of: a) Hexagonal right prism: $S = ap + hp$ b) Right circular cylinder: $S = 2 \pi rh + 2 \pi r^2$ c) Sphere: $S = 4 \pi r^2$ d) Right circular cone, lateral surface only: $S = \frac{1}{2} cs$ e) Pyramids (sauare, triangular, hexagonal base), lateral surface only: $S = \frac{1}{2} ps$	 Square Pyramid $V = \frac{1}{3} Bh$ Lateral Surface $= \frac{1}{2} ps$ (not base) $(p = 4n)$ (s = alt. each face)  Right Circular Cone $V = \frac{1}{3} Bh$ or $V = \frac{1}{3} \pi r^2 h$ Lateral Surface $= \frac{1}{2} cs$ (Doesn't include base) (c = circum. of base) (s = slant height)

Strand IV, Business Arithmetic - - - - Grade 8

Strands And Topics	Content And Competencies To Be Developed	Suggested Background And Resource Material
IV. Business Arithmetic A. Budgets.	1. Short study about budgets and why they are useful.	
	2. Pupil explores personal budgets and demonstrates ability to: a) Compute amount of a student's allowance on basis of need, use, and family income. b) Use percent to compute amount out of a given income which should be spent on separate items.	2. Suggested budget practice a) Pupils use own situation for this computation. b) Use several levels of income with variation in the recommended percentages. NOTE: Use U. S. Government printing office publications on family income budgeting.
	3. a) Study of the need for and advantages of household budgets. b) Pupil demonstrates ability to compute actual amounts spent on various items from representative salaries by use of established percentages.	3. b) List divisions - food, shelter, household and operating, furnishings, clothing, health, education, recreation, personal, automobile, gifts, insurance and taxes, savings.
	4. Enrichment - Exploring a business budget and setting up sample records.	4. This item is very general and should be treated as the community situation permits.
B. Social Security, Income Tax	1. Learning about basic regulations currently governing payment of social security and income taxes.	
	2. Computing some social security taxes and benefit payments based on various age and salary factors.	2. & 3. Use actual forms put out by U. S. Government. This would be a good time to utilize qualified outside resource person.
	3. Brief introduction of income tax payments based on sample salaries.	

Strands And Topics	Content And Competencies To Be Developed	Suggested Background And Resource Material
C. Retail Buying 1. Comparative Shopping	1. Study of sample sale labels with change to "per unit" cost.	1. See Strand IV Appendix.
	2. Pupil can evaluate a "best buy" price per unit using ratio concepts.	2. Is the "better buy" 10 oz. for 65¢ or 15 oz. for 90¢. Rate pair: 10/65 ≈ 2/13 or 2 oz. for 13¢. Rate pair: 15/90 ≈ 1/6 ≈ 2/12 or 2 oz. for 12¢ ←Better "buy"!
	3. Brief introduction to the concept of "hidden costs".	3. Example: What is the hidden cost in the purchase of a sweater which will require dry cleaning as opposed to hand washing?
	4. Class evaluation of the convenience of home town shopping versus savings from mail order houses or trips to large cities for shopping.	
2. Time Payments	1. Class discussion of the usefulness of "time payments".	
	2. Pupil computes the "cost" of time payments, by using interest, carrying charge, insurance, and other possible charges.	2. True interest rate $r = \frac{2mI}{B(n+1)}$ r = annual rate as decimal; B = unpaid balance at beginning m = payments in one year; n = number of payments called for excluding down payment. I – carrying charges
	3. Pupil evaluates cost of time payments in terms of convenience (usefulness).	
3. Automobiles	1. Class considers cost and depreciation, resale, etc. of automobiles	1. By using the convenient payment plan, Mr. Jones bought a bicycle for his son. He paid $7.50 down and 15 weekly payments of $2.00 each. By paying cash he would have paid $35.00. What did the "convenience" cost him?
4. Houses	1. Class discussion of home mortgages	
	2. Pupil observes printed guide from a bank to determine amount of "safe" loan on homes.	
	3. Enrichment: Pupil learns of amortized vs fixed mortgage.	

Strand V, Ratios, Relations, And Graphs - - - - Grade 8

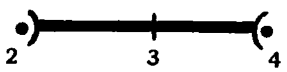
Strands And Topics

V. Ratios, Relations Graphs
  A. Graphs
    1. Number Line Graphs

Content And Competencies To Be Developed

1. Graphing compound conditions using the connectives "and" and "or" based on the meanings that in a compound condition made up of two simple conditions:
   a) The connective "and", $\wedge$, means that each solution must satisfy both conditions.
   b) The connective "or", $\vee$, means that a solution must satisfy one or the other or both of the conditions.

NOTE: When a number is not to be included in the truth set, its coordinate on the number line is marked as •) or (•. A truth set which excludes both 2 and 4 but which includes all intervening values could be indicated as



Some texts denote excluded endpoints in other ways.

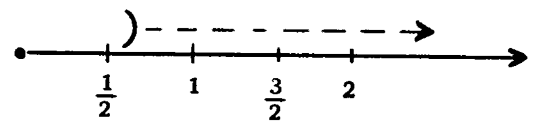
2. Use of positive real number line to graph solution sets of:
   a) Simple conditions
   b) Compound conditions

3. Recognizing and locating the solution set on number lines using intersection and union of sets

Suggested Background And Resource Material

1, 2, 3. Examples Universe = positive reals

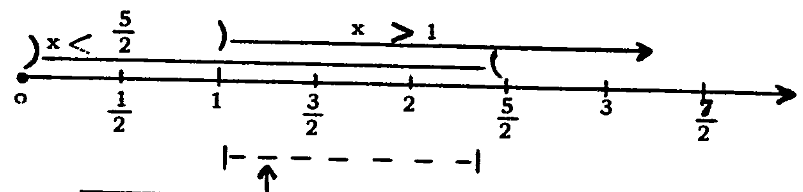
a) Given: $x > \frac{1}{2}$



Simple Condition
Solution set represented by dotted graph line.

b) $x < \frac{5}{2} \wedge x > 1$

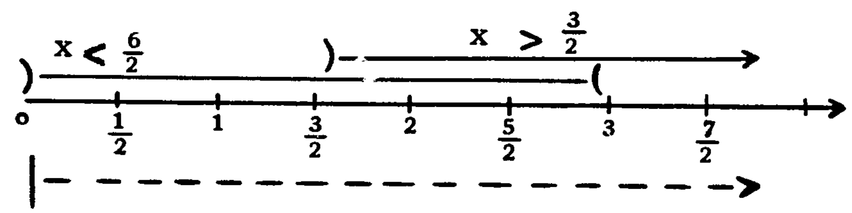
Compound Condition
(with connective "and", $\wedge$)



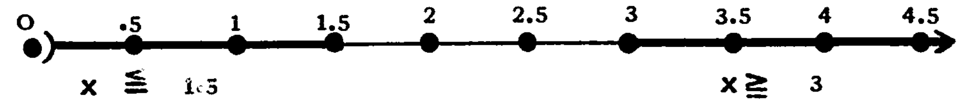
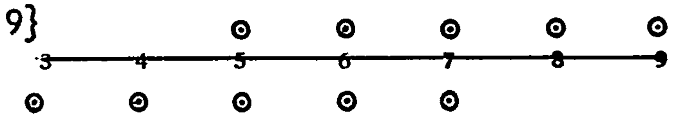
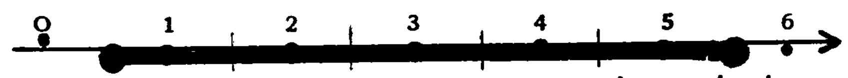
Intersection of two sets of values graphed above is solution set.

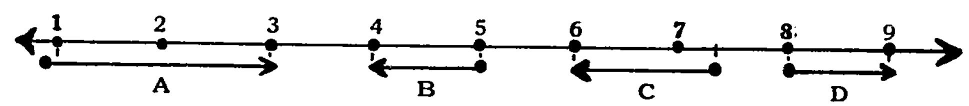
c) $x < \frac{6}{2} \vee x > \frac{3}{2}$

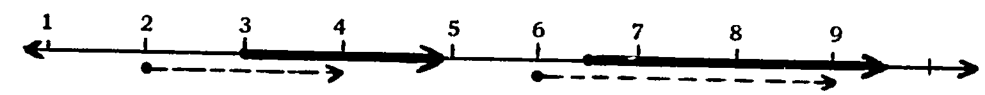
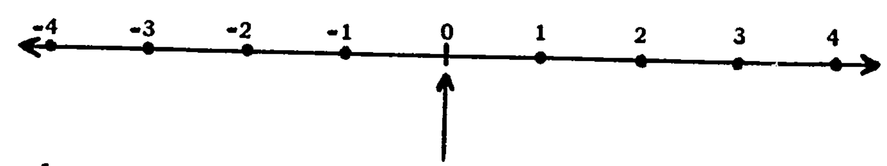
Compound Condition
(with connective "or", $\vee$)

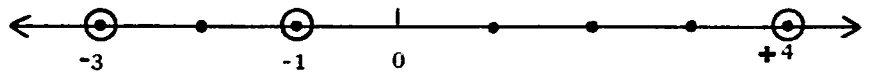
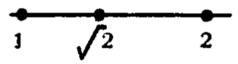
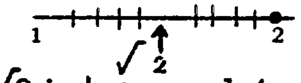
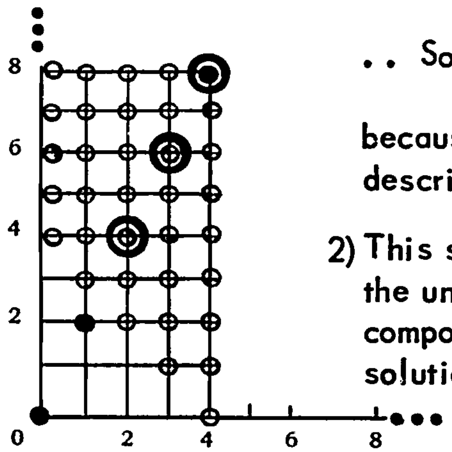


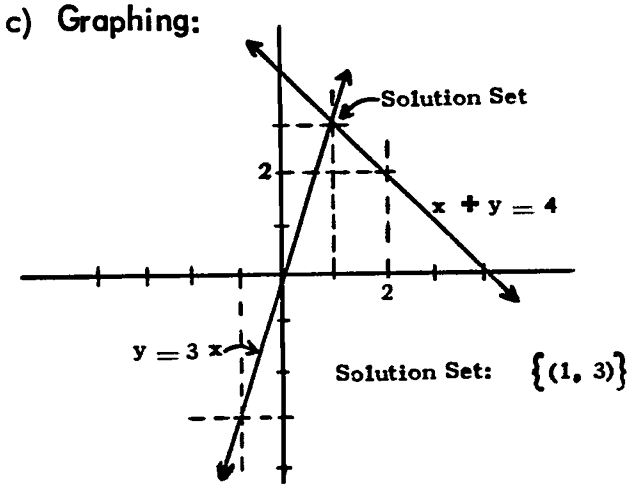
Union of two sets of values graphed above is solution set.

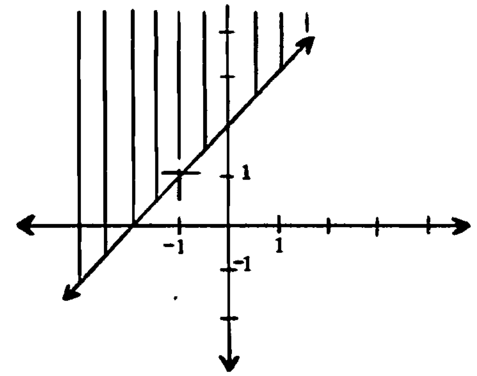
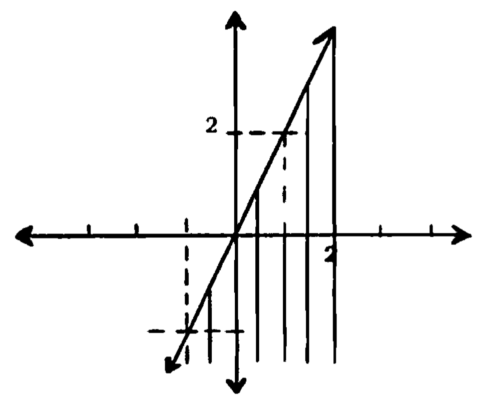
Strands And Topics	Content And Competencies To Be Developed	Suggested Background And Resource Material
A. Graphs (cont.)	4. Distinguishing between true and false statements and deciding whether solution sets are possible or impossible	
	5. Graphing "less than or equal to" and "more than or equal to" conditions	5. U = positive rationals 
	6. Tabulating complements from a number line Definition: The complement of a given subset is the set that contains all the members of the universe, and only those members, that do not belong to the given subset.	6. U = $\{3, 4, 5 \cdots 9\}$ Given: $x > 4 \wedge x < 8$  Solution Set = $\{5, 6, 7\}$ $\overline{\{5,6,7\}} = \{3,4,8,9\}$ — complement tabulated Read: Complement of set whose members are 5,6,7 is equal to the set whose members are 3,4,8,9.
	7. Graphing intervals on line representing positive real numbers a) Closed intervals with limits Definition: The set of real numbers that contains two given positive real numbers and all the real numbers between the two given numbers is a closed interval in positive real numbers.	7. U = all positive real numbers a) Given: $\frac{1}{2} \leqq x \leqq 5\frac{1}{2}$ or $x \geqq \frac{1}{2} \wedge x \leqq 5\frac{1}{2}$  A closed interval contains the two given numbers which are called the "limits" of the interval.

Strands And Topics	Content And Competencies To Be Developed	Suggested Background And Resource Material
A. Graphs (cont.)	b) Half-open interval with limits Definition: The set of numbers that contains a given positive real number and all the real numbers between that number and another given positive real number, is a half-open interval in positive real numbers.	b) Given $3 \leqq x < 6$ or $x \geqq 3 \wedge x < 6$  A half-open interval contains only one of its "limits." This limit is contained in interval This limit is not contained in interval
	8. NOTE: Textbooks develop the "oppositeness" of graphs of positive and negative integers in various ways. Teachers are urged to alert pupils to this fact. The following development is included because of the great usefulness of the "ordered - pair" idea. Developing the meaning of a directed segment on a number line as the graph of an ordered pair of numbers which represents: a) A positive real number if the segment extend in an agreed-upon direction along the number line b) A negative real number if the segment extends in oppoaite direction from a) above	8. NOTE: Let us agree that for this problem, graphs of positive real numbers extend to the right.  Positively directed segments: A, D Negatively directed segments: B, C
	9. Comparing directed segments which differ only in direction.	9. See 8 above: Length of segment B = length of segment D Positive and negative properties come from oppositeness of direction on number line.

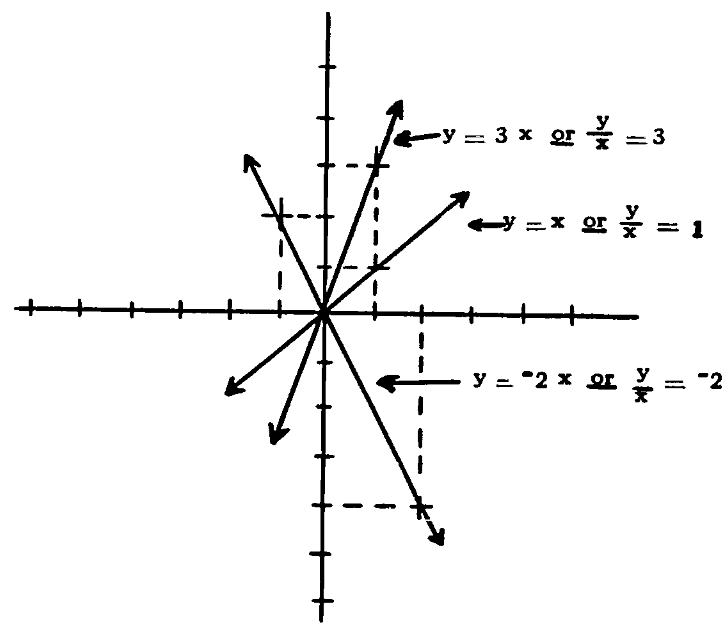
Strands And Topics	Content And Competencies To Be Developed	Suggested Background And Resource Material
A. Graphs (cont.)	10. Recognition of equivalent directed segments and equivalent ordered pairs indicating directed segments	10.  Let us agree that 1st no. of ordered pair represents the terminal point and that the second no. represents the beginning point. From above we see: (4,2) (5,3) also: (9, 6) $(9\frac{1}{2}, 6\frac{1}{2})$ We see that $4 + 3 = 2 + 5$ and that $9 + 6\frac{1}{2} = 6 + 9\frac{1}{2}$ If the sum of one set of first and last components equals the sum of the other set of first and last components of two ordered pairs representing directed segments, then the ordered pairs are equivalent. ∴ If two ordered pairs for directed segments are equivalent, the indicated directed segments are equivalent.
	11. Ordering the integers on the number line NOTE: a) Non-negative integers may be represented by the set (0,1,2,3, . . .) b) Negative integers have an "exactly opposite" meaning and so are represented as $(\ldots {}^-3, {}^-2, {}^-1)$	11.  Set of negative integers = $\{\ldots {}^-3, {}^-2, {}^-1\}$ See each integer as 1 unit less than its predecessor moving left from zero on number line. NOTE: Zero is an integer but is <u>not</u> positive or negative. Set of positive integers = $\{1,2,3, \ldots\}$ See each integer as 1 unit larger than its predecessor moving right from zero on number line.

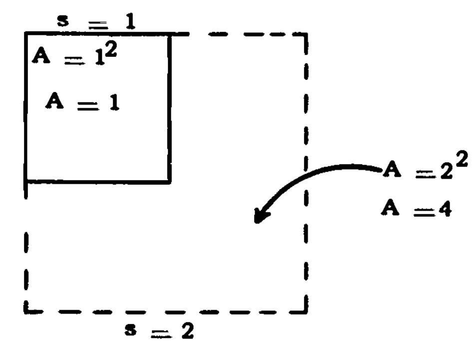
A. <u>Graphs</u> (cont.)	12. Graphing integers on number line	12. To graph integers: -3, + 4, -1 
	13. Locating irrational numbers such as $\sqrt{2}$ on the number line	13. To locate $\sqrt{2}$ on number line: ($\sqrt{2}$ is an infinite, non-repeating decimal) Let us take just three of many steps. $\sqrt{2} \approx 1.4142$ a)  $\sqrt{2}$ is between 1 and 2. b)  $\sqrt{2}$ is between 1.4 and 1.5 c)  $\sqrt{2}$ is between 1.41 and 1.42
2. Coordinate Plane Graphs With Universe N X N	1. Graphing Cartesian set and locating solution set of compound condition in two variables 2. Ability to distinguish between finite and infinite sets	1,2,3. Universe for (x,y) = N X N (Read: N cross N) 1) To graph the compound condition $x + y > 3 \wedge y = 2x$ ooo shows condition of $x + y > 3$ . . . shows condition of $y = 2x$ = Solution set of $x + y > 3 \wedge y = 2x$ . . Solution set = {(2,4), 3,6), (4,8)} because this is intersection of sets described by simple conditions  2) This solution set is infinite because the universe is infinite and the compound condition doesn't <u>limit</u> the solution to a maximum size.

Strands And Topics	Content And Competencies To Be Developed	Suggested Background And Resource Material
2. Coordinate Plane Graphs With Universe N X N (cont.)	3. Developing concept of incomplete graphs as representing part of a Cartesian plane or part of a solution set NOTE: Concept is especially useful in work with infinite sets.	3. The three dots at the end of the axes in no. 1 above indicate that the graph of N X N is an infinite (unending) set. The graph of the solution set in no. 1 is called an incomplete graph because the entire graph of the solution set is not shown.
3. Coordinate Plane Graphs (Universe Is Set of all Real Numbers)	1. Graphing Cartesian set using real numbers Definition: Real numbers can be expressed by infinite decimals, repeating or non-repeating.	1. Suggestion: a) Review meaning and use of incomplete planes and graphs. See nos. 1, 2, 3 above. b) Explore the need for "axes" as referrant or beginning lines. c) Develop need for representation and use of "quadrants" in describing position.
	2. Learning the need for two graph lines when representing a compound condition in two variables 3. Learning to locate the intersection of graph lines on a cordinate plane for the solution set of a compound condition	2, 3. Example: a) Algebric solution of condition in two variables $y = 3x \wedge x + y = 4$ $y = 3x \wedge x + 3x = 4$ $y = 3x \wedge x = 1$ $y = 3 \wedge x = 1$ b) To locate intersection of lines: Sketch $y = 3x$ using points $(^-1, ^-3)$ and $(0,0)$ Sketch $x + y = 4$ using points $(2,2)$ and $(1,3)$ c) Graphing: 

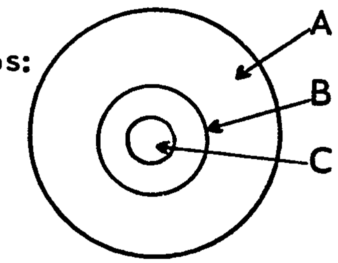
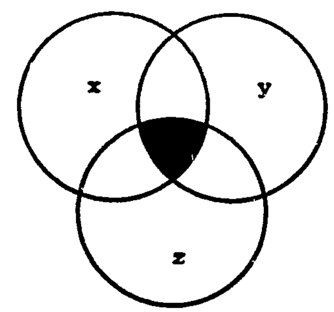
Strands And Topics	Content And Competencies To Be Developed	Suggested Background And Resource Material
3. Coordinate Plane Graphs With Universe the Set of Real Numbers (cont.)	4. Learning to illustrate sections of coordinate plane represented by inequalities	4. a) To show $\{(x,y) \mid y > x + 2\}$  Graphed set: $\{(^-1, 1), (0, 2)\}$ Graph of inequality is indicated by lined region but does <u>not</u> include graph line. Why not? b) To show $\{(x, y) \mid y < 2x\}$  Graphed set: $\{(^-1, ^-2), (1, 2)\}$ Graph of inequality is indicated by lined region but does <u>not</u> include graph line. Why not?

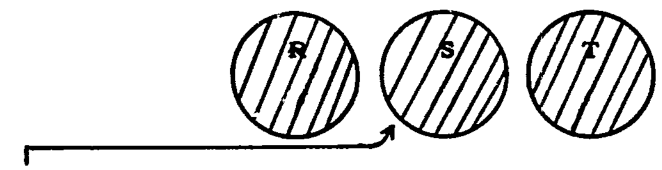
Strands And Topics	Content And Competencies To Be Developed	Suggested Background And Research Material
B. Relations 1. Ratios and Equivalences	1. Setting up and solving "rate pair" problems by use of ratios and equivalences	1. See Strand V Appendix
	2. Learning to set up and solve compound conditions by use of equivalences	2. Example: Bill wants to buy a motor-scooter. He can buy the machine at either of 2 stores for \$180. In one store he can pay $\frac{1}{9}$ of the cost each month. At the other store he can pay $\frac{1}{6}$ of the cost each month. What can his monthly payments be? $\frac{1}{9} : 1 \sim x : 180 \vee \frac{1}{6} : 1 \sim x : 180$ $x = \frac{180}{9}$ or $x = \frac{180}{6}$ $x = \$20$ payment $x = \$30$ payment
	3. Setting up and solving compound conditions involving complex fractions by use of equivalences	3. Pete and Joe work together mowing lawns in the summer. Pete works half as fast as Joe so they know that Pete does $\frac{1}{3}$ of each job while Joe does $\frac{2}{3}$ of it. They can mow a large lawn in $\frac{1}{2}$ day. At the end of $\frac{1}{4}$ day, one boy is called away. What part of the job has each boy finished by that time? $\frac{\frac{1}{3}}{\frac{1}{2}} \sim \frac{x}{\frac{1}{4}} \wedge \frac{\frac{2}{3}}{\frac{1}{2}} \sim \frac{x}{\frac{1}{4}}$ Soln: $\frac{1}{2}x = \frac{1}{12}$ (Pete) $\frac{1}{2}x = \frac{2}{12}$ (Joe) $x = \frac{1}{6}$ of job $x = \frac{4}{12}$ or $\frac{1}{3}$ of job

Strands And Topics	Content And Competencies To Be Developed	Suggested Background And Resource Material
1. Ratios and Equivalences (cont.)	4. Setting up and solving conditions involving three variables	4. Example: The sum of 3 natural numbers is 3. The sum of the first and second numbers is less than 3. The second number is less than 2. Compound Condition: $x + y + z = 3 \wedge x + y < 3 \wedge y < 2$ $U = N$ Using given conditions $y = 0$ or 1, $x = 0, 1, 2,$ $z = 1, 2, 3$ Solution set of ordered triples for (x, y, z) is $\{(0, 0, 3), (1,0,2), (2,0,1), (0,1,2), (1,1,1)\}$
	5. Developing some knowledge of changes in graphs of $y = K X$ on coordinate plane as K changes. (K is a constant.) NOTE: If $Y = K X$, then $\frac{y}{x} = K$ if $x \neq 0$. Thus we are considering how the graph of $\frac{y}{x} = K$ changes with variations in K.	5. Example: 

Strands And Topics	Content And Competencies To Be Developed	Suggested Background And Resource Material
1. Ratios and Equivalences (cont.)	6. Class explores role of constants in proportional geometric figures.	6. Example: Consider two squares, one with side 1, the other with side 2.  $A = 1^2$, $A = 2^2$ — constants Doubling the side of a square results in quadrupling the area. That is, if ratio of sides of two squares is 1:2, the ratio of of their areas is 1:4.
	7. Construction of simple maps with same ratio on map as in reality.	7. Example: Map scale of 50′ to 1″ would mean $1\frac{4}{5}$″ to represent 90′.

Strands And Topics	Content And Competencies To Be Developed	Suggested Background And Resource Material
VI. Sets A. Sets of Points In Geometry	1. Learning that geometric figures are defined as the union of the sets of points which comprise the line segments known as "sides " See Grade 7, Strand II, Geometry	1. Example: A plane "triangle" is frequently defined as, "The union of the three segments determined by any three noncollinear points," "Segment" may be defined as "a set of points whose members are two given points and all the points between these two points."
B. Complement of a Set	1. Exploration of the meaning of the complement of a given subset of a universe as being the set that contains all the members of the universe, and only those members, that do not belong to the given subset.	1. Example: If the universal set is the alphabet and if V is the set of vowels, then the complement of V is the set of letters that are not vowels. The complement of V is written V' or $\overline{V}$ and is read "The complement of V" or "V bar."
	2. By exploration discovering the meaning of specific set complements as: a) The complement of an empty set will be the universal set. b) The complement of the universal set will be the empty set. c) The complement of a complementary set will be the given set.	2. a) If $U = \{1,2,3,4\}$ and $A = \{\ \}$, then $\overline{A} = \{1,2,3,4\}$ b) If $U = \{1,2,3,4\}$ and $A = \{1,2,3,4\}$ then $\overline{A} = \{\ \}$ c) If $U = \{1,2,3,4\}$ and $A = \{1,2,3\}$, then $\overline{A} = \{4\}$ The complement of $\overline{A}$ will be all the members of the universe that are not in $\overline{A}$ or $\{1,2,3\}$ which is A. $\therefore$ the complement of a complementary set is the given set.
C. Closure Property of Sets	1. Development of pupil recognition that in operations with sets the closure property holds as: a) The union of 2 sets results in the set composed of all objects which are members of either or both of the given sets. b) The intersection of 2 sets results in the set composed of only those objects which are members of both of the given sets.	1. a) If $A = \{1,2,3,4\}$ and $B = \{3,4,5,6\}$ then $A \cup B = \{1,2,3,4,5,6\}$ b) If $A = \{1,2,3\}$ and $B = \{7,8,9\}$, then $A \cap B = \{\ \}$

Strands And Topics	Content And Competencies To Be Developed	Suggested Background And Resource Material
C. Closure Property of Sets (cont.)	c) In set complementation: 1) The complement of given subset is the set composed of all members of the universe which are not members of the given subset. 2) The complement of a complementary set is the given subset.	c) If U = $\{1,2,3,4,5,6\}$ and A = $\{1,3,5\}$, then $\overline{A} = \{2,4,6\}$ If $\overline{A} = \{2,4,6\}$ then the complement of $\overline{A} = \{1,3,5\}$, which is Set A.
D. Relationships Between Three Sets	1. Exploration by means of Venn diagrams, of: a) Various subset relationships. b) A possible relationship under the intersection of three sets.	1. a) Subset relationships:  A = Set of all boys in New Mexico B = Set of all boys in this school C = Set of all boys in this math class. b) Let x = set of girls in this school y = set of all girls named Mary. z = set of all girls who are 14 years old. Shaded are represents set of girls in this school who are named Mary and who are 14 years of age. 

Strands And Topics	Content And Competencies To Be Developed	Suggested Background And Resource Material
D. Relationships Between Three Sets (cont.)	c) Possible relationships under the union of three sets.	c) The union of three sets shown in a) above, would be Circle A. The union of three sets in b) above would be the total membership of sets x, y and z.  The union of disjoint sets R, S, T would be the total membership of R, S, T
E. Cartesian Set And Compound Conditions	1. Pupil learns necessary basic concepts: a) The symbol " $\in$ " means "is a member or element of a given set." b) Compound conditions require that each element of a solution set be selected in the light of two or more conditions. See Grade 7, Strand VII, C.	1. a) The symbol " $\in$ " is one form of the Greek letter "epsilon." b) A frequent compound condition is the operation of set intersection. If $K = \{3,5,7,9\}$ and $L = \{1,3,7,5,9,11\}$, then $K \cap L =$ $\{3,5,7,9\}$ The members of the new intersection set belong to both given sets.
	c) Meaning of and importance of Cartesian set in determining solution set for separate conditions of a compound condition. See Grade 8 Strand VII, A	c) See Grade 7, Strand VI, L for explanation and example of Cartesian set.
	2. Attainment of reasonable skill in stating and solving compound-condition problems a) Recognition that connective "and" (symbolized by $\wedge$) requires each member of solution set to meet both given conditions	2. a) $x + y < 3 \wedge x > 1$, $U = N \times N$ Soln. set for $x + y < 3$ is $\{(0,0), (0,1), (0,2), (1,0), (1,1), (2,0)\}$ but the second condition, $x > 1$, indicates soln. set for both conditions is $\{(2,0)\}$

Strands And Topics	Content And Competencies To Be Developed	Suggested Background And Resource Material
E. Cartesian Set And Compound Conditions (cont.)	b) Recognition that condition "or" (symbolized by V) requires each member of solution set to meet at least one of the given conditions	b) $x + y < 3 \ \vee \ x + y > 5$ means the total solution set would contain the six members listed in 2 a) and the infinite solution set for $x + y > 5$
F. Set Sequences (Enrichment Material)	NOTE: This material is suggested for the able class, or as enrichment for a group within a class.	
	1. Meanings of "sequence vocabulary" as: a) Defn.: A "sequence" is a set of objects arranged in a definite order so that there is a first member, a second member, etc. b) Defn.: "Terms of a sequence" are the members of the sequence. c) Defn.: "Successor of a given term" is a term that immediately follows a given term in a sequence.	1. Examples: a) $x = \{10,8,6,4,2\}$ is a sequence of even numbers arranged in the order of decreasing size. b) Terms of the above sequence are: 10, 4, 6, 8, 2 c) In the sequence in a) above, 8 is the successor of 10, 2 is the successor of 4, 6 is the successor of 8, and 4 is the successor of 6.
	2. Recognition of the difference between sequence types as: a) Defn.: A finite sequence is a finite set. b) Defn.: An infinite sequence is an infinite set.	2. a) A finite sequence is a countable number of objects arranged in a definite order. $\{15, 18, 21, 24 \cdots 39\}$ is a finite sequence. b) $\{2,4,6,8,10 \cdots\}$ is an infinite sequence because it is a set which continues endlessly, in which members are arranged in definite order.

Strands And Topics	Content And Competencies To Be Developed	Suggested Background And Resource Material
VII. Mathematical Sentences A. Simple and Compound Conditions	1. Extension of pupil ability to express and solve simple and compound conditions in one and two variables and to tabulate their solution sets.	1. NOTE: Simple conditions express a single idea about a quantity such as, $x + 2 < 6$, $x + 2 = 6$, $x + 2 > 6$, $x + 2 \neq 6$

Suggested Background And Resource Material (continued)

Compound conditions express two or more ideas about a quantity as: $x + 2 > 6 \wedge 6 + x < 12$

Examples of problems, solutions, tabulation of solution sets:

a) In seven years Kathy will be more than 18 years old. Six years ago she was less than 9 years old. How old can Kathy be now? Let x = no. of years that Kathy's present age can be

$$\text{Then } x + 7 > 18 \wedge x - 6 < 9$$

$$x + 7 > 18 \qquad x - 6 < 9$$

$$x = \{12, 13, 14, \ldots\} \qquad x = \{14, 13, 12, 11, 10, 9, 8, 7, 6\}$$

Each member of solution set of a compound condition must satisfy <u>both</u> simple conditions.

$\therefore$ Solution Set = $\{12, 13, 14\}$

b) Greg and Joe went hunting together. They killed fewer than 5 rabbits. How many rabbits could each of the boys have killed?

Let x = no. of Greg's rabbits and y = no. of Joe's rabbits
Then: $x + y < 5$ We see that there are several combinations for this sentence so we make a table.

Possible no. of Greg's rabbits → row x
Possible no. of Joe's rabbits → rows y

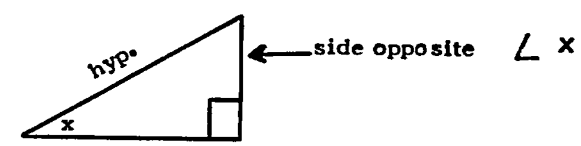
x	0	1	2	3	4
y	0	0	0	0	0
	1	1	1	1	
	2	2	2		
	3	3			
	4				

Soln. Set = $\{(0,0), (0,1), (0,2), (0,3), (0,4), (1,0), (1,1), (1,2), (1,3), (2,0), (2,1), (2,2), (3,0), (3,1), (4,0)\}$

Strands And Topics	Content And Competencies To Be Developed	Suggested Background And Resource Material
A. Simple and Compound Conditions (cont.)	2. Extension of pupil facility in use of number systems as: a) C represents counting numbers. b) C X C (C cross C) means the matching of each member of the counting numbers, in turn, with each member of the counting numbers. c) N means the natural numbers. d) N X N (N cross N) means the matching of each member of the natural numbers, in turn, with each member of the natural numbers. e) R_a is used by some authors as the name of the set of all possible rational numbers of arithmetic.	2. a) $C = \{1,2,3 \cdots\}$ b) $C \times C = \{(1,1), (1,2), (1,3) \cdots (2,1), (2,2), (2,3) \cdots\}$ c) $N = \{0,1,2 \cdots\}$ d) $N \times N = \{(0,0), (0,1), (0,2) \cdots (1,0), (1,1), (1,2) \cdots\}$ e) A set of a few $R_a = \{\frac{0}{3}, \frac{1}{6}, \frac{4}{4}, \frac{19}{3}, \frac{5}{1}\}$ Denominators $\neq 0$
	3. Learning meanings of the connectives: a) Defn: V represents the connective "or". If "or" is used as the connective, the solution set of the ccompound condition is the union of the two sets described by the simple conditions.	3. a) $x + 2 > 6 \vee 4 + x < 2$ $U = C$ For $x + 2 > 6$, solution set $= \{5,6,7 \cdots\}$ For $4 + x < 12$, solution set $= \{7,6,5,4,3,2,1\}$ $\therefore$ for $x + 2 > 6 \vee 4 + x < 12$, solution set $= \{1,2,3 \cdots\}$
	b) ~ is used to express the idea, "not."	b) Notice that the same symbol is used to represent "not" and to indicate equivalence. The conditions expressed by a problem and/or the way the symbol is used in a sentence determine whether ~ expresses the idea of not or the idea of equivalence.
	4. Writing and solving conditions using positive and negative integers	4. Example: On the first down, the (Artesia) football team gained 9 yards. On the second down (Artesia) lost 6 yards. How many yards did the team gain on the two downs? x = no. yds. gained on 2 downs Then: $9 + (^{-}6) = x$ $3 = x$ or 3 yds. gained on 2 downs

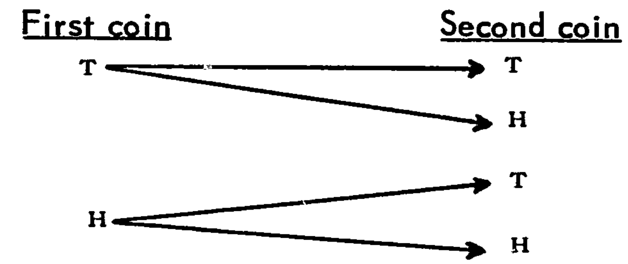
Strands And Topics	Content And Competencies To Be Developed	Suggested Background And Resource Material
B. Conditions For Equivalence	1. Extension of pupil ability to write and solve mathematical sentences involving conditions of equivalence in one and two variables, and to tabulate solution sets	1. Example: Mary took a math test and had 5 times as many correct as incorrect answers. She had at most 15 correct answers. How many incorrect answers did she have? Let x = no. of correct answers and y = no. of incorrect answers. Then $\frac{5}{1} \sim \frac{x}{y} \wedge x \leqq 15$ We see that the $\frac{5}{1}$ ratio indicates the proportional relation $\{\frac{5}{1}, \frac{10}{2}, \frac{15}{3}, \frac{20}{4}\}$ Since Mary had 15 or fewer correct answers, we see that $\frac{x}{y} = \{\frac{5}{1}, \frac{10}{2}, \frac{15}{3}\}$ $\therefore$ y (no. incorrect answers) = $\{1,2,3\}$
	2. Pupil uses rate pairs and equivalent rate pairs in writing mathematical sentences. NOTE: See Grade 8, Strand V, Section B.	2. Examples: a) If 4 cans of milk cost 42¢, what is the cost of 1 can? Let x = cost of 1 can milk Then: $\frac{4}{42} \sim \frac{1}{x}$ $4x = 42¢$ $x = 10\frac{1}{2}¢$ and $S = \{10\frac{1}{2}¢\}$ b) How many centimeters in 16 in.? (.394 in. ≈ 1 cm.) Let x represent no. of cm. Then: $\frac{1}{.394} \sim \frac{x}{16}$ $.394x = 16$ and $x = 40.6$ cm. 16 in. ≈ 40.6 cm.

Strands And Topics	Content And Competencies To Be Developed	Suggested Background And Resource Material
C. Conditions of Direct and Inverse Variation	1. Learning that direct variation is expressed as conditions of the general forms: $y = K x$ and $y = K x^2$	1. NOTE: In $y = K x$, $I = 12 f$ — constants Here I changes or varies directly as f changes. The 12 does not change so it is called the constant of variation.
	2. Learning that inverse variation is expressed as conditions of the general forms: a) $y = K \bullet \frac{1}{x}$ or $y = \frac{K}{x}$ b) $y = K \bullet \frac{1}{x^2}$ or $y = \frac{K}{x^2}$	2. Notice that $\frac{1}{x}$ is the multiplicative inverse of x. Therefore, we say that y varies inversely as x varies. As x grows very large, the value of $\frac{1}{x}$ becomes very small and the value of y becomes small. If $x = 2$, then $y = 3 \bullet \frac{1}{x} \rightarrow y = 3 \bullet \frac{1}{2}$ or $y = \frac{3}{2}$ If $x = 1796$, then $y = 3 \bullet \frac{1}{x} \rightarrow y = 3 \bullet \frac{1}{1796}$ or $y = \frac{3}{1796}$
	3. Learning to write and solve conditions for direct and inverse variation	3. Examples: a) The distance traveled by a falling object, starting from rest, varies directly as the square of the time it has been falling. A rock falling from the top of a cliff fell 300 ft. in the first 5 seconds of fall. How many feet did the rock travel in the first 10 seconds of fall? Let x = time and y = distance fallen $y = k x^2$ $300 = k \bullet 25$ $12 = k$ → For the 10 seconds of falling $y = k x^2$ $y = 12 \bullet 100$ $y = 1200$ ft. fall in 10 seconds

Strands And Topics	Content And Competencies To Be Developed	Suggested Background And Resource Material
C. Conditions of Direct and Inverse Variation (cont.)		b) Three boys can wax a car in $2\frac{3}{4}$ hours. In how many hours can 2 boys wax the same car? Let x = no. of boys and let y = no. hours it takes to wax the car. We see that fewer boys will need a longer time so we have an inverse variation. For 3 boys: $y = \frac{k}{x}$; $2\frac{3}{4} = \frac{k}{3}$; $8\frac{1}{4} = k$ For 2 boys: $y = \frac{8\frac{1}{4}}{2}$; $y = 4\frac{1}{8}$ hrs. for 2 boys to wax car.
D. Formulas	1. Pupil recognition that: a) Formulas are mathematical sentences which indicate how to solve various types and classes of problems. b) The frequency of their use in mathematics and in business make some formulas especially important.	1. a) $d = rt$ (distance) b) $p = s_1 + s_2 + s_3$ (perimeter of scalene triangle) c) $\frac{\sin \angle x}{1} \sim \frac{\text{side opposite } \angle x}{\text{hypotenuse}}$  d) $I = prt$ (interest) e) $F = \frac{9}{5}(c + 32)$ (centigrade to Farenheit) f) $c = \pi d$ c and d are variables representing circumference and diameter. π represents the ratio of circumference to diameter.

Strands And Topics	Content And Competencies To Be Developed	Suggested Background And Resource Material
VIII. Probability A. Meaning	1. Learning that probability is the mathematical science that deals with predicting the likelihood (or chance) of the happening of an event.	1. Class discussion: If the names of 18 boys and 12 girls in a math class are placed on separate cards in a box, mixed thoroughly, and a card is drawn "sight-unseen," what is the chance that the name will be that of a girl?
B. Basic Concepts	1. Some study of likelihood of specific event occurring with attention on each of the following definitions: a) "Chance event" or "event" is possible happening or occurrence. b) "Outcome" is the result of some action. c) "Favorable (or successful) outcome" is the result whose likelihood is being considered. d) "Unfavorable (or unsuccessful) outcome" is any result other than the desired result. e) "Probability" is the measure of chance that a specific outcome will occur.	1. a) Discussion: In how many ways can two tossed coins turn up? Class conclusion - These possible ways of turning up are called "chance events" since they may result from the tossing: heads-heads, heads-tails, tails-heads, tails-tails. b) If one marble is to be chosen from a box containing 1 red marble, and 1 white marble, there are two possible "outcomes"; taking the red marble or taking the white marble. c) In b, above, if we wish to take the red marble and do take it, the choice is called a "favorable outcome." d) In b, above, if we wish to take the red marble but take the white one, the choice is called an "unfavorable outcome."
C. "Equal Chance" Probability	1. Development of the ideas that: a) "Equal chance" probability refers to those situations in which equal likelihood is assumed for all possible outcomes of an action.	1. a) Suggestion: See Probability Appendix Discussion: Is the particular outcome of an action described below equally likely with other possible outcomes of the action? Obtaining a head on a toss of an honest coin. Seeing the sun rise on looking east, at the proper time tomorrow morning. Having a test on meeting of this class on Tuesday.

Strands And Topics	Content And Competencies To Be Developed	Suggested Background And Resource Material
C. "Equal Chance" Probability (cont.)	b) The probability of a "favorable outcome," A, is written as the quotient: $P(A) = \frac{\text{no. of ways a favorable outcome can occur}}{\text{(total no. ways either a favorable or an unfavorable outcome can occur)}}$ Sometimes it is more convenient to speak of the probability of the occurrence of a favorable event, E. Then: $P(E) = \frac{\text{no. of ways the favorable event can occur}}{\text{(total no. of ways the event can occur, favorably or unfavorably)}}$	b) In tossing a coin for which we desire heads to turn up, we write: $P(\text{heads}) = \frac{1}{2}$ ← No. of ways favorable outcome can occur / ← Total no. of ways either favorable or unfavorable outcome can occur
	c) The probability of an "unfavorable outcome" (not A), is written as the quotient: $P(\text{not } A) = \frac{\text{no. of ways unfavorable outcome can happen}}{\text{(total no. of ways either favorable or unfavorable outcome can happen)}}$ It may be convenient to speak of the probability of the non-occurrence of a certain event, E. Then: $P(\text{not } E) = \frac{\text{no. of ways unfavorable event can occur}}{\text{(total no. of ways event can occur, favorably or unfavorably)}}$	c) In tossing a coin for which the turning up of tails is regarded as an "unfavorable outcome" (not A) we write: $P(\text{not } A) = \frac{1}{2}$ ← No. of ways unfavorable outcome can occur / ← Total no. of ways either favorable or unfavorable outcome can occur.
	d) If an event, E, is certain to happen, the probability is written: $P(E) = 1$ e) If an event (E) cannot occur, the probability is written: $P(E) = 0$	d) Since a "certain" event must happen, there is only one possible outcome. ∴ P (event certain to happen) = 1 e) If a specific event cannot occur, we know there is no probability that it will occur. ∴ P (impossible event) = 0 In drawing a marble out of a bag of green marbles: P (drawing a green marble) = 1, but P (drawing a red marble) = 0

Strands And Topics	Content And Competencies To Be Developed	Suggested Background And Resource Material
C. "Equal Chance" Probability (cont.)	2. Development of the generalization: The probability of an event, A, is a ratio whose value is between 0 and 1, inclusive. ∴ $0 \leq P(A) \leq 1$	2. Class discussion: See Probability Appendix for array of events for which probability varies from 0 to 1.
D. Determing Possible Outcomes	1. Introduction to the "branching" or "routes of occurrence" property of multiple events NOTE: See Probability Appendix	1. a) Possible outcomes of single coin toss: T H ∴ 2 outcomes, no "branching" b) Possible outcomes of 2-coin toss:  ∴ 4 possible outcomes as result of "branching."
	2. Recognition of the general pattern of the Fundamental Counting Property. a) Definition: "If a first event can occur in s ways, and if, after the first event has occurred, a second event can occur in t ways, then the two successive events can occur in s x t ways." b) The pattern of the Fundamental Counting Property can be extended to any number of successive events.	2. a) In the coin problem, above, we see that: Possible outcomes from tossing a single coin = 2 Possible outcomes from tossing 2 coins = 2 x 2 or 4 b) See Probability Appendix

Strands And Topics	Content And Competencies To Be Developed	Suggested Background And Resource Material
D. Determining Possible Outcomes (cont.)	3. Reasonable facility in use of "sample space" concepts as: a) Definition: The sample space of an experiment is the set whose elements are all the possible outcomes of the experiment. b) Definition: If the elements of a sample space are ordered pairs, then the elements of the sample space are called sample points. c) Definition: An event is a subset of the sample space.	3. Example: Four green cards numbered 1, 2, 3, 4 respectively are placed in one pile and three white cards numbered 1, 2, 3 respectively are placed in a second pile. What is the probability that a blindfolded, simultanous choice of 1 card from each pile will result in the numeral 2 on the white card? (see array below)
E. Empirical Probability	1. Reasonable grasp on part of pupils of basic concepts of "empirical" probability as: a) The measure of probability determined for situations in which it is not practical to list all outcomes. b) A measure based on information about a "sample" of the population. c) A reliable measure, if sample is of appropriate size and selection.	1. See Probability Appendix a) and b) Is it possibie to make reasonably accurate predictions 1) of baseball player's next batting performance if his batting average for the season is known? 2) of tomorrow's weather on the basis of today's and yesterday's weather? c) Lead to intuitive conclusion that size and selection of sample should vary with situation.

Total array is sample space for drawing a white card followed by a green card!

W \ G	1	2	3	4
1	(1,1)	(1,2)	(1,3)	(1,4)
2	(2,1)	(2,2)	(2,3)	(2,4)
3	(3,1)	(3,2)	(3,3)	(3,4)

← Sample Space for drawing a green card

Sample Point → (1,4)

Hatched region (row 2) = event, A, or desired outcome

We see that $U = W \times G$ and event $A = \{ (W,G) \mid W = 2 \}$

$\therefore P(A) = \frac{4}{12}$ ← 4: favorable outcomes (sample points); 12: total possible outcomes, (total no. of sample points in the sample space for the successive events)

Strands And Topics	Content And Competencies To Be Developed	Suggested Background And Resource Material
E. Empirical Probability (cont.)	2. Recognition that empirical probability is written as a ratio: $P\ (\text{event}) = \frac{\text{no. times event appeared in sample}}{\text{total no. elements in sample}}$	2. Example: The record of a local weather station shows that in the past 100 days its weather prediction has been correct 74 times. On the basis of this "sample" we assume that: P (tomorrow's prediction being correct) $= \frac{74}{100} \approx \frac{3}{4}$
	3. Some knowledge of importance of; a) Sampling b) Appropriate size of sample c) Random selection of sample	3. See Probability Appendix.
F. Probability of Events In Relation To Each Other	1. Learning that the "equal chance" probability of an event, A, is stated: $P\ (A) = \frac{\text{no. of ways event can occur}}{\text{no. of all possible outcomes of experiment}}$	1. See section C, 1, above.
	2. Learning that the probability of one out of two mutually exclusive (disjoint) events, A or B, is stated: $P\ (A \cup B) = P\ (A) + P\ (B)$	2. "Mutually exclusive" means the two events can't happen at same time. This means A and B are disjoint. See Probability Appendix.
	3. Learning that probability of occurrence of both of two independent events, A and B is stated $P(A \cap B) = P\ (A) \bullet P\ (B)$	3. In tossing 2 coins, probability that both will show heads comes from: 1st. coin. P (heads) $= \frac{1}{2}$ and 2nd. coin, P(heads) $= \frac{1}{2}$ $\therefore$ P (heads and heads) $= \frac{1}{2} \bullet \frac{1}{2} = \frac{1}{4}$ by formula. This can be checked by considering that possible outcomes of two-coin toss are: (H,H), (H,T), (T,H), (T,T). Hence, by considering the sample space, P (heads and heads $= \frac{1}{4}$

Mathematics Terms And Concepts Used In Guide

Strand I

Factor, exponent, multiple, lowest common multiple, greatest common factor

Number and numeral, natural or whole numbers, number base, place value, sustem of numeration, expanded notation

Operation and inverse operation

Prime number, composite number, unique prime factorization

Properties of closure, commutativity, associativity, distributivity, identity element, inverse element

Positive and negative integers, number line, absolute value

Ratio, rational number, reciprocal

Strand II

Altitude, diagonal, perimeter, area, volume

Angles: complementary, supplementary, corresponding, central, vertical, adjacent

Betweenness, bisector, median

Circle: diameter, radius, arcs, chord, secant, tangent

Congruence, similarity

Converse

Finite, infinite

Geometric concepts: point, line, segment, ray, plane, space, collineat point, co-planar points, separation, boundary, half-line, half-plane, half-space, interior and exterior regions

Geometric figures: simple closed curves with interior and exterior regions, triangles, quadrilaterals, other plane polygons, cone, cylinder, prisms, pyramid

Geometric property, theorem

Hypotenuse, Pythagorean theorem

Lines: parallel, perpendicular, intersecting, skew, transversals

Strand III

Discrete and continuous sets

Formulas: circumference, perimeter, area, volume

Measurement properties: motion, comparison, matching, subdivision

Measurement terms: standard unit, greatest possible error, tolerance, significant digits, precision, relative error, accuracy

Measurement relationships: metric mass and weight, basic metric and English units of measure

Strand IV

Banking terms, investment, interest

Budget

Consumer, comparative shopping, unit cost, discount

Gross and net weight, gross and net income

Strand V

Horizontal and vertical number scales, graphing, incomplete graphs, tabulating and rounding off data

Irrational numbers, real numbers

Number-line relationships: closed and half-open intervals, directed segments

Rate pair relationships in problems, equivalent and complex fractions, per cents

Sets of ordered pairs of numbers, coordinate diagrams, Cartesian products, complement of a set

Simple and compound conditions: variable, equations and inequality, equivalence in contrast to equation

Mathematics Terms And Concepts Used In Guide (continued)

Strand VI

Conditions (simple and compound), equations, inequalities, solution sets

Description and tabulation of sets

Elements (members) of a set

Equal and equivalent sets

Finite and infinite sets, disjoint sets

Ordered pairs, Cartesian set

Sets (meaning and symbolization), subsets, universe

Union, intersection, complementation of sets

Strand VII

Conditions of equality (simple and compound), inequalities and equivalences, solution sets

Equivalent rate pairs, ratio, proportional relations, direct and inverse variation

Open, closed, true and false mathematical sentences

Strand VIII

Array

Branching on routes of occurrence of events

Data (datum)

Favorable and unfavorable outcomes

Frequency tally

Fundamental counting property

Histogram

Measures of central tendency: arithmetic mean, median, mode

Measures of spread (scatter) of data: range, average deviation

Probability and empirical probability

Sample space and sample points

Suggested Teaching Equipment For A Junior High School Mathematics Room

abacus

base two computer or binary counter

cardboard or lucite models to show intersection of lines and planes

chalkboard with Cartesian coordinate section (or large, separate, rigid coordinate section)

chalkboard drawing tools: compass, protractor, T-square

Colored chalk, colored blocks, colored marbles with 12 each of 3 colors for use in probability

deck of cards, dice for use in probability

demonstration slide rule

display areas: pegboard display section, bulletin board

equal-volume models (plastic or cardboard)

flannel board (magnetic board) with appropriate symbols for sets, inequalities, etc.

fraction wheel

filmstrips (selection based on pre-viewing)

hexstat (use in probability)

mathematical puzzles (large size, wooden or plastic)

needle, thread, heavy colored paper for curve-stitching

overhead projector and screen

pegboard and vari-colored golf tees for illustrating Cartesian plane and graphs

TEACHING – UNIT APPENDICES

The following appendices are presented as specific teaching units.

It is hoped that they will save time for the busy instructor by:

a) Indicating instructional objectives and pupil conclusions (outcomes) which can be quickly judged to be desirable (or undesirable) for a specific class.

b) Suggesting helpful problem situations and for teaching techniques.

c) In some cases providing background information which can significantly assist in the development of future pupil understandings.

Strand I Appendix: Modular Arithmetic ---------- p. 105

Strand III Appendix: The Meaning and Usefulness of Pi ---------- p. 109

Strand IV Appendix: A Lesson In Comparative Shopping ---------- p. 112

Strand V Appendices: A) Help With Ordered Pairs and Cartesian Products ---------- p. 114

B) Per Cents (Percents) As Ratios ---------- p. 117

Strand VIII Appendices: A) Measures of Central Tendency ---------- p. 120

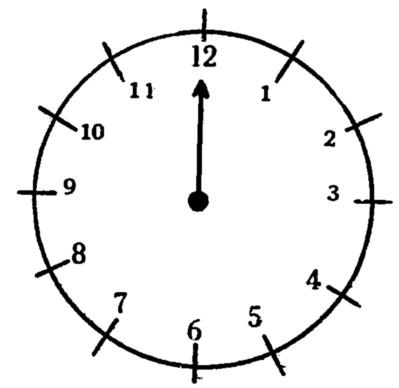
B) Measures of Spread or Dispersion ---------- p. 122

Strand I Appendix - Clock or Modular Arithmetic

Objective: To provide practice in one type of search for mathematics patterns

I. Class discussion of:

A. Clock arithmetic as a "minature number system" which consists of only twelve members.



From experience in telling time we know that:

4 + 10 = 2 (Ten hours after four o'clock is 2 o'clock.)
8 + 7 = 3 (Seven hours after eight o'clock is 3 o'clock.)
12 + 5 = 5 (Five hours after twelve o'clock is five o'clock.)

B. What is the difference between clock addition and ordinary arithmetic addition? Examine these tables:

ORDINARY ARITHMETIC	CLOCK ARITHMETIC
8 + 9 = 17	8 + 9 = 5
12 + 2 = 14	12 + 2 = 2
10 + 6 = 16	10 + 6 = 4

NOTE: In clock arithmetic if the ordinary sum exceeds 12, we subtract 12 from that sum, and the result is the clock sum.

C. How does multiplication work in clock arithmetic?

ORDINARY ARITHMETIC	CLOCK ARITHMETIC	OBSERVATION
3 x 3 9	3 x 3 = 9	Same
3 x 5 = 15	3 x 5 = 3	15 − 3 = 12
4 x 8 = 32	4 x 8 = 8	32 − 8 = 24
7 x 12 = 84	7 x 12 = 12	84 − 12 = 72

NOTE: It appears that the product of two numbers in clock arithmetic is either equal to the ordinary product, or it is equal to the ordinary product diminished by some multiple of 12.

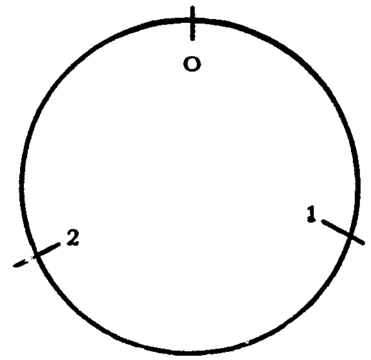
D. The clock arithmetic that we use every day is really an example of a modular or finite arithmetic. Because our clock system contains twelve elements, it is called a modulo 12 system.

II. Practice material

A. 1. Since the modulo 12 system has twelve elements, how many elements would we expect to find in a modulo 3 system?

2. List the elements of the modulo 3 system. - - - - - - - - (0, 1, 2)

B. 1. Using the modulo 3 clock, complete this addition table.



+	0	1	2
0			
1			
2			

+	0	1	2
0	0	1	2
1	1	2	0
2	2	0	[illegible]

2. Notice these differences:

Usual Arithmetic	Modulo 3 Arithmetic
0 + 1 = 1	0 + 1 = 1
1 + 1 = 2	1 + 1 = 2
1 + 2 = 3	1 + 2 = 0
2 + 2 = 4	2 + 2 = 1

NOTE: In each case where the sums are different, the modulo 3 sum is 3 less than the corresponding sum is usual Arithmetic.

3. Let us divide each whole number by 3 and write the remainder below the original number.

Whole number: 1, 2, 3, 4, 5, 6, 7, 8, 9, . . .
Remainder: 1, 2, 0, 1, 2, 0, 1, 2, 0, . . .

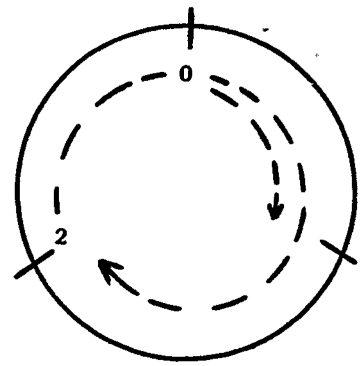
NOTE: We see that if whole numbers are divided by three, every whole number will fall into a certain "residue class" according to its remainder. The whole number divisor is called the modulus.

4. If 2+1 = 3 = 0 (mod. 3), correctly complete these statements:

1 + 1 = ___ (mod. 3), 0 + 2 = ___ (mod. 3), 1 + 2 = ___ (mod. 3), 2 + 2 = ___ (mod. 3)

II. Practice material (cont.)

C. We may also multiply in a modulo 3 system.



To show 2 x 2 on a modulo 3 clock, we start at 0 and move 2 spaces two consecutive times. We arrive at 1. We see that 2 x 2 (mod.3) = 1 (mod.3)

1. Complete this modulo 3 multiplication table:

x	0	1	2
0			
1			
2			

Ans.

x	0	1	2
0	0	0	0
1	0	1	2
2	0	2	1

2. Are both the multiplication and addition tables of modulo 3 symmetrical with respect to the diagonal from the upper left to the lower right?

   NOTE: If both tables are symmetrical with regard to their respective diagonals, then we know that in the modulo 3 system, addition and multiplication are commutative.

D. Addition and multiplication are associative.

$$(0+1) + 2 = 0 + (1+2) \qquad (0+2) + 1 = 0 + (2+1)$$
$$1 + 2 = 0 + 0 \qquad 2 + 1 = 0 + 0$$
$$0 = 0 \qquad 0 = 0$$

By listing every possible combination of the elements as illustrated above, we find that the residue classes, modulo 3, are <u>associative</u> with respect to addition. In similar manner one could show multiplication of residue classes, modulo 3, to be <u>associative</u>.

E. Multiplication distributes over addition:

$$2 \cdot 2 = 2 \cdot (1 + 1) = (2 \cdot 1) + (2 \cdot 1) = 2 + 2$$
$$1 \text{ (mod.3)} = 1 \text{ (mod. 3)}$$

By this type of testing we see that the elements of the residue classes, modulo 3, behave according to the distributive law, also.

II. Practice materials (cont.)

F. Identity elements: zero is the identity element with respect to addition because $N+O=O+N=N$.
one is the identity element with respect to multiplication because $1\cdot N=N\cdot 1=N$ This is true for every N in the system.

G. Each element in the system has an inverse.

1. The above statement means that each element can be combined with some other element to produce the identity element.
2. The inverse of 1 in the addition table is 2, because $1+2=0$ and 0 is the identity element.
3. Find the inverse of 0 in the addition table. (Ans: zero is its own additive inverse.)
   Find the inverse of 2 in the multiplication table. (Ans.: 2 because $2\cdot 2=1$ (mod 3) and 1 is identity element)
   Does 0 have an inverse in the multiplication table? (Ans.: No, because $0\cdot N=0$, not 1. This is the only exception to the inverse property of multiplication.)

III. Summary of learnings

A. A "minature number system" has a countable number of members.
B. The number of members in the system is known as the modulus or the modulo number.
C. A "miniature number system" may be tested to see if its elements obey the basic laws of a number system.

Strand III Appendix - Discovering the Meaning and Usefulness of Pi

Objective: To develop a beginning sense of the meaning and usefulness of the "irrational number" commonly symbolized by π.

I. Class discussion of:

A. How does the measure of the circumference of a circle compare with the measure of its diameter? Will the circumference-diameter ratio be the same for a small circle as for a larger circle?

1. Find several objects that are circular and measure the diameter of each with a ruler. (half-dollar, top of tin can, etc.)

2. Using the same objects, find the circumference of each. To find the circumference wrap a piece of paper tightly and smoothly around the object. Make a pencil mark where the paper overlaps the starting edge. Lay the paper out flat and measure the distance from the starting edge to the pencil mark. This is the circumference of the circle being measured.

3. Find the quotient of each circumference divided by the corresponding diameter and determine that the value is approximately 3.14.

4. If pupils apply this experiment to several different sets of circular planes, they will discover that the ratio of the measure of the circumference of a circle to the measure of its diameter remains constant, regardless of variations in the size of the diameter.

B. Will the circumference–diameter relationship discovered in A, above, hold for measurements taken in a different way?

1. Use a circlometer or a meter stick and discs of different diameters.

2. Using discs of different diameters, roll discs along a common yardstick to determine the measure of the circumference of each circle. Divide the measure of each circumference by the measure of that circle's diameter. (Regardless of the unit in which both circumference and diameter are expressed, the ratio of circumference to diameter remains approximately 3.14 or $3\frac{1}{7}$.)

B. Circumference–diameter relationship (cont.)

NOTE: The symbol π(pi) is used to represent the number (3. 14, $3\frac{1}{7}$, $\frac{22}{7}$, etc.) which expresses the ratio of the measure of the circumference of a circle to the measure of that circle's diameter.

C. Mathematical sentences of special importance to circles: $\frac{C}{d} = \pi$ $\quad C = \pi \cdot d \quad C = 2 \cdot \pi \cdot r$

NOTE: "20 π units" is a quite satisfactory answer to the question, "What is the length of a circle with a radius of 10 units?"

D. What is the <u>exact</u> value of pi?

Suggestion: a special report by an able student can contribute facts of this type:

1. Pi is an irrational number. <u>Definition</u>: An irrational number is a nonrepeating, endless sequence of digits.

   NOTE: Pi is <u>not</u> a repeating decimal, <u>nor</u> is it a terminating (ending) decimal.

2. A decimal expression for pi developed to 10,000 places has been published.

3. The value of pi developed to fifteen decimal places is: 3.14159 26534 89793...(Three dots indicate that the expression continues endlessly.) NOTE: Nonrepeating and nonterminating

4. The approximate value of pi most often used is 3.14 or $3\frac{1}{7}$ or $\frac{22}{77}$

E. Can the value of pi be located easily on the number line?

1. Teacher–and–pupil development of the division of the measure of any circle's circumference by the measure of its diameter to an ever-increasing number of decimal places will show that:

   $\pi > 3$ and $\pi < 4$; $\pi > 3.1$ and $\pi < 3.2$; $\pi > 3.14$ and $\pi < 3.15$; ...

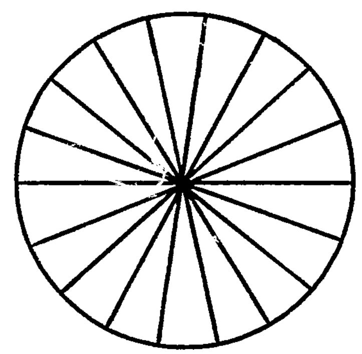
2. The above procedure can be continued an infinite number of times, but each step designates a <u>smaller</u> segment on the number line as containing the point, pi.

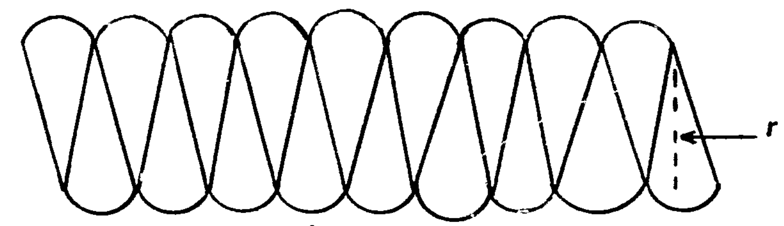
II. Practice materials

A. You know that the circumference of a circle equals pi times d or $C = \pi \cdot d$. You also know that the area of a parallelogram equals b times h. In the drawing below the triangular sections of circle X have been fitted together to form figure Y, which is almost a true parallelogram. This drawing will help the students understand why the area of a circle is equal to $\pi \cdot r \cdot r$.

II. Practice materials (cont.)

Circle X (18 parts)- - - - - - - - - - - - - - - - -becomes- - - - - - - - - - - - - - - -Parallelogram Y (18 parts).





Base (b) of figure Y equals $\frac{1}{2}$ of circumference of circle X.

∴ $b = \frac{1}{2}C = \frac{1}{2}\pi d = \pi r$

Height of figure Y equals radius (r) of circle X.

B. For any parallelogram, $A = bh$

For parallelogram Y, $A = b \times r$ For b let us substitute $\pi \cdot r$

Then, $A = \pi r \times r$ or $A = \pi r^2$

III. Summary of learnings

A. An irrational number is a nonrepeating, endless sequence of digits.

B. The ratio of the measure of the circumference of any circle to the measure of its diameter, is an irrational number which has the approximate value of $3\frac{1}{7}$ or $\frac{22}{7}$ or 3.14.

C. The value of the circumference–diameter relationship of circles is so useful that it has been designated as pi, which is represented by the symbol π.

Strand IV Appendix—Comparative Shopping

Objectives:

A. To make the student aware of the idea of comparative shopping
B. To help the student arrive at some guidelines to follow in shopping
C. To provide further practice in the oral and written use of basic mathematics skills

I. Class practice and discussions:

A. Open the lesson with oral calculation drill of the type:
   1. Make two flash cards for each product; 3 x 5 file cards work nicely. Ask the student to determine the unit cost of each item. (cost per pound, per item, per can, etc.)

   2. Sample cards might be:

TUNA		PICKLES		PENCILS	
2 for 69¢	35¢/can	2 for 74¢	35¢/jar	47¢/dozen	$3\frac{1}{2}$¢ each

JUICE		PEARS		BALL MITT	
6 cans for $1.00	20¢/can	3 lbs. for $1.00	32¢/lb.	$5.00 each	4 for $20.00

BLANKETS		TROUSERS		TENNIS RACKET	
2 for $15.00	$7.49 each	$10.00 each	$9.95 each	4 for $35.00	$8.50 each

B. Class discussion and "follow-up"

1. Following the drill, ask the students to name as many reasons as they can why the cheapest product might not be the best buy. Look for such answers as difference in size of can, what kind of material makes up the product, is it a brand name, where was it packaged, how is it to be used, how will it wash, etc. These items should be listed where all can see, such as overhead projector, blackboard, etc.

2. Class discussion on, "How can I, as the buyer, decide whether or not I am making the best possible purchase?" Opposite the list in No. 2, make another list to show how they can arrive at the answers. The list might include: look on the label, be able to figure %, what is it to be used for, can it be used immediately, etc.

II. Practice materials

A. After the discussion, the class may be divided into small groups or work individually. Pass out product labels, or have the students bring them in, or make sample labels on the mimeograph. Let each group or student compare two or more labels for similar products and arrive at a conclusion as to the best buy. Be sure they understand terms on labels.

Sample labels:

PEARS		BLANKETS	
PEARS (Broken halves & pieces) Net wt. 1 lb.	PEARS (fancy halves) Net wt. 13 oz.	BLANKET 100% wool Full size 80" by 90" wt. $3\frac{1}{2}$ lbs.	BLANKET 100% orlon Full size 80" by 90" wt. $3\frac{1}{2}$ lbs.
43¢	45¢	$14.47	$13.75

B. Each group or individual should report his conclusion in brief oral or written form.

III. Summary of learnings:

A. In comparative shopping, the shopper should first determine the unit cost.
B. Secondly, the shopper must evaluate the cost in terms of his need, the quality of the product, how and when buyer will use article.

Strand V Appendix A–Ordered Pairs and Cartesian Products

Objectives: Development of the necessary understandings and skills to enable pupils to:

A. Grasp the meaning of ordered pairs and write them in correct notation.
B. Locate the single point on a coordinate plane which establishes a one-to-one correspondence between it and a specific ordered pair.
C. Recognize that an ordered pair is not commutative.

I. Class discussion

A. If we were given set A = {1,2,3} and set B = {a,b} could we use the elements of the given sets to write a third set in which each element is composed of a "pair" of numbers?

B. Suppose we agree to choose number "pairs" so that the first part of each pair comes from set A and the second part of each pair comes from set B.

1. Does (1,a) seem to be a sensible way to choose and designate the first pair?
2. Is (1,b) a good choice for a second number-pair?
3. Sticking to our agreement about using sets A and B, what other pairs can we write? (2,a), (2,b), (3,a), (3,b)
4. Since all our number pairs have a first component (part) from A and a second component (part) from B, can we say that the "order" within our pairs is the same in all cases?
5. Does it seem sensible to call our number pairs, "ordered pairs"?
6. Shall we agree to refer to a pair of objects as an "ordered pair", if the objects appear in some special order?
7. In writing ordered pairs of numbers, is it sensible to write the first components as the left numerals in left-to-right arrangements like these? (1,a), (1,b), (2,a), (2,b), (3,a), (3,b)
   How would you describe the first component of each of the above ordered pairs? How would you describe the second component?

C. Mathematicians call our total set of ordered pairs a "Cartesian set" or a "Cartesian product".

1. In forming our Cartesian product did we match each member of set A in turn with each member of set B?
2. We can write this matching arrangement as AXB which we read as "A cross B" or as "the Cartesian set A cross B".
3. Can we correctly describe the matching in our ordered pairs by this set notation?

$$A \times B = \{(a,b) \mid a \text{ is in } A \text{ and } b \text{ is in } B\}$$

<u>or</u>

$$A \times B = \{(a,b) \mid a \in A \text{ and } b \in B\}$$

II. Practice material

A. Given: set $A = \{1, 2, 3\}$ and set $B = \{a, b\}$

Suppose we agree to write all possible ordered pairs by choosing the <u>first</u> component from B and the <u>second</u> component from A.

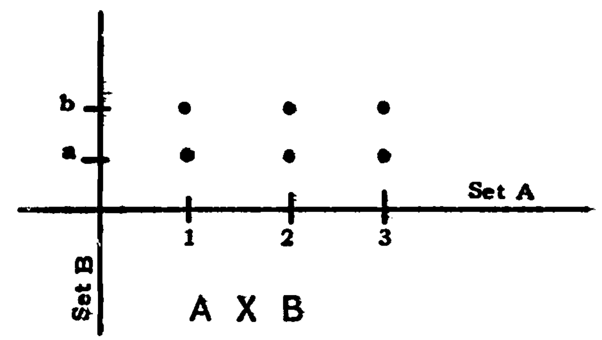
1. Now what shall we write for pairs?
   Class "constructs" the pairs: (a,1), (a,2), (a,3), (b,1), (b,2) (b,3)
2. This Cartesian set (or Cartesian product) looks different from our first one. Since all our first components came from set B, to indicate a Cartesian product what shall we use for these parts? ____ X ____
3. If we use set notation, we write this Cartesian product as:

$$B \times A = \{(a,1), (a,2), (a,3), (b,1), (b,2), (b,3)\}$$

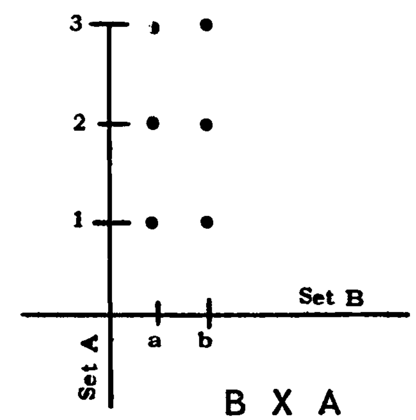
B. If we can follow a correct way of graphing Cartesian products do you guess that A X B will give us the same picture as B X A?
Lets try these ideas:
1. The Cartesian product of any two sets may be represented pictorially using perpendicular lines.
2. Along the horizontal axis we label equally-spaced points with the elements of the first set.
   Along the vertical axis we label equally-spaced points with elements of the second set.

II. Practice material (continued)

3. We read along the horizontal axis first and then along the vertical axis.



A X B



B X A

Is each ordered pair matched to one and only one point?

Is each ordered pair matched to one and only one point?

4. Note that our two Cartesian sets gave us different graphs. Now complete this statement with the correct symbol: A X B B X A (= or ≠)

∴ We see that an ordered pair is <u>not</u> Commutative. (1,a) and (a,1) denote two different points.

III. Summary of learnings

A. An ordered pair of numbers is a pair of numbers arranged in a special way, or "order".
B. The Cartesian product of set A and set B is the set of <u>all ordered pairs</u>, (a,b), in which the first element, a, is an element of set A and the second element, b, is an element of set B.
C. An ordered pair is not commutative. If $x \neq y$, then (x,y) does not designate the same point as (y,x) on the Cartesian plane.

Strand V Appendix B–Per Cents as Ratios and Ordered Pairs

Objectives:

- **A.** To apply the concepts of "rate pair" and "ratio" in solving per cent problems.
- **B.** To develop pupil understanding that:
  1. The relationship expressed by N% may also be written $\frac{N}{100}$, recognized as a rate pair.
  2. In the $\frac{N}{100}$ rate pair of per cent, N represents the "part of" element and 100 represents the "all" element.

I. Class discussion and practice–Per cents as ratios or rate pairs

A. Suppose Joe has \$3 for each \$2 that Bill has.
What special name do we give to the "comparison numeral", 3 : 2 or $\frac{3}{2}$?
Since the numeral shows the comparison (or rate) between two quantities, can we think of the ratio as a "rate pair"?

B. Does the ratio $\frac{23}{25}$ equal the ratio $\frac{92}{100}$? $\frac{23}{25} \stackrel{?}{=} \frac{92}{100}$ (Yes. Statement indicates two ways of naming same quantity).

What is the special name for a rate pair like $\frac{92}{100}$? (92%)

C. What are equivalent ratios (rate pairs) and per cents for these situations?

1) 11 of each 50 items means $\frac{\square}{50}$ or $\frac{\square}{100}$ or ______ %

2) $70\% = \frac{\square}{100} = \frac{\square}{50}$

3) 13 out 25 means $\frac{\square}{25}$ or $\frac{\square}{100}$ or ______ %

4) $120\% = \frac{\square}{100} = \frac{6}{\square}$

Can we say that each per cent expresses a ratio whose second term is 100?

D. Use equivalent ratios and cross products to solve these per cent problems:
Hint: <u>Write the per cent ratio first.</u> Its second term is always ______ .

1) If 42 is 84% of N, then N = ?

Soln.: $\frac{84}{100} = \frac{42}{N}$

$(84 \times N) = (42 \times 100)$ cross-products

$84N = 4200$

$N = \frac{4200}{84}$

$N = 50$

Now we see that N = 50

D. Equivalent ratios (cont.)

2) 50 is what per cent of 25?
Rewrite: 50 is P% of 25.

Soln.:

$$\frac{P}{100} = \frac{50}{25}$$

$$(25 \times P) = (50 \times 100)$$

$$25P = 5000$$

$$P = \frac{5000}{25}$$

$$P = 200$$

Now we see that:

P% = 200%

50 = 200% of 25

3) 210% of 40 is N

What is N?

Soln.:

$$\frac{210}{100} = \frac{N}{40}$$

$$(100 \times N) = (40 \times 210)$$

$$100N = 8400$$

$$N = \frac{8400}{100}$$

$$N = 84$$

Now we see

that N = 84

II. Class discussion and practice – Relationship of "Part" to "All" in per cent

A. What numbers shall we compare to determine the per cent of students in this class today?
Is <u>this</u> comparison model correct?

$$\frac{\text{the number present (Part of)}}{\text{total number enrolled in class (All)}} = \frac{N \text{ (Part of)}}{100 \text{ (All)}}$$

NOTE: Comparison of Part to All <u>is equivalent to</u> Comparison of Part to All

B. If a student missed 4 questions on a test of 50 questions, what per cent of the questions did he miss? Can we use <u>ordered pairs</u> in comparing "Part" to "All"?

Is <u>this</u> comparison correct? (4 missed (Part of), 50 (All)) = (P (Part of), 100 (All))

$$50P = 400$$

$$P \text{ (per cent)} = 8$$

Now we see that: 4 questions are 8% of 50 questions.

II. Relationship of "Part" to "All" (continued)

C. In example B, what per cent of the questions were correct? What numbers do we compare?

NOTE: Since the number missed was 4, the number correct was 46.

Is this comparison correct? (46 correct (Part of), 50 (All)) = (P (Part of), 100 (All))

50 P = 4600

P = 92

Now we see that 46 questions are 92% of 50 questions.

III. Summary of learnings

A. Any per cent may be written as a ratio or rate pair in which the second term is 100.

B. A per cent may be expressed as an ordered pair in which the second component is 100.

C. In solving per cent problems we can use either equivalent ratios or ordered pairs to show the "Part" to "All" relationship.

Strand VIII, Appendix A–Measures of Central Tendency

Objective:
To develop a beginning sense of the usefulness of the arithmetic mean, the median, and the mode as single elements which may summarize large groups of data.

I. Class discussion of:

A. Examples of the school-time use of a single representative for a group of facts or data by:

1. The principal of this school–(average daily attendance, average enrollment, etc.)
2. The teacher of this class–(average achievement of the class as measured by semester scores, the middle of the ordered grades made on a daily quizz)
3. A pupil in this class–(average of his "weekly quizz" grades, his grade position in comparison to the middle grade in an ordered set of grades)

B. What would you choose as the single element most representative of the total group in each of the following situations?

1. Four boys whose respective heights are 5′10″, 4′10″, 5′3″, and 5′7″.

   Review meaning of arithmetic mean and how to obtain it. See grade 7, Strand VIII, C. Hoped-for pupil generalization: Choice of 5′3″ "representative" boy since his height is just $1\frac{1}{2}$″ from arithmetic mean of 5′ $4\frac{1}{2}$″ of heights of 4 boys.

2. A set of 27 math test grades arranged in order from the highest grade of 89 to the lowest grade of 59. Discussion of arrangement "in order". Hoped-for conclusion: Neither of the extremes is truly representative of the group. A set of 27 ordered scores allows the finding of an exact middle point, called median, with the same number of scores preceding and following it. This is one way of representing the group.

3. Ten boys wear these shoe sizes: 11, $7\frac{1}{2}$, 10, $8\frac{1}{2}$, $9\frac{1}{2}$, 8, 9, $7\frac{1}{2}$, 10, $10\frac{1}{2}$ Discussion of most-commonly-worn size as a good representative of the group. Hoped-for conclusion: There are two "most fashionable" or most frequently worn sizes in the group. These are called "modes". Note the meaning of "modish" and "modes-of-the-times" in people's clothing and cars as being typical of, or representing, these items now.

II. Practice materials

A. Can you make up a problem involving five numbers in which the average, median, and mode are 24?

B. Can you make up an average, median, mode problem about this class?

C. In a certain oil company ten employees received these salaries last year: $4,500; $6,500; $13,000; $5,500; $7,500; $6,000; $5,000; $5,500; $7,000; $5,500.

1. Find the mean of the data. ($6,600)

2. How many salaries are greater than the mean? (Three)

3. How many salaries are less than the mean? (Seven)

4. Does the mean seem to be a fair way to describe the typical salary of these employees? (No)

5. Find the median of the set of data. ($5,750)

6. Does the median seem to be a fair average to use for this data? (Reasonably fair, since there are five salaries above the median and five salaries below it.

7. If the data has a mode, what is it? (Mode is $5,500).

III. Summary of learnings
(To be stated in acceptable pupil-wording)

A. The <u>mode</u> is the element which occurs most frequently in a set of data.

B. The <u>arithmetic mean</u> (or <u>average</u>) is a single representative of a set of elements found by dividing the sum of the elements by the number of the elements.

C. The <u>median</u> is the middle element when an <u>odd</u> number of elements are arranged in order of size. The <u>median</u> is the average of the two middle elements when an <u>even</u> number of elements are arranged in order of size.

Strand VIII, Appendix B–**Measures of Spread or Dispersion**

Objective:

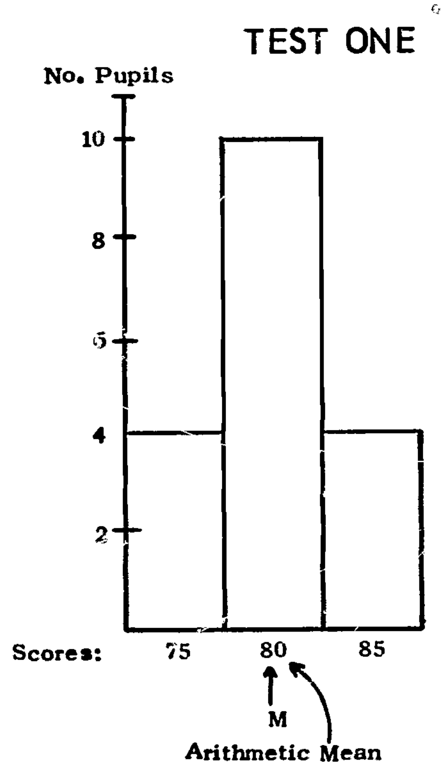
To develop the beginning of a pupil-sense of the usefulness of the ability to judge whether any given group of data differs widely or very little between its extremes.

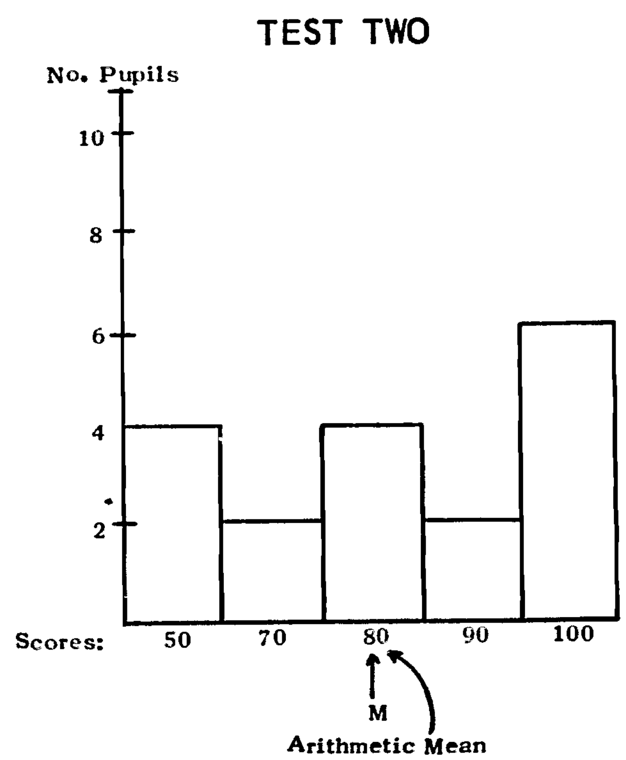
I. Class discussion of:

A. 1. To describe two sets of data in a fair manner, do we really need more than the single representative element which is either a center element of the group (arranged in order of size), or the most frequently used element?

2. Does the average, the median, or the mode tell us whether we have any very large or very small elements in the set?

B. These histograms (type of graph which shows number or quantity of each item) give us the records made by a certain class on two tests. Notice that on both tests the average and median scores are the same.





I. Class discussion of: (continued)

1. Do you think that the class showed equally good understanding of content on the two tests?
2. On which test do you think the class made a better record? Can you tell why you think so?
3. To compare class understanding in a fair manner on these two tests, must we consider highest and lowest scores?

NOTE: Mathematicians use the word "range" to refer to the difference between the highest and the lowest elements of a set of numerical data.

C. In Histogram One above:

1. What is the difference between the lowest score and the median?
2. What is the difference between the highest score and the median?
3. Would it be fair to average the differences above and below the median this way?

$$\frac{\text{(no. pupils x no. points below M)} + \text{(no. pupils x no. points above M)}}{\text{total no. pupils in the class}}$$

NOTE: Mathematicians use "average deviation" as the name of the average of the individual differences of all scores from the arithmetic mean.

4. Find the average deviation of the test scores in Histogram One. (answer: $\frac{(5 \times 5) + (5 \times 5)}{20} = 2\frac{1}{2}$)

II. Practice materials

A. Referring to Histogram Two, above, complete this table and find the average deviation of the scores on Test Two.

Score	Diff. from Arith. mean of 80	No. pupils (Frequency)	Freq. x Diff.
50	-30	4	120
70	-10	___	___
80	0	___	___
90	___	___	___
100	___	___	___

Ans: $\frac{120+30+0+30+120}{20} = \frac{300}{20} = 15$ average deviation

II. Practice Materials (continued)

B. Do the histogram "pictures" of Test Two above, agree with our findings that the average deviation (difference) of class scores from the arithmetic mean was much greater in Test 2 than in Test 1?

Check all the correct descriptions of class performance on Tests 1 and 2:

_______1. Class performance was about the same on both tests.

_______2. Test 2 shows a greater range of achievement than does Test 1.

_______3. Test 1 appears to show about the same understanding on the part of all pupils.

_______4. Test 1 shows that some pupils have complete understanding of test content.

_______5. By the grading standards of our school, Test 2 shows that some pupils have poor understanding of the test's content.

C. Using the kind of table shown in Problem A of this Section II, find the range, arithmetic mean, and average deviation (to the nearest tenth) of this set of scores: 70, 100, 85, 90, 100, 60, 90.

answers: range = 40, mean = 85, average deviation = 11.4

III. Summary of learnings

(To be stated in acceptable pupil-wording).

A. To summarize a set of data fairly we should use, in addition to a "center element" in an order-of-size arrangement, some measures which show how the elements are scattered from largest to smallest in the set, and a measure which shows how elements are scattered from the center element.

B. The range is the difference between the largest and the smallest elements of a set of data.

C. The average deviation is the average of the individual differences of each element from the arithmetic mean.