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TABLES FOR COMPARING RELATED-SAMPLE PERCENTAGES AND FOR THE MEDIAN TEST.

BY- TATE, MERLE W.  BROWN SARA M.
PENNSYLVANIA UNIV., PHILADELPHIA,GRAD.SCH.OF EDUC.

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TO STUDY THE EXACT DISTRIBUTIONAL DIFFERENCES BETWEEN SMALL, RELATED-SAMPLE PERCENTAGES AND THE ACCURACY OF LARGE-SAMPLE TESTS, SOME SIX HUNDRED SAMPLING DISTRIBUTIONS WERE CONSTRUCTED BY RANDOMIZATION. EXACT PROBABILITY FIGURES IN THE .204-.005 RANGE WERE TABULATED. THE TABLE OF SIGNIFICANCE VALUES EXTENDS TO SAMPLES YIELDING MATRICES VARYING IN SIZE FROM TREE COLUMNS AND 12 ROWS TO SIX COLUMNS AND FIVE ROWS, INCLUDING ALL COMBINATIONS OF ROW TOTALS. THE TWO-SAMPLE PROBLEM WAS TREATED AS A SPECIAL CASE OF THE SIGN TEST. THE USE AND RELIABILITY OF THE Q-TEST IS DISCUSSED. THE USE AND ACCURACY OF THE MEDIAN TEST IN THE TWO-WAY CLASSIFICATION OF QUANTITATIVE DATA IS DESCRIBED. (PS)

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MERLE W. TATE
SARA M. BROWN



**Graduate School of Education**
*UNIVERSITY of PENNSYLVANIA*
Philadelphia, Pennsylvania

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University of Pennsylvania

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## FOREWORD

Our interest in the related-sample proportion or percentage test was aroused by the practical question of whether large-sample tests were appropriate for several small samples in hand. In comparing the approximate and exact sampling distributions, it became apparent that the number of samples and the amount of correlation between them, as well as their size, affected the accuracy of the approximations.

By the time we found out that no simple or general rules regarding minimum sample size could be made, we had constructed rather numerous exact distributions. We decided to continue until we had complete tables of significance levels for samples yielding matrices varying in size from (a) three columns and twelve rows to six columns and five rows, including all combinations of row totals, and (b) seven columns and six rows to sixteen columns and three rows, with row totals equal to half the number of columns, if that number is even, or half the number of columns less one-half, if that number is odd. The tables are presented as Tables A and B, respectively, of this report.

The work was carried out under the Cooperative Research Program of the United States Office of Education. We wish to acknowledge the advice and help of two members of that Office: Howard Hjelm and Glenn Boerrigter. A preliminary study was supported by the Committee on the Advancement of Research of the University of Pennsylvania.

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## INTRODUCTION

The problem of comparing two or more related-sample proportions or percentages arises where an investigator has dichotomous data classifiable in $r$ rows and $c$ columns. Such data originate in situations where each of $r$ subjects is observed under each of $c$ treatments or conditions. If a success or presence of some characteristic is observed on the $i^{th}$ subject under the $j^{th}$ condition, 1 is recorded in the $ij$ cell; if a failure or absence of the characteristic is observed, 0 is recorded. This is typically the situation in test-item analysis where each of $r$ examinees passes or fails each of $c$ test items. Or, an investigator may have $c$ samples of blood from each of $r$ patients known to have a certain disease and may apply a diagnostic test to each sample, recording 1 or 0 for the $i^{th}$ sample under the $j^{th}$ test, depending on whether the result is positive or negative. As a third situation, an investigator may have $r$ groups of $c$ matched or related individuals and may record 1 or 0 for the individual in the $ij$ cell, depending on whether the individual responds or does not respond to a given task in a certain way.

The data and schema for comparing related-sample percentages are shown in Table 1. It will be noticed that the table is similar to that used in conventional two-way analysis of variance in the $r$ by $c$ table, one observation per cell. When dichotomous data are

Table 1. Schema and data for comparing related-sample percentages.

| Subject ($i$) | Treatment or Condition ($j$) 1 | 2 | 3 | 4 | . . . | c | Total |
|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | . . . | 1 | $u_1$ |
| 2 | 1 | 1 | 1 | 1 | . . . | 0 | $u_2$ |
| 3 | 1 | 1 | 1 | 0 | . . . | 0 | $u_3$ |
| 4 | 1 | 1 | 0 | 0 | . . . | 0 | $u_4$ |
| . | . | . | . | . | . . . | . | . |
| . | . | . | . | . | . . . | . | . |
| . | . | . | . | . | . . . | . | . |
| r | 0 | 0 | 0 | 0 | . . . | 0 | $u_r$ |
| Total | $T_1$ | $T_2$ | $T_3$ | $T_4$ | . . . | $T_c$ | $\sum_i u_i = \sum_j T_j$ |

classified as in Table 1, the question of interest usually is whether the proportions or percentages of 1's in the columns are significantly different. To answer the question, the null hypothesis that the $c$ population percentages are equal is tested against the alternative hypothesis that one or more is different from the others.

Although several large-sample tests of the hypothesis have been developed, no exact test is available. It is our purpose to present tables of exact significance levels for comparing percentages in small related samples and tables for the Brown-Mood median test, a special case of the related-percentage test. We begin with a review of large sample tests.

## LARGE SAMPLE TESTS

The c (>2) Sample Case. Cochran (1950) gives a statistical test, the Q test, for the significance of differences among proportions or percentages in $c$ samples when $r$ is large. He shows that the limiting distribution of the quantity Q,

$$Q = \frac{c(c-1)\sum_j (T_j - \bar{T}.)^2}{c\sum_i u_i - \sum_i u_i^2}, \qquad (1)$$

as $r$ increases, is the $\chi^2$ distribution with (c-1) degrees of freedom, where $c$ is the number of columns (samples), $T_j$ the sum of 1's in the $j^{th}$ column, $\bar{T}.$ the mean of the $T_j$, and $u_i$ the number of 1's in the $i^{th}$ row. Cochran notes, p. 259,

> . . . The only restriction needed is a rather obvious one, to guard against the possibility that as the number of rows tends to infinity, the value of $u_i$ might be $c$ or 0 in all but a finite number of rows. If this happens, the size of the population is still finite in the limit, because permutations within rows having $u_i = c$ or 0 do not generate any new cases. This situation is avoided by stipulating that for at least one intermediate value of $u_i$, the number of rows having that value must tend to infinity.

In considering the exact sampling distribution of Q, Cochran again notes that rows in which $u_i$ = c or 0 may be added to the basic table without affecting the value of Q.*

---

*Failure to take into account the fact that rows containing only 1's or only 0's do not generate new cases, i.e., do not affect the sampling distribution of Q, has led to misleading statements about sample size. McNemar (1962, pp. 227-228) says that "The sampling distribution of [Q] follows the $\chi^2$ distribution . . . for N large (N>30, presumably.)" Siegel (1956, p. 162) says, ". . . if the number of rows is not too small [Q] is distributed approximately as chi-square. . ." In his illustrative example, one-third of the rows contain only 1's or only 0's. Hays (1963, p. 629) says, "For relatively large K [number of rows] this [Q] is distributed approximately as chi-square. . ." In his illustrative example, Hays includes a row of 1's and a row of 0's. Statements about sample size appropriate for the Q test are meaningless unless they exclude rows of 1's and rows of 0's, and illustrations of the Q test are misleading unless such rows are deleted.

The Q test is easy to apply. Ordinarily the most convenient computational formula is,

$$Q = \frac{(c-1)[c\sum_j T_j^2 - (\sum_j T_j)^2]}{c\sum_i u_i - \sum_i u_i^2}. \qquad (2)$$

To apply the formula to the data of Table 2, where the problem is to determine whether the three items vary significantly in difficulty, we have $c = 3$; $\sum_j T_j^2 = (12)^2 + (5)^2 + (10)^2 = 269$; $\sum_j T_j = \sum_i u_i = 27$; $\sum_i u_i^2 = (2)^2 + (2)^2 + \ldots + (0)^2 + (3)^2 = 57$, so that

$$Q = \frac{2[3(269) - (27)^2]}{3(27) - 57} = 6.50.$$

Table 2. Responses of 19 subjects to three _Miller Analogies Test_ items involving mathematical concepts.

| Subject | Item 1 | Item 2 | Item 3 | $u_i$ |
|---|---|---|---|---|
| 1 | 1 | 0 | 1 | 2 |
| 2 | 1 | 0 | 1 | 2 |
| 3 | 0 | 0 | 1 | 1 |
| 4 | 0 | 0 | 0 | 0 |
| 5 | 1 | 0 | 1 | 2 |
| 6 | 1 | 1 | 0 | 2 |
| 7 | 0 | 0 | 0 | 0 |
| 8 | 0 | 0 | 0 | 0 |
| 9 | 1 | 0 | 0 | 1 |
| 10 | 1 | 1 | 1 | 3 |
| 11 | 1 | 0 | 0 | 1 |
| 12 | 1 | 1 | 1 | 3 |
| 13 | 1 | 0 | 0 | 1 |
| 14 | 0 | 0 | 1 | 1 |
| 15 | 0 | 1 | 1 | 2 |
| 16 | 1 | 0 | 0 | 1 |
| 17 | 1 | 0 | 1 | 2 |
| 18 | 0 | 0 | 0 | 0 |
| 19 | 1 | 1 | 1 | 3 |
| Total, $T_j$ | 12 | 5 | 10 | $\sum_j T_j = \sum_i u_i = 27$ |

The corresponding P in a table of $\chi^2$ at two degrees of freedom is $.05 > P > .025$. The hypothesis that the items are of equal difficulty in the population can be rejected at the

5 per cent level.*

When the rows having sums of 3 and 0 are deleted in Table 2 (Rows 4, 7, 8, 10, 12, 18, and 19), $\sum_j T_j^2 = 134$, $\sum_j T_j = \sum_i u_i = 18$, and $\sum_i u_i^2 = 30$. Substituting in (2), we have

$$Q = \frac{2[3(134) - (18)^2]}{3(18) - 30} = 6.50,$$

as before. A moment's reflection will convince one that rows containing c 1's or c 0's do not affect the value of Q. Rows containing c 0's obviously have no effect on Q in formula (2). Rows containing c 1's do not affect the numerator of (2) since they increase the $T_j$ by a constant. Nor do they affect the denominator of (2), since each such row will merely add $c^2$ to both terms.

The conventional F test in two-way classification, one score per cell, was applied to item scores, such as those of Table 2, as early as 1941 by Hoyt. However, Hoyt was mainly concerned with the estimates of reliability and measurement error yielded by the F ratio and remainder variance. Cochran (1950) suggests the F test, as a method alternative to the Q test, for determining whether differences between columns (samples) are significant. He compares the results from the Q and F tests with the results obtained from the exact sampling distributions of Q in eight small samples. His comparisons will be elaborated in a later section. Cochran points out that the F test, unlike the Q test, is affected by rows where $u_i = c$ or 0.**

When the conventional F test is applied to the data of Table 2, $F_{2,36}$ turns out to be 3.72, with P = .036. However, when the rows whose sum is 3 or 0 are deleted, $F_{2,22}$ = 4.09, and the corresponding probability is .030.

Blomqvist (1951) derives several tests for dichotomized data. His test statistic, S $[= \sum_j (T_j - \bar{T}.)^2]$, is related to Cochran's Q statistic through the expression,

$$S = \frac{Q(c\sum_i u_i - \sum_i u_i^2)}{c(c - 1)}, \qquad (3)$$

*If a definite probability figure is desired, it may be obtained from the normalizing transformation of $\chi^2$. Since $\chi^2/n = F_{n,\infty}$, Paulson's (1942) normalizing equation for the variance ratio becomes

$$z = [\sqrt[3]{\chi^2/n} + 2/(9n) - 1] / \sqrt[2]{2/(9n)}$$

where z is a normal deviate and n is the number of degrees of freedom for $\chi^2$.

**Johnson and Jackson (1950) apparently overlook this point in their statement, p. 142, "The Q statistic Cochran proposed can, in fact, be expressed in terms of the quantities used in the usual analysis of variance test and would seem to possess no inherent advantage over the latter."

as is apparent in formula (1), above. One of the tests Blomqvist proposes is equivalent to the Q test. As the basis for another test, he shows that as the number of columns c increases, r and $u_i$ remaining fixed, the limiting distribution of S is normal. The normal approximation is difficult to apply, except where the row sums are equal. It is particularly tractable where the $u_i$ are each equal to c/2, when c is even, or to (c-1)/2, where c is odd. We shall return to it in connection with the median test.

The Two-Sample Case. If there are only two columns (samples) and if the rows which sum to 2 or 0 are deleted, then each row sums to 1, and formula (1) reduces to $Q = (T_1 - T_2)^2/r$, with one degree of freedom. Hence $z = (T_1 - T_2)/\sqrt{r}$, where z is a normal deviate, $T_1$ and $T_2$ are the column sums, and r is the number of rows having sums of 1.

Now the sum $(T_1 + T_2)$ is equal to r, while the difference $(T_1 - T_2)$ is the difference between the number of rows having a 0-1 sequence and the number having a 1-0 sequence. If one of these numbers is designated h, the other may be written (r-h). Hence, $z = (2h - r)/\sqrt{r}$, which corrected for continuity, becomes,

$$z = \frac{|2h - r| - 1}{\sqrt{r}} \qquad (4)$$

where h is either of the two column totals.

The test embodied in formula (4) is commonly known as the sign test. It is equivalent to the test for related-sample proportions proposed by McNemar (1946); however, it does not require that the data be classified in a fourfold table. All that is necessary is to delete rows which sum to 2 or 0, count the number of remaining rows, find the sum of the 1's in either column, and substitute in (4). For a two-sided test, ordinarily appropriate, the probability corresponding to z is doubled.

Accuracy of Large-Sample Tests. The minimum sample size (the minimum number of rows whose sum is other than c or 0) for which the large-sample tests are appropriate is not known. Since an increase in the number of samples (columns) also affects the distributions of Q and S, the number of samples as well as their size affects the accuracy of the approximations.

Both Cochran and Blomqvist consider the question of accuracy. Cochran constructed the exact sampling distributions of Q for the eight samples,

| No. of Columns, c | No. of Rows, r | Row Totals | No. of Columns, c | No. of Rows, r | Row Totals |
|---|---|---|---|---|---|
| 3 | 10 | 5(2), 5(1)* | 4 | 6 | 5(3),1(2) |
| 3 | 10 | 1(2), 9(1) | 4 | 9 | 3(3),3(2),3(1) |
| 3 | 11 | 1(2),10(1) | 4 | 10 | 3(3),3(2),4(1) |
| 3 | 16 | 1(2),15(1) | 5 | 8 | 2(4),2(3),2(2),2(1), |

*The line is read, "There are three columns (samples) and ten rows (sample sizes), five of the rows having sums of 2 and five sums of 1."

by the familiar method of randomization. He compared the P values obtained from uncorrected and corrected $\chi^2$ and from corrected F with the exact P's over the .005-.200 range. He concludes, p. 263,

> None of the methods is free from bias. $\chi^{2'}$ tends to overestimate and F' to underestimate. Over the range as a whole $\chi^2$ comes off fairly well with 23 overestimates and 32 underestimates, but it appears that a negative bias in the region of 0.2 to 0.1 is being counteracted by a positive bias in the region of 0.02 to 0.005. For practical uses $\chi^2$ is preferable to F', since it is slightly easier to calculate, though the possible application of F' to more complex tables should be borne in mind. . . . At the true 5% level, average errors of about 14% are to be anticipated, which means that the tabular approximations might give a value of 0.057 or 0.043 instead of 0.05. At the 1% level the corresponding figures are about 0.012 and 0.008. These results appear close enough for routine decisions. For true probabilities below 0.005, all methods go to pieces. F' may give values only one-quarter of the true probability, while the two $\chi^2$ values may be six or eight times too high.

Blomqvist gives the exact sampling distributions of S for the samples,

| No. of Columns, *c* | No. of Rows, *r* | Row Totals | No. of Columns, *c* | No. of Rows, *r* | Row Totals |
|---|---|---|---|---|---|
| 4 | 3 | 3(2) | 6 | 5 | 5(3) |
| 4 | 4 | 4(2) | 8 | 3 | 3(4) |
| 4 | 5 | 5(2) | 8 | 4 | 4(4) |
| 4 | 6 | 6(2) | 10 | 3 | 3(5) |
| 4 | 7 | 7(2) | 10 | 4 | 4(5) |
| 4 | 8 | 8(2) | 12 | 3 | 3(6) |
| 6 | 3 | 3(3) | 14 | 3 | 3(7) |
| 6 | 4 | 4(3) | 16 | 3 | 3(8), |

in which the row sums are each equal to $c/2$. In this special case, the distribution of the statistic, $4(c-1)S/cr$, approaches the $\chi^2$ distribution with $(c-1)$ degrees of freedom as *r* increases; while the distribution of S, itself, approaches the normal distribution with mean $cr/4$ and variance $c^2r(r-1)/8(c-1)$, as *c* increases. He makes seven comparisons of the P's obtained from uncorrected $\chi^2$ and from the normal approximation, corrected for continuity, with the exact P's nearest the 5% point. The median percentage error for $\chi^2$ is 47 with range 4 to 129; that for the normal approximation, 40 with range 14 to 55. As would be expected, the normal approximation improves rapidly as the number of columns increases.

## THE EXACT DISTRIBUTION OF Q

__The *c* (>2) Sample Case.__ As Cochran (1950) shows, the exact sampling distribution of Q may be generated by randomization, the observed sum $\underline{u}_i$ of the 1's in the $\underline{i}^{th}$ row being regarded as fixed.* Under the null hypothesis, all different arrangements of the 1's in the $\underline{i}^{th}$ row have the same probability. Hence, the possible results in the $\underline{i}^{th}$ row consist of the $\binom{c}{u_i}$ ways in which the $\underline{u}_i$ can be distributed among the $\underline{c}$ columns. The exact distributions are constructed row by row, the observations in all rows except the first being permuted, as illustrated in the Appendix. Three distributions of Q are shown in Table 11, p. 16.

Table A, pp.23-35, includes selected probability figures obtained from the exact sampling distributions of Q in samples ranging from two columns and twenty rows to six columns and five rows for the various combinations of row totals. The largest probability, P, shown in any line of the table is the largest P less than .204 in the exact distribution; the smallest P in any line is the smallest P not less than .005 in that distribution. The intermediate P's are those nearest the 10%, 5%, and 1% points, respectively. Where fewer than five P's are shown in a line, they constitute all of the P's between .204 and .005 for that distribution. When an observed sum of squares of column totals is less than the first recorded sum of squares for given $\underline{c}$, $\underline{r}$, and row totals in Table A, the corresponding probability is greater than .204; when the observed sum is greater than the last recorded sum, the corresponding probability is less than .005.

The table is entered at the number of columns, number of rows, and row totals. For example, in Table 2, after deletion of rows having sums of 3 or 0, there are 12 rows, 6 of which have sums of 2 and 6 sums of 1; hence, Table A is entered at 3; 12; 6(2),6(1). The sum of squares of column totals in Table 2 is 134. The exact probability of a sum of squares of column totals this large, if the null hypothesis is true, is .051, as shown in the parentheses after 134 in the row 3; 12; 6(2),6(1) of Table A.

The sums of squares of column totals are tabulated in Table A, rather than the actual Q's, at the various levels of significance. Where a Q is desired, it may be readily obtained from formula (2) and the information given in the table. For example, the value of Q corresponding to the sum of squares of 36 in the distribution, c = 4, r = 5, with row totals 2(3), 1(2), 2(1), is

$$Q = \frac{3[4(36) - (10)^2]}{4(10) - 24} = 8.25,$$

since $\sum_i u_i^2 = 2(3)^2 + 1(2)^2 + 2(1)^2$ or 24.

The use of Table A is further illustrated in testing for differences among the columns of Table 3. There are four columns and, after deleting the rows which sum to

*Both Siegel (1956) and Tatsuoka and Tiedeman (1963) state in effect that, under the null hypothesis, the 1's are randomly distributed in the rows and columns of the table. It would be difficult to construct the sampling distributions of Q if randomness prevailed in both rows and columns.

Table 3*. Reports of 9 patients of relief, 1, or of little or no relief, 0, under treatment for asthma.

| Patient | Drug | Drug | Placebo | Placebo |
|---|---|---|---|---|
| 1 | 1 | 0 | 0 | 0 |
| 2 | 0 | 1 | 0 | 1 |
| 3 | 1 | 1 | 1 | 0 |
| 4 | 1 | 1 | 1 | 1 |
| 5 | 1 | 0 | 0 | 0 |
| 6 | 0 | 0 | 0 | 0 |
| 7 | 1 | 1 | 1 | 1 |
| 8 | 1 | 0 | 0 | 0 |
| 9 | 1 | 1 | 0 | 0 |

*From Tate and Brown (1964).

four or zero (Rows 4, 6, and 7), six rows. One of the rows sums to 3, two to 2, and three to 1, so that the row totals are 1(3), 2(2), and 3(1). The column totals are 5, 3, 1, and 1, whose squares sum to 36. Entering Table A at 4; 6; 1(3), 2(2), 3(1) we find that the probability corresponding to 36 is .105. This is the probability of a sum of squares of column totals as large as 36, if the null hypothesis is true.

It should be noted that certain of the distributions under a given _c_ and _r_ in Table A can be paired. The distribution for 5; 4; 3(4), 1(1), for example, differs from the distribution for 5; 4; 1(4), 3(1) only in location, i.e., values of sums of squares of column totals or values of Q. Many such pairs exist in Table A. Both members of the pair are included to simplify entry into the table.

_The Two-Sample Case_. Although an exact test for the difference between two related-sample percentages may be constructed by the method described above, it is not necessary. An exact test may be made by means of a binomial probability table, such as that compiled under the direction of Aiken (1955).

As noted in the preceding section, when all rows whose sum is 2 or 0 are deleted, the column totals sum to _r_, i.e., $T_1 + T_2 = r$. Whether $T_1$ (or $T_2$) differs sufficiently from expectation, (r/2), to discredit the null hypothesis may be determined from the binomial $(.5 + .5)^r$. For example, if $T_1 = 2$ and $T_2 = 10$ so that $r = 12$, we consult the cumulative binomial table at sample size 12, p = .5, and find that as few as 2 or as many as 10 successes have a probability of .0193 + .0193 or .039. In terms of sums of squares of column totals, a sum as large as $(10)^2 + (2)^2$ or 104 has a probability of .039, if the null hypothesis is true.

Applying the approximation of formula (4) to the above data, we have,

$$z = \frac{2(10) - 12 - 1}{\sqrt{12}} = 2.02.$$

The corresponding P is 2(.0217) or .043.

In general, the probability obtained from formula (4) will be very close to the true probability in samples where r is about 20 or more.

## ACCURACY OF THE Q TEST

Cochran's comparisons, mentioned earlier, indicate that uncorrected $\chi^2$ is a somewhat better approximation than F to the exact sampling distribution of Q. Blomqvist's comparisons suggest that, at least when the number of columns is not more than six, the uncorrected $\chi^2$ approximation is better than the normal. Intuitively, one would expect $\chi^2$ to be the best of the approximations.

For these reasons and because the normal approximation is laborious, except where row sums are equal, we studied only the accuracy of the $\chi^2$ approximation, as applied to the distributions of Table A. For each of the sums of squares of Table A, we computed Q ($\chi^2$) and obtained its definite P value by use of the normalizing transformation. We then determined the percentage error in each P from $\chi^2$ taken as an estimate of the exact P. Some of the results are summarized in Tables 4, 5, and 6.

Table 4. Median and range of percentage errors in the $\chi^2$ approximation over two regions of significance for selected distributions.

| Distribution* Columns | Rows | Median Percentage Error .204 - .005 | .100 - .020 | Range of Errors .100 - .020 |
|---|---|---|---|---|
| 3 | 6 | 25% | 21% | 8% - 50% |
| 3 | 12 | 18 | 20 | 6 - 33 |
| 4 | 4 | 29 | 28 | 2 - 39 |
| 4 | 8 | 16 | 13 | 0 - 35 |
| 5 | 3 | 38 | 17 | 0 - 44 |
| 5 | 5 | 20 | 12 | 0 - 50 |
| 6 | 3 | 32 | 32 | 10 - 68 |
| 6 | 5 | 21 | 11 | 0 - 55 |

*All distributions having indicated numbers of columns and rows.

It is apparent from the median percentage errors of Table 4 that the approximation becomes better as the number of rows increases for both regions of significance levels shown, particularly when c is greater than three. The ranges of errors, however, indicate that at no time, within sample sizes considered, is the approximation necessarily close.

The medians and ranges of errors at the .204-.101 and .019-.005 regions (not shown in the table) are different from those at the .100-.020 region. The former tend to be somewhat smaller than those at the .100-.020 region; the latter, substantially larger. The median of the 158 percentage errors over the .019-.005 region was 54 in the c=6, r=5 distributions. In particular, the approximation tends to be very poor when c is greater than three and exact P less than .010. Then, the approximation nearly always gives P's that are too large, frequently more than twice the size of the exact P's. In Table 10, p.15 , P's from $\chi^2$ are compared with the exact P's in fourteen distributions. It will be seen that in all but one case $\chi^2$ overestimates exact P's near the .01 level.

Table 5. Distributions of percentage errors in the $\chi^2$ approximation over two regions of significance for selected distributions.

| Error Distribution | Region and Distribution* .204 - .005 | | | | .100 - .020 | | | |
|---|---|---|---|---|---|---|---|---|
| | c=3,r=12 | c=4,r=8 | c=5,r=5 | c=6,r=5 | c=3,r=12 | c=4,r=8 | c=5,r=5 | c=6,r=5 |
| 0 - 4 | 9 | 24 | 16 | 54 | | 16 | 14 | 51 |
| 5 - 9 | 4 | 27 | 28 | 62 | 2 | 12 | 24 | 54 |
| 10 - 14 | 11 | 48 | 31 | 62 | 8 | 18 | 20 | 44 |
| 15 - 19 | 13 | 34 | 32 | 55 | 4 | 14 | 8 | 29 |
| 20 - 24 | 14 | 22 | 20 | 54 | 6 | 4 | 9 | 24 |
| 25 - 29 | 5 | 14 | 25 | 53 | 3 | 4 | 5 | 17 |
| 30 - 34 | 7 | 41 | 22 | 18 | 5 | 13 | 6 | 6 |
| 35 - 39 | | 4 | 6 | 21 | | 1 | 2 | 6 |
| 40 - 44 | 2 | | 11 | 11 | | | 2 | 2 |
| 45 - 49 | | | | 10 | | | | 4 |
| 50 - 54 | | 2 | 10 | 17 | | | 2 | |
| 55 - 59 | | 2 | 1 | 9 | | | | 2 |
| 60 - 64 | | 5 | | 6 | | | | |
| 65 & Above | | 2 | 14 | 71 | | | | |
| Median | 18 | 16 | 20 | 21 | 20 | 13 | 12 | 11 |

*All distributions having indicated numbers of columns and rows.

The error distributions of Table 5, show that, as numbers of rows and columns increase, the range of error gets considerably broader. What appears to be the explanation of this curious circumstance is the tendency of a few in the family of distributions to take on atypical irregularities as c and r increase but remain small. Inspection of the original distributions of Q failed to find a consistent relationship between numbers of columns, numbers of rows, and composition of rows which would enable one to anticipate poor approximations. At both the .204-.101 and .100-.020 regions, the approximation appeared to be somewhat more trustworthy in distributions having intermediate row sums $[(c-1) > u_i > 1]$, but there were numerous exceptions. It may be that the Q test, as approximated by $\chi^2$ can never be wholly trusted. The Q distributions are occasionally unruly. (See Fig. 1, p.17). However, with increasing sample size, the median error in the approximation to exact P decreases, and poor approximations should become relatively few in number.

The under- and overestimates of Table 6 indicate that the $\chi^2$ approximation nearly always underestimates exact P in the .204-.101 region and, when c is greater than three, overestimates exact P most of the time in the .019-.005 region. The approximation appears to be unbiased, when c is greater than three, in the .100-.020 region. These considera-

Table 6. Number of times P from $\chi^2$ was greater than (+), equal to (0), or less than (-) exact P in selected distributions.

| Distributions* | Region of Exact P: .204 - .101 | | | .100 - .020 | | | .019 - .055 | | |
|---|---|---|---|---|---|---|---|---|---|
| | + | 0 | - | + | 0 | - | + | 0 | - |
| c=3, r=12 | | | 14 | | | 28 | 10 | 8 | 5 |
| c=4, r=8 | | 1 | 63 | 31 | 9 | 42 | 72 | 5 | 2 |
| c=5, r=5 | | | 60 | 41 | 8 | 43 | 53 | 5 | 6 |
| c=6, r=5 | 2 | | 104 | 107 | 19 | 113 | 153 | 2 | 3 |

*All distributions having the indicated numbers of columns and rows.

tions may be helpful in interpreting the large-sample Q test, assuming that the same tendencies exist in samples beyond the sizes considered. The assumption seems plausible and receives some support from the error comparisons of Tables 9 and 10.

## MEDIAN TEST IN TWO-WAY CLASSIFICATION OF QUANTITATIVE DATA

__Exact Median Test.__ Mood (1950) and Brown and Mood (1951) give a test for column (or row) effects in the _c_ by _r_ table, one observation per cell. The test, known as a median test, is analogous to parametric analysis of variance in two-way classification.

To test for column effects, i.e., differences between columns, the observations above the median of a row are replaced by 1's; those at or below the median by 0's. The resulting table is a _c_ by _r_ table of dichotomous data, with row sums each equal to c/2 or (c-1)/2, depending on whether _c_ is even or odd. Hence, the median test for column effects in the _c_ by _r_ table, one observation per cell, is a special case of the Q test.

Consider the data of Table 7. The question of first interest is whether the

Table 7. Average number of contributions to medicine by ages of contributors.*

| Field | Age Interval: 20-24 | 25-29 | 30-34 | 35-39 | 40-44 | 45-49 | 50-54 | 55-59 | 60-64 |
|---|---|---|---|---|---|---|---|---|---|
| Bacteriology | .010 | .030 | .040 | .050 | .025 | .035 | .011 | .022 | .023 |
| Pathology | .006 | .024 | .031 | .055 | .033 | .026 | .019 | .034 | .021 |
| Anatomy | .022 | .031 | .044 | .053 | .036 | .034 | .005 | .028 | .009 |
| Pharmacology | .046 | .054 | .074 | .047 | .034 | .040 | .031 | .006 | .000 |

*Abridged from Lehman (1953).

differences between columns (ages of contributors) are significant. When the values in each row are replaced by 1's (above row median) and 0's (at or below row median), the 9-column and 4-row matrix results, as shown below, with row sums each equal to 4. The

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 0 | 1 | 1 | 1 | 0 | 1 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 1 | 0 | 0 | 1 | 0 |
| 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 |
| 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |

column totals are 1, 2, 4, 4, 2, 2, 0, 1, 0, whose squares sum to 46. Going to Table B, p.36, at 9; 4; 4(4), we find that the sum of squares, 46, corresponds to a probability of .033.

To test for differences between rows, the roles of columns and rows are reversed. When 1's and 0's are assigned to the values in the respective columns, the resulting table is thought of as having 4 columns and 9 rows, with row sums equal to 2. When this is done, the column totals are 6, 6, 4, 2, whose squares sum to 92. According to Table B, this total corresponds to a probability greater than .204, since it is less than the smallest total entered under 4; 9; 9(2).

Table B was constructed in the same way as Table A. (See p.21) Twenty-five of the distributions of Table B also appear in Table A; they are repeated in Table B for ready reference.

Although Brown and Mood discuss several advantages of the median test, they say little about its power. Since it reduces quantitative to qualitative data, it sacrifices information and would be expected to be less powerful than parametric analysis of variance or the Friedman rank test. It is attractive mainly because of its freedom from assumptions and ease of application.

Approximate Median Tests. As noted above, the median test is a special case of the Q test; hence, formulas (1) and (2) may be used in the median test where sample size is beyond Table B. Mood gives an approximation which is equivalent to that of formula (1) or (2). Applying formula (2) to the data of Table 7 where $c=9$, $r=4$, $\sum_i u_i = 16$, $\Sigma T_j^2 = 46$, and $\Sigma u_i^2 = 64$, we get $Q = 15.8$ with 8 degrees of freedom. The corresponding P is $.05 > P > .025$.

Blomqvist's normal approximation, adapted to the Q statistic where the $u_i$ are equal to $c/2$, when c is even, or to $(c-1)/2$, when c is odd, turns out to have a mean of $(c-1)$ and a standard deviation of $\sqrt{2(c-1)(r-1)/r}$. In using the approximation, the difference $Q-(c-1)$ should be corrected for continuity by subtracting the quantity $4(c-1)/rc$, when c is even, or $4c/r(c+1)$, when c is odd. The corrections are embodied in the formulas below,

$$z = \frac{Q - (c-1) - \frac{4(c-1)}{rc}}{\sqrt{2(c-1)(r-1)/r}} \quad \text{when } c \text{ is even, or} \tag{5}$$

$$z = \frac{Q - (c-1) - \frac{4c}{r(c+1)}}{\sqrt{2(c-1)(r-1)/r}} \quad \text{when } c \text{ is odd.} \tag{6}$$

For the data of Table 7, where $Q = 15.8$, $c = 9$, and $r = 4$, we have,

$$z = \frac{15.8 - (9-1) - \frac{4(9)}{4(9+1)}}{\sqrt{2(9-1)(4-1)/4}} = \frac{6.9}{3.46} = 1.99.$$

The corresponding normal probability is .023, the critical region being on the right side only.

Accuracy of the Approximate Median Tests. For each of the sums of squares shown in Table B, we computed Q and $z$, obtaining definite P values for the Q's by means of the normalizing transformation of $\chi^2$. The median error of the 192 $\chi^2$ approximations to exact P's was 22, with range from 0 to 600. Thirteen of the errors were overestimates by 100% or more; seven by 200% or more. None of the underestimates was in error by more than 57%. The median error of the normal approximation was 32, with range from 2 to 99, the upper half of the errors being well distributed over the 32 to 99 interval.

To get a better idea of how the approximations might work in samples larger than those of Table B, we made the comparisons summarized in Tables 8, 9, and 10. The medians and ranges of Table 8 indicate that the $\chi^2$ approximation is better than the normal where there are twelve or fewer columns, but that the normal is better where there are thirteen or more columns--with numbers of rows as given. The over- and underestimates of Table 9

Table 8. Median and range of percentage errors in the $\chi^2$ and normal approximations at two regions of significance levels for selected distributions of Table B.

| Distributions | Median Percentage Error | | | | Range of Errors | | | |
|---|---|---|---|---|---|---|---|---|
| | .204 - .005 | | .100 - .020 | | .204 - .005 | | .100 - .020 | |
| | $\chi^2$ | Normal | $\chi^2$ | Normal | $\chi^2$ | Normal | $\chi^2$ | Normal |
| c=3,4,5,6,7 r=4 | 24% | 37% | 17% | 47% | 3% - 159% | 8% - 76% | 3% - 51% | 9% - 76% |
| c=8,9,10,11,12 r=4 | 26 | 31 | 14 | 26 | 1 - 165 | 3 - 73 | 1 - 57 | 3 - 45 |
| c=13,14,15,16 r=3 | 47 | 17 | 40 | 13 | 1 - 225 | 2 - 44 | 11 - 88 | 5 - 24 |

indicate that the $\chi^2$ approximation has a negative bias in the .204 - .101 region and a positive bias in the .100 - .020* and .019 - .005 regions. The normal approximation shows a consistent negative bias in the .100 - .020 and .019 - .005 regions.

The comparisons are sharpened in Table 10, where are shown the exact and approximate P's at about the .10, .05, and .01 levels of the exact P's for each of the largest samples at the given numbers of columns of Table B.

*The positive bias in the .100 - .020 region disappears rapidly as the number of rows increases. For five or more rows, the approximation gives about as many under as over estimates.

Table 9. Number of times P from the $\chi^2$ and normal approximations was greater than (+), equal to (0), or less than (-) exact P in selected distributions of Table B.

| Distributions | Region of Exact P | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | .204 - .101 | | | | | | .100 - .020 | | | | | | .019 - .005 | | | | | |
| | $\chi^2$ | | | Normal | | | $\chi^2$ | | | Normal | | | $\chi^2$ | | | Normal | | |
| | + | 0 | - | + | 0 | - | + | 0 | - | + | 0 | - | + | 0 | - | + | 0 | - |
| c=3,4,5,6,7 r=4 | | | 4 | 2 | | 2 | 6 | | 1 | | | 7 | 2 | | 1 | | | 3 |
| c=8,9,10,11,12 r=4 | | | 4 | 2 | | 2 | 9 | | 2 | | | 11 | 7 | | | | | 7 |
| c=13,14,15,16 r=3 | | | 3 | 2 | | 1 | 7 | | | | | 7 | 4 | | | | | 4 |
| Total | 0 | 0 | 11 | 6 | 0 | 5 | 22 | 0 | 3 | 0 | 0 | 25 | 13 | 0 | 1 | 0 | 0 | 14 |

The various comparisons suggest that, when there are fewer than about twelve columns and more than three rows, the $\chi^2$ is better than the normal approximation at the .100-.020 region of exact P in the median test. Whether or not the biases persist at the .204-.101 and .019-.005 regions, the approximation should become better at all levels as the number of rows increases. The statement is supported both by the comparisons and the nature of $\chi^2$. As a matter of fact, the $\chi^2$ approximation should give better results regardless of the number of columns, where there are five or more rows.

When there are more than about twelve columns and fewer than four rows, the normal approximation appears to be the better of the two. The seven exact P's over the .204-.005 region in the distributions having two rows and thirteen or more columns of Table B and the corresponding approximate P's are shown below. In six of the seven comparisons,

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Exact P: | .025 | .143 | .015 | .100 | .009 | .061 | .005 |
| P($\chi^2$) : | .062 | .136 | .050 | .115 | .043 | .095 | .035 |
| P(z) : | .031 | .152 | .020 | .108 | .012 | .073 | .008 |

the normal approximation gives better results than the $\chi^2$. Its median percentage error is 24, while that of the $\chi^2$ approximation is 148.

Where there are four rows and more than about twelve columns, there are neither comparisons nor theoretical considerations to suggest which of the two approximations is preferable. However, in this case the exact P should usually fall between P($\chi^2$) and P(z), when P($\chi^2$) is about .10 or less.

As rules of thumb, in the median test, (1) given five or more rows, use the $\chi^2$ approximation, (2) given more than twelve columns and fewer than four rows, use the normal approximation, (3) given four rows and more than twelve columns, take P($\chi^2$) as the upper limit of exact P and P(z) as the lower limit, when P($\chi^2$) is about .10 or less.

Table 10. Exact P's nearest .10, .05, and .01 levels and corresponding P's from approximations to selected distributions of Table B.

| Distribution Columns | Rows | Exact | Probability P $\chi^2$ | Normal |
|---|---|---|---|---|
| 3 | 12 | .115 | .104* | .149 |
| | | .048 | .038* | .013 |
| | | .012 | .009* | .0002 |
| 4 | 10 | .091 | .084* | .078 |
| | | .062 | .050* | .026 |
| | | .011 | .013* | .001 |
| 5 | 8 | .122 | .097 | .106* |
| | | .048 | .050* | .029 |
| | | .010 | .012* | .002 |
| 6 | 6 | .075 | .075* | .062 |
| | | .050 | .048* | .027 |
| | | .008 | .013* | .001 |
| 7 | 6 | .074 | .074* | .061 |
| | | .048 | .048* | .027 |
| | | .008 | .013 | .004* |
| 8 | 5 | .087 | .082* | .072 |
| | | .045 | .051* | .030 |
| | | .010 | .019 | .003* |
| 9 | 4 | .085 | .081* | .071 |
| | | .033 | .046 | .023* |
| | | .016 | .024* | .006 |
| 10 | 4 | .117 | .108 | .111* |
| | | .055 | .063* | .044 |
| | | .008 | .019 | .004* |
| 11 | 4 | .098 | .095* | .087 |
| | | .046 | .055* | .034 |
| | | .008 | .017 | .003* |
| 12 | 4 | .135 | .123 | .129* |
| | | .067 | .074* | .057 |
| | | .013 | .024 | .007* |
| 13 | 3 | .091 | .123 | .087* |
| | | .031 | .048 | .024* |
| | | .008 | .023 | .005* |
| 14 | 3 | .121 | .117 | .117 |
| | | .043 | .061 | .038* |
| | | .013 | .030 | .009* |
| 15 | 3 | .066 | .078 | .059* |
| | | .021 | .040 | .016* |
| | | .006 | .019 | .003* |
| 16 | 3 | .086 | .095 | .081* |
| | | .031 | .051 | .025* |
| | | .009 | .025 | .006* |

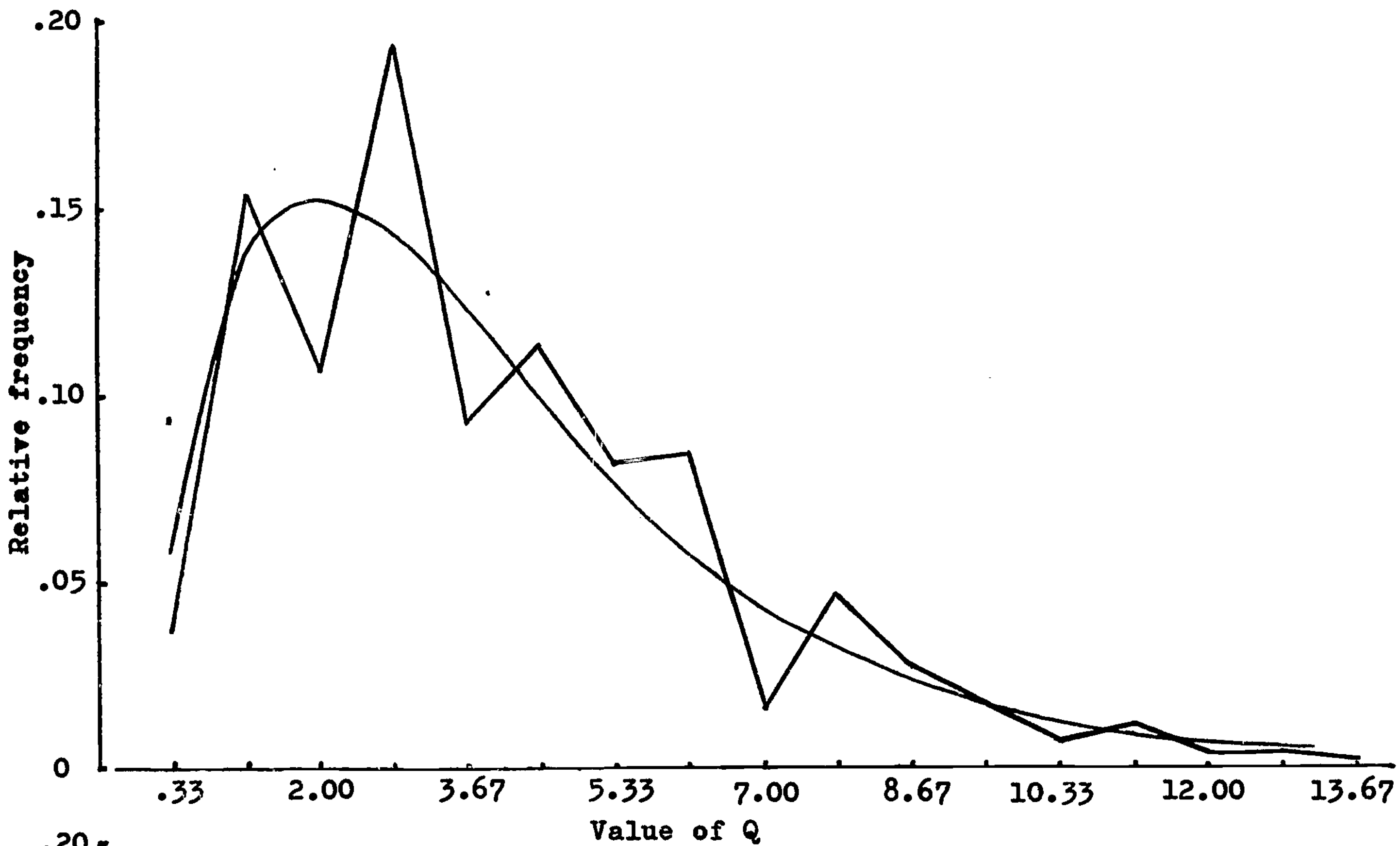
*Best of the two approximations.

Data for comparing the $\chi^2$ and normal distributions with three exact distributions of Q are included in Table 11, and one comparison is made in Figure 1.

Table 11. Distributions of Q for c=5, r=4, 4(2)*; c=5, r=8, 8(2); c=11, r=4, 4(5) and corresponding $\chi^2$ and normal relative frequencies.

| c=5, r=4, 4(2)* | | | | | c=5, r=8, 8(2) | | | | | c=11, r=4, 4(5) | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Q | f | Relative Frequency | | | Q | f | Relative Frequency | | | Q | f | Relative Frequency | | |
| | | Q | $\chi^2$ | Normal | | | Q | $\chi^2$ | Normal | | | Q | $\chi^2$ | Normal |
| 1.00 | 204 | .204 | .252 | .189 | .33 | 372,960 | .037 | .058 | .109 | 1.50 | 347,580 | .004 | .006 | .025 |
| 2.67 | 261 | .261 | .292 | .231 | 1.17 | 1,534,680 | .153 | .136 | .072 | 3.33 | 4,013,118 | .041 | .055 | .044 |
| 4.33 | 288 | .288 | .206 | .264 | 2.00 | 1,070,580 | .107 | .153 | .093 | 5.17 | 11,242,800 | .114 | .128 | .087 |
| 6.00 | 96 | .096 | .124 | .193 | 2.83 | 1,930,320 | .193 | .143 | .112 | 7.00 | 15,920,580 | .161 | .173 | .138 |
| 7.67 | 108 | .108 | .068 | .090 | 3.67 | 932,400 | .093 | .122 | .124 | 8.83 | 19,206,360 | .195 | .175 | .182 |
| 9.33 | 18 | .018 | .036 | .027 | 4.50 | 1,128,456 | .113 | .100 | .127 | 10.67 | 16,344,325 | .166 | .149 | .183 |
| 11.00 | 24 | .024 | .018 | .006 | 5.33 | 807,093 | .081 | .077 | .108 | 12.50 | 13,584,624 | .138 | .112 | .152 |
| 16.00 | 1 | .001 | .004 | .000 | 6.17 | 829,920 | .083 | .058 | .091 | 14.33 | 8,334,000 | .084 | .078 | .102 |
| | | | | | 7.00 | 169,848 | .017 | .044 | .065 | 16.17 | 5,144,400 | .052 | .050 | .053 |
| | | | | | 7.83 | 478,800 | .048 | .032 | .045 | 18.00 | 2,548,800 | .026 | .031 | .023 |
| | | | | | 8.67 | 269,136 | .027 | .023 | .026 | 19.83 | 1,153,800 | .012 | .018 | .008 |
| | | | | | 9.50 | 170,016 | .017 | .017 | .015 | 21.67 | 537,300 | .005 | .010 | .002 |
| | | | | | 10.3 | 65,520 | .007 | .012 | .007 | 23.50 | 144,200 | .002 | .006 | .001 |
| | | | | | 11.17 | 107,688 | .011 | .008 | .003 | 25.34 | 77,850 | .001 | .003 | .000 |
| | | | | | 12.00 | 36,120 | .004 | .006 | | | | | | |
| | | | | | 12.83 | 44,352 | .004 | .004 | | 27.17–40.00 | 21,391 | .000 | .005 | |
| | | | | | 13.67 | 18,816 | .002 | .003 | | | | | | |
| | | | | | 14.50–32.00 | 33,295 | .003 | .004 | .002 | | | | | |
| Total | 1,000 | 1.000 | 1.000 | 1.000 | | 10,000,000 | 1.000 | 1.000 | .999 | | 98,611,128 | 1.001 | .999 | 1.000 |

*c=5, r=4, 4(2) indicates a basic table of 5 columns and 4 rows, with row sums each equal to 2; c=5, r=8, 8(2) and c=11, r=4, 4(5) are read similarly.



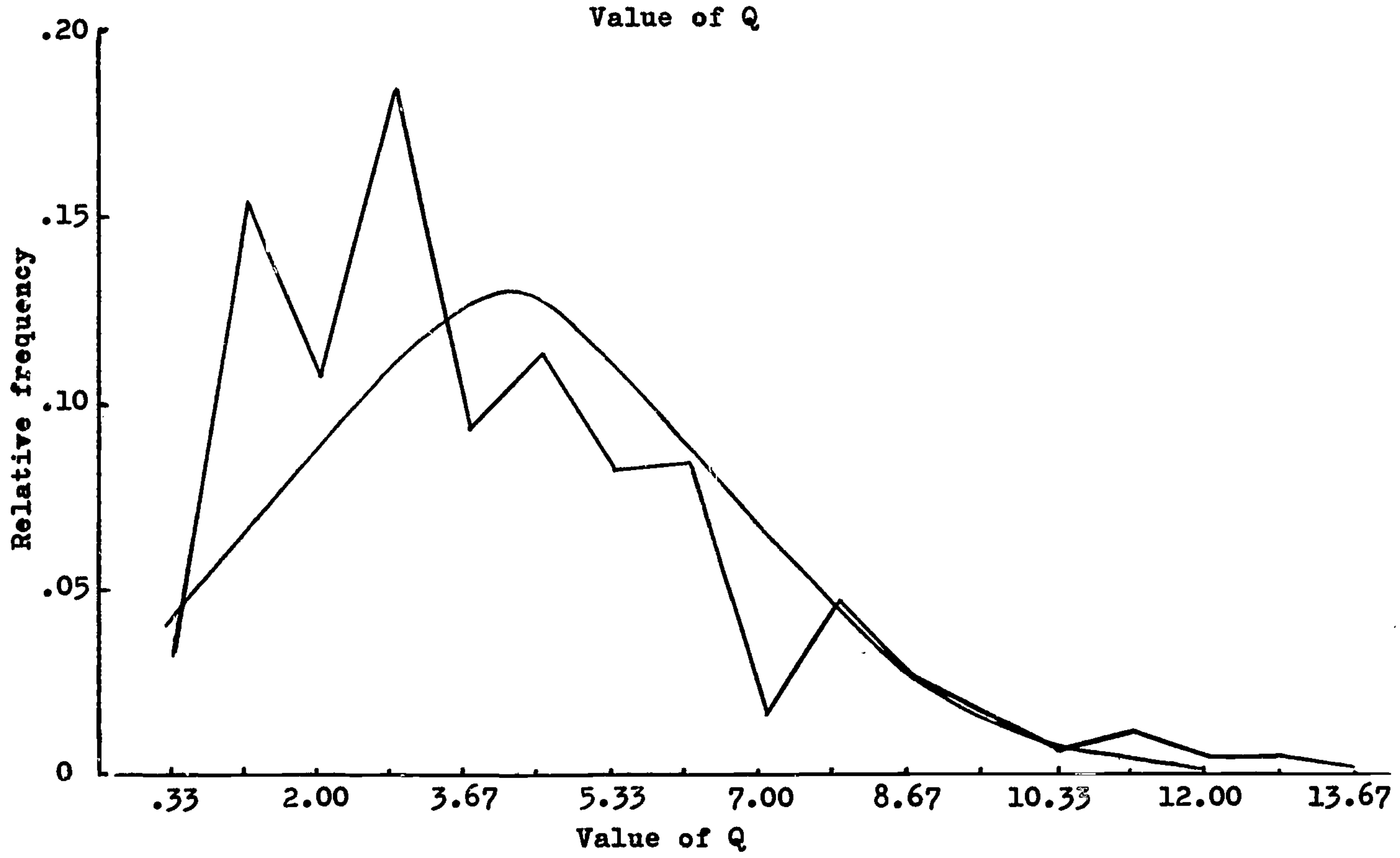




Fig. 1. Distribution of Q where c=5, r=8, 8(2), and fitted chi-square and normal curves.

# CONCLUDING REMARKS

In this study of the exact sampling distributions of the differences between small related-sample percentages and the accuracy of large-sample tests, some six hundred sampling distributions were constructed by randomization, and exact probability figures in the .204-.005 region were tabulated. The table of significance values extends to samples yielding matrices varying in size from three columns and twelve rows to six columns and five rows, including all combinations of row totals. (The two-sample problem was treated as a special case of the well-known sign test.)

Comparisons of exact P's with approximate P's obtained from Cochran's $\chi^2$ approximation, the Q test, indicated that the approximation shows consistent and marked improvement as the number of rows increases and some, though irregular, improvement as the number of columns increases. The median errors in the approximate P's for the largest distributions constructed for four or more columns were less than 15 per cent of the exact P in the .100-.020 region. However, the ranges of errors indicated that at no time, within the scope of sample size considered, was the approximation necessarily closer than 33 per cent of exact P. Since the ranges tended to get broader as the numbers of rows and columns increased, it may be that the Q test is never wholly trustworthy.

Where there are four or more columns, the Q test nearly always underestimated exact P in the .200-.101 region and almost as frequently overestimated exact P in the .019-.005 region. In the latter, the median error of the approximate P's was 56 per cent of the exact P's in the largest distributions considered.

For samples beyond the sizes included in the table, the Q test, as a rule, should be satisfactory for practical purposes in the .100-.020 region. As size and number of samples (rows and columns of the matrix) increase, both the median percentage error and the frequency of poor approximations decrease. However, in even relatively large samples, there is no certainty that a particular approximate P is a good estimate of the exact P. No consistent relationship between the number of columns, number of rows, and row totals was found which would enable one to anticipate poor approximations. Where the true significance level is needed, it would seem necessary to construct the exact sampling distribution.

In the special cases where row totals of the matrix are each equal to half the number of columns, when that number is even, or to half the number of columns less one-half, when that number is odd, the Q test is equivalent to the Brown-Mood median test for column (or row) effects in two-way classification, one observation per cell. For those cases, distributions were constructed for samples yielding matrices varying from three columns and twelve rows to sixteen columns and three rows, and the exact probability figures in the .204-.005 region were tabulated.

Comparisons of the exact P's with the P's from the $\chi^2$ and normal approximations indicated that the Q test worked as well or better than the normal *z* test, except where there were more than twelve columns and fewer than four rows. Then, the normal approximation appeared to be distinctly preferable.

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## APPENDIX: THE EXACT DISTRIBUTION OF Q

The exact distribution of Q may be built up, row by row, by permuting the observations in all rows except the first. Consider the case where there are four columns and three rows, with row totals 2, 2, and 1. The procedure is,

| | | | | | | |
|---|---|---|---|---|---|---|
| First Row: | 1 1 0 0 | 1 1 0 0 | 1 1 0 0 | 1 1 0 0 | 1 1 0 0 | 1 1 0 0 |
| Permutations, 2nd Row: | 1 1 0 0 | 1 0 1 0 | 1 0 0 1 | 0 1 0 1 | 0 1 1 0 | 0 0 1 1 |
| Column Totals: | 2 2 0 0 | 2 1 1 0 | 2 1 0 1 | 1 2 0 1 | 1 2 1 0 | 1 1 1 1 |

At this point, the distribution of column totals, under the null hypothesis, is (2 2 0 0) with probability 1/6, (2 1 1 0) with probability 4/6, and (1 1 1 1) with probability 1/6. When the permutations in the third row (1 0 0 0, 0 1 0 0, 0 0 1 0, and 0 0 0 1) are added to each of the column totals, and the results collected, the final distribution is,

| Column Totals | Frequency |
|---|---|
| 3 2 0 0 | 2 |
| 3 1 1 0 | 4 |
| 2 2 1 0 | 10 |
| 2 1 1 1 | 8 |
| | 24 |

This is the distribution of possible column totals (from which Q or S is readily obtained) in a sample where c=4, r=3, with row totals 2(2) and 1(1). The probability of such a sample having column totals of (3 2 0 0) is 2/24 or .083, if the null hypothesis is true. The sum of squares of the column totals (3 2 0 0) is 13, and the entry in Table A is,

4 3 2(2), 1(1) 13(.083).

Although the above procedure could be used in constructing the distributions of Q for samples of any size, it would soon become extremely laborious. For example, given six columns and five rows, each row containing three 1's, as in the median test for column effects, there would be $(20)^4$ or 160,000 sets of column totals. However, only 32 of the sets would be different, and the 32 would yield only 15 different sums of squares of column totals or 15 different values of Q.

Unfortunately, the procedure cannot be efficiently programmed for the computer; however, it can be greatly simplified for manual computation. Suppose it is required to find the distribution of Q where c=6, r=4, with row totals 1(4), 1(3), 2(2). Since there are six columns, four rows, and eleven 1's, the possible column totals, arranged in descending order of sums of squares of column totals, are as shown under Distribution A below. The frequencies of the column totals of Distribution A may be obtained from the distribution for c=6, r=3, 1(4), 1(3), 1(2) or from any other distribution for c=6, r=3 having three of the four row totals of Distribution A.

The 6; 3; 1(4), 1(3), 1(2) distribution is shown under Distribution B below. The purpose of the colons is to separate groups of like digits and thereby to facilitate the enumeration of permutations. Distribution B was obtained from a c=6, r=2 distribution, the latter having been obtained by combining the permutations in the second row with the first row. That is, each distribution having <u>c</u> columns and <u>r</u> rows is built up from a similar distribution having <u>c</u> columns and r-1 rows.

To obtain the frequencies for Distribution A from the frequencies and column totals of Distribution B, we must combine the permutations of a row containing two 1's and four 0's with the column totals of Distribution B. We proceed by subtracting each of the column totals of Distribution B from each of the column totals of Distribution A to see

**Distribution A**

[6;4;1(4),1(3),2(2)]

| Column Totals |
|---|
| 4 4 3 0 0 0 |
| 4 4 2 1 0 0 |
| 4 3 3 1 0 0 |
| 4 4 1 1 1 0 |
| 4 3 2 2 0 0 |
| 4 3 2 1 1 0 |
| 3 3 3 2 0 0 |
| 4 3 1 1 1 1 |
| 3 3 3 1 1 0 |
| 4 2 2 2 1 0 |
| 4 2 2 1 1 1 |
| 3 3 2 2 1 0 |
| 3 2 2 2 2 0 |
| 3 3 2 1 1 1 |
| 3 2 2 2 1 1 |
| 2 2 2 2 2 1 |

**Distribution B**

[6;3;1(4),1(3),1(2)]

| Column Totals | Frequency |
|---|---|
| 3 3:2:1:0 0 | 3 |
| 3 3:1 1 1:0 | 3 |
| 3:2 2 2:0 0 | 3 |
| 3:2 2:1 1:0 | 24 |
| 3:2:1 1 1 1 | 11 |
| 2 2 2 2:1:0 | 11 |
| 2 2 2:1 1 1 | 20 |

whether the larger can be obtained from the smaller by combining a row containing two 1's and four 0's, and, if so, in how many ways. Four of the 112 subtractions are shown below. The first and third subtractions indicate that the larger totals cannot be ob-

| | | | |
|---|---|---|---|
| 4 4 3 0 0 0 | 4 4 2 1 0 0 | 4 2 2 1 1 1 | 4 3 2 1 1 0 |
| 3 3 :2: 1 :0 0 | 3 3 :2: 1 :0 0 | 2 2 2: 1 1 1 | 3 3 :1 1 1: 0 |
| 1 1 :1:-1 :0 0 | 1 1 :0: 0 :0 0 | 2 0 0: 0 0 0 | 1 0 :1 0 0: 0 |

tained from the smaller by adding a row containing 1's and 0's. The second subtraction indicates that the larger totals may be obtained from the smaller in only one way, but since the totals (3 3 2 1 0 0) have a frequency of 3, their contribution to the frequency of the larger totals is 3. The fourth subtraction indicates that the larger totals may be obtained from the smaller in 2(3) ways, since the first group of digits permits two permutations and the second three. Since the totals (3 3 1 1 1 0) have a frequency of 3, their contribution to the frequency of the larger totals is 6(3) or 18.

The work is systematized in the following chart. If the column totals of Distribution B are written on slips of paper, the calculation of their contribution to the frequencies of Distribution A proceeds rapidly. The subtractions are made mentally, and the numbers of permutations at the various row totals are entered in the chart.

| Distribution A | Frequency (Distribution B) | | | | | | | Frequency (Distribution A) |
|---|---|---|---|---|---|---|---|---|
| Column Totals | 3 | 3 | 3 | 24 | 11 | 11 | 20 | |
| 4 4 3 0 0 0 | | | | | | | | 0 |
| 4 4 2 1 0 0 | 1 | | | | | | | 3 |
| 4 3 3 1 0 0 | 2 | | | | | | | 6 |
| 4 4 1 1 1 0 | | 1 | | | | | | 3 |
| 4 3 2 2 0 0 | 2 | | 3 | | | | | 15 |
| 4 3 2 1 1 0 | 4 | 6 | | 2 | | | | 78 |
| 3 3 3 2 0 0 | 1 | | 3 | | | | | 12 |
| 4 3 1 1 1 1 | | 2 | | | 1 | | | 17 |
| 3 3 3 1 1 0 | 2 | | | 1 | | | | 30 |
| 4 2 2 2 1 0 | | | 2 | 2 | | | | 54 |
| 4 2 2 1 1 1 | | | | 1 | 4 | | | 68 |
| 3 3 2 2 1 0 | 2 | 3 | 6 | 4 | | 6 | | 195 |
| 3 2 2 2 2 0 | | | | 1 | | 4 | | 68 |
| 3 3 2 1 1 1 | 1 | 3 | | 2 | 4 | | 3 | 164 |
| 3 2 2 2 1 1 | | | 1 | 2 | 6 | 4 | 9 | 341 |
| 2 2 2 2 2 1 | | | | | | 1 | 3 | 71 |
| Totals | 15 | 15 | 15 | 15 | 15 | 15 | 15 | 1,125 |

## TABLE A*

### SUMS OF SQUARES OF COLUMN TOTALS AND CORRESPONDING PROBABILITIES FOR THE RELATED-PERCENTAGE TEST

| No. of Columns | No. of Rows | Row Totals | Sums of Squares and (Probabilities) | | | | |
|---|---|---|---|---|---|---|---|
| 2 | 4 | 4(1) | 16(.125) | | | | |
| 2 | 5 | 5(1) | 25(.062) | | | | |
| 2 | 6 | 6(1) | 36(.031) | | | | |
| 2 | 7 | 7(1) | 37(.125) | 49(.016) | | | |
| 2 | 8 | 8(1) | 50(.070) | 64(.008) | | | |
| 2 | 9 | 9(1) | 53(.18[illegible]) | 65(.039) | | | |
| 2 | 10 | 10(1) | 68(.109) | 82(.021) | | | |
| 2 | 11 | 11(1) | 85(.065) | 101(.012) | | | |
| 2 | 12 | 12(1) | 90(.146) | 104(.039) | 122(.006) | | |
| 2 | 13 | 13(1) | 109(.092) | 125(.022) | | | |
| 2 | 14 | 14(1) | 116(.180) | 130(.057) | 148(.013) | | |
| 2 | 15 | 15(1) | 137(.118) | 153(.035) | 173(.007) | | |
| 2 | 16 | 16(1) | 160(.077) | 178(.021) | | | |
| 2 | 17 | 17(1) | 169(.143) | 185(.049) | 205(.013) | | |
| 2 | 18 | 18(1) | 194(.096) | 212(.031) | 234(.008) | | |
| 2 | 19 | 19(1) | 205(.167) | 221(.064) | 241(.019) | | |
| 2 | 20 | 20(1) | 232(.115) | 250(.041) | 272(.012) | | |
| 3 | 3 | 3(2) | 18(.111) | | | | |
| 3 | 3 | 3(1) | 9(.111) | | | | |
| 3 | 4 | 4(2) | 32(.037) | | | | |
| 3 | 4 | 3(2),1(1) | 25(.074) | | | | |
| 3 | 4 | 2(2),2(1) | 20(.074) | | | | |
| 3 | 4 | 1(2),3(1) | 17(.074) | | | | |
| 3 | 4 | 4(1) | 16(.037) | | | | |
| 3 | 5 | 5(2) | 42(.136) | 50(.012) | | | |
| 3 | 5 | 4(2),1(1) | 35(.123) | 41(.025) | | | |
| 3 | 5 | 3(2),2(1) | 30(.123) | 32(.049) | 34(.025) | | |
| 3 | 5 | 2(2),3(1) | 25(.123) | 27(.049) | 29(.025) | | |
| 3 | 5 | 1(2),4(1) | 20(.123) | 26(.025) | | | |
| 3 | 5 | 5(1) | 17(.136) | 25(.012) | | | |
| 3 | 6 | 6(2) | 56(.177) | 62(.053) | | | |
| 3 | 6 | 5(2),1(1) | 49(.177) | 51(.095) | 53(.049) | 61(.008) | |
| 3 | 6 | 4(2),2(1) | 42(.189) | 44(.074) | 46(.049) | 50(.016) | 52(.008) |
| 3 | 6 | 3(2),3(1) | 35(.181) | 41(.058) | 45(.008) | | |
| 3 | 6 | 2(2),4(1) | 30(.189) | 32(.074) | 34(.049) | 38(.016) | 40(.008) |
| 3 | 6 | 1(2),5(1) | 25(.177) | 27(.095) | 29(.049) | 37(.008) | |
| 3 | 6 | 6(1) | 20(.177) | 26(.053) | | | |
| 3 | 7 | 7(2) | 76(.136) | 78(.078) | 86(.021) | | |
| 3 | 7 | 6(2),1(1) | 67(.106) | 69(.078) | 73(.037) | 75(.019) | |
| 3 | 7 | 5(2),2(1) | 62(.093) | 66(.019) | 72(.005) | | |
| 3 | 7 | 4(2),3(1) | 51(.123) | 53(.082) | 57(.030) | 59(.022) | 61(.011) |
| 3 | 7 | 3(2),4(1) | 44(.123) | 46(.082) | 50(.030) | 52(.022) | 54(.011) |
| 3 | 7 | 2(2),5(1) | 41(.093) | 45(.019) | 51(.005) | | |

* Use of the table is discussed on page 7.

TABLE A (Con.)

| No. of Columns | No. of Rows | Row Totals | Sums of Squares and (Probabilities) | | | | |
|---|---|---|---|---|---|---|---|
| 3 | 7 | 1(2),6(1) | 32(.106) | 34(.078) | 38(.037) | 40(.019) | |
| 3 | 7 | 7(1) | 27(.136) | 29(.078) | 37(.021) | | |
| 3 | 8 | 8(2) | 96(.142) | 98(.110) | 102(.059) | 104(.033) | 114(.008) |
| 3 | 8 | 7(2),1(1) | 89(.129) | 93(.033) | 99(.014) | 101(.007) | |
| 3 | 8 | 6(2),2(1) | 76(.167) | 78(.114) | 82(.048) | 86(.026) | 90(.007) |
| 3 | 8 | 5(2),3(1) | 67(.155) | 69(.114) | 73(.055) | 77(.017) | 81(.008) |
| 3 | 8 | 4(2),4(1) | 62(.131) | 66(.036) | 72(.014) | 74(.008) | |
| 3 | 8 | 3(2),5(1) | 51(.155) | 53(.114) | 57(.055) | 61(.017) | 65(.008) |
| 3 | 8 | 2(2),6(1) | 44(.167) | 46(.114) | 50(.048) | 54(.026) | 58(.007) |
| 3 | 8 | 1(2),7(1) | 41(.129) | 45(.033) | 51(.014) | 53(.007) | |
| 3 | 8 | 8(1) | 32(.142) | 34(.110) | 38(.059) | 40(.033) | 50(.008) |
| 3 | 9 | 9(2) | 122(.166) | 126(.050) | 132(.025) | 134(.014) | |
| 3 | 9 | 8(2),1(1) | 109(.146) | 113(.070) | 117(.042) | 121(.014) | 129(.005) |
| 3 | 9 | 7(2),2(1) | 96(.189) | 102(.080) | 104(.057) | 110(.016) | 114(.010) |
| 3 | 9 | 6(2),3(1) | 89(.167) | 93(.054) | 99(.025) | 101(.016) | |
| 3 | 9 | 5(2),4(1) | 76(.198) | 82(.076) | 84(.059) | 90(.016) | 94(.007) |
| 3 | 9 | 4(2),5(1) | 67(.198) | 73(.076) | 75(.059) | 81(.016) | 85(.007) |
| 3 | 9 | 3(2),6(1) | 62(.167) | 66(.054) | 72(.025) | 74(.016) | |
| 3 | 9 | 2(2),7(1) | 51(.189) | 57(.080) | 59(.057) | 65(.016) | 69(.010) |
| 3 | 9 | 1(2),8(1) | 46(.146) | 50(.070) | 54(.042) | 58(.014) | 66(.005) |
| 3 | 9 | 9(1) | 41(.166) | 45(.050) | 51(.025) | 53(.014) | |
| 3 | 10 | 10(2) | 146(.178) | 150(.093) | 154(.059) | 166(.010) | 168(.006) |
| 3 | 10 | 9(2),1(1) | 133(.178) | 137(.106) | 141(.039) | 149(.018) | 153(.006) |
| 3 | 10 | 8(2),2(1) | 122(.202) | 126(.073) | 132(.039) | 134(.025) | 140(.007) |
| 3 | 10 | 7(2),3(1) | 109(.180) | 113(.099) | 117(.053) | 125(.012) | 129(.009) |
| 3 | 10 | 6(2),4(1) | 98(.180) | 102(.103) | 106(.047) | 114(.015) | 118(.007) |
| 3 | 10 | 5(2),5(1) | 89(.202) | 93(.074) | 99(.039) | 101(.026) | 107(.007) |
| 3 | 10 | 4(2),6(1) | 78(.180) | 82(.103) | 86(.047) | 94(.015) | 98(.007) |
| 3 | 10 | 3(2),7(1) | 69(.180) | 73(.099) | 77(.053) | 85(.012) | 89(.009) |
| 3 | 10 | 2(2),8(1) | 62(.202) | 66(.073) | 72(.039) | 74(.025) | 80(.007) |
| 3 | 10 | 1(2),9(1) | 53(.178) | 57(.106) | 61(.039) | 69(.018) | 73(.006) |
| 3 | 10 | 10(1) | 46(.178) | 50(.093) | 54(.059) | 66(.010) | 68(.006) |
| 3 | 11 | 11(2) | 178(.132) | 180(.098) | 182(.053) | 190(.026) | 194(.010) |
| 3 | 11 | 10(2),1(1) | 165(.094) | 171(.054) | 173(.036) | 179(.011) | 185(.007) |
| 3 | 11 | 9(2),2(1) | 150(.123) | 152(.103) | 158(.036) | 166(.015) | 168(.008) |
| 3 | 11 | 8(2),3(1) | 137(.130) | 139(.101) | 141(.061) | 153(.012) | 155(.005) |
| 3 | 11 | 7(2),4(1) | 126(.095) | 132(.055) | 134(.038) | 140(.012) | 146(.007) |
| 3 | 11 | 6(2),5(1) | 113(.127) | 115(.103) | 121(.038) | 129(.014) | 131(.007) |
| 3 | 11 | 5(2),6(1) | 102(.127) | 104(.103) | 110(.038) | 118(.014) | 120(.007) |
| 3 | 11 | 4(2),7(1) | 93(.095) | 99(.055) | 101(.038) | 107(.012) | 113(.007) |
| 3 | 11 | 3(2),8(1) | 82(.130) | 84(.101) | 86(.061) | 98(.012) | 100(.005) |
| 3 | 11 | 2(2),9(1) | 73(.123) | 75(.103) | 81(.036) | 89(.015) | 91(.008) |
| 3 | 11 | 1(2),10(1) | 66(.094) | 72(.054) | 74(.036) | 80(.011) | 86(.007) |
| 3 | 11 | 11(1) | 57(.132) | 59(.098) | 61(.053) | 69(.026) | 73(.010) |
| 3 | 12 | 12(2) | 210(.115) | 216(.070) | 218(.048) | 224(.017) | 230(.012) |
| 3 | 12 | 11(2),1(1) | 193(.147) | 197(.091) | 201(.048) | 209(.023) | 211(.013) |
| 3 | 12 | 10(2),2(1) | 178(.156) | 180(.123) | 186(.052) | 196(.009) | 204(.005) |
| 3 | 12 | 9(2),3(1) | 165(.117) | 171(.071) | 173(.050) | 179(.018) | 185(.011) |
| 3 | 12 | 8(2),4(1) | 150(.151) | 154(.087) | 158(.051) | 168(.012) | 174(.005) |
| 3 | 12 | 7(2),5(1) | 137(.153) | 141(.082) | 145(.052) | 155(.010) | 161(.006) |
| 3 | 12 | 6(2),6(1) | 126(.117) | 132(.072) | 134(.051) | 140(.018) | 146(.011) |

TABLE A (Con.)

| No. of Columns | No. of Rows | Row Totals | Sums of Squares and (Probabilities) | | | | |
|---|---|---|---|---|---|---|---|
| 3 | 12 | 5(2),7(1) | 113(.153) | 117(.082) | 121(.052) | 131(.010) | 137(.006) |
| 3 | 12 | 4(2),8(1) | 102(.151) | 106(.087) | 110(.051) | 120(.012) | 126(.005) |
| 3 | 12 | 3(2),9(1) | 93(.117) | 99(.071) | 101(.050) | 107(.018) | 113(.011) |
| 3 | 12 | 2(2),10(1) | 82(.156) | 84(.123) | 90(.052) | 100(.009) | 108(.005) |
| 3 | 12 | 1(2),11(1) | 73(.147) | 77(.091) | 81(.048) | 89(.023) | 91(.013) |
| 3 | 12 | 12(1) | 66(.115) | 72(.070) | 74(.048) | 80(.017) | 86(.012) |
| 4 | 2 | 2(2) | 8(.167) | | | | |
| 4 | 3 | 3(3) | 27(.062) | | | | |
| 4 | 3 | 2(3),1(2) | 22(.125) | | | | |
| 4 | 3 | 2(3),1(1) | 17(.188) | | | | |
| 4 | 3 | 1(3),2(2) | 19(.083) | | | | |
| 4 | 3 | 1(3),2(1) | 11(.188) | | | | |
| 4 | 3 | 3(2) | 18(.028) | | | | |
| 4 | 3 | 2(2),1(1) | 13(.083) | | | | |
| 4 | 3 | 1(2),2(1) | 10(.125) | | | | |
| 4 | 3 | 3(1) | 9(.062) | | | | |
| 4 | 4 | 4(3) | 42(.203) | 48(.016) | | | |
| 4 | 4 | 3(3),1(2) | 37(.125) | 41(.031) | | | |
| 4 | 4 | 3(3),1(1) | 34(.047) | | | | |
| 4 | 4 | 2(3),2(2) | 34(.083) | 36(.021) | | | |
| 4 | 4 | 2(3),1(2),1(1) | 27(.156) | 29(.062) | | | |
| 4 | 4 | 2(3),2(1) | 24(.047) | | | | |
| 4 | 4 | 1(3),3(2) | 29(.097) | 33(.014) | | | |
| 4 | 4 | 1(3),2(2),1(1) | 24(.083) | 26(.042) | | | |
| 4 | 4 | 1(3),1(2),2(1) | 19(.156) | 21(.062) | | | |
| 4 | 4 | 1(3),3(1) | 18(.047) | | | | |
| 4 | 4 | 4(2) | 24(.134) | 26(.079) | | | |
| 4 | 4 | 3(2),1(1) | 21(.097) | 25(.014) | | | |
| 4 | 4 | 2(2),2(1) | 18(.083) | 20(.021) | | | |
| 4 | 4 | 1(2),3(1) | 13(.125) | 17(.031) | | | |
| 4 | 4 | 4(1) | 10(.203) | 16(.016) | | | |
| 4 | 5 | 5(3) | 63(.180) | 67(.062) | | | |
| 4 | 5 | 4(3),1(2) | 58(.133) | 60(.039) | 66(.008) | | |
| 4 | 5 | 4(3),1(1) | 51(.105) | 57(.012) | | | |
| 4 | 5 | 3(3),2(2) | 51(.146) | 55(.031) | 57(.016) | 59(.005) | |
| 4 | 5 | 3(3),2(1) | 37(.199) | 39(.105) | 41(.035) | 43(.012) | |
| 4 | 5 | 3(3),1(2),1(1) | 44(.125) | 46(.070) | 48(.023) | 50(.016) | |
| 4 | 5 | 2(3),3(2) | 44(.174) | 46(.111) | 48(.035) | 50(.028) | 52(.007) |
| 4 | 5 | 2(3),2(2),1(1) | 39(.141) | 41(.057) | 43(.031) | 45(.010) | |
| 4 | 5 | 2(3),1(2),2(1) | 34(.141) | 36(.047) | 38(.016) | | |
| 4 | 5 | 2(3),3(1) | 27(.199) | 29(.105) | 31(.035) | 33(.012) | |
| 4 | 5 | 1(3),4(2) | 39(.185) | 41(.083) | 43(.056) | 45(.021) | |
| 4 | 5 | 1(3),2(2),2(1) | 29(.141) | 31(.057) | 33(.031) | 35(.010) | |
| 4 | 5 | 1(3),3(2),1(1) | 34(.187) | 36(.076) | 38(.028) | 42(.007) | |
| 4 | 5 | 1(3),1(2),3(1) | 24(.125) | 26(.070) | 28(.023) | 30(.016) | |
| 4 | 5 | 1(3),4(1) | 21(.105) | 27(.012) | | | |
| 4 | 5 | 5(2) | 36(.109) | 38(.047) | 42(.016) | | |
| 4 | 5 | 4(2),1(1) | 29(.185) | 31(.083) | 33(.056) | 35(.021) | |
| 4 | 5 | 3(2),2(1) | 24(.174) | 26(.111) | 28(.035) | 30(.028) | 32(.007) |
| 4 | 5 | 2(2),3(1) | 21(.146) | 25(.031) | 27(.016) | 29(.005) | |

TABLE A (Con.)

| No. of Columns | No. of Rows | Row Totals | Sums of Squares and (Probabilities) | | | | |
|---|---|---|---|---|---|---|---|
| 4 | 5 | 1(2),4(1) | 18(.133) | 20(.039) | 26(.008) | | |
| 4 | 5 | 5(1) | 13(.180) | 17(.062) | | | |
| 4 | 6 | 6(3) | 90(.180) | 92(.062) | 98(.019) | | |
| 4 | 6 | 5(3),1(2) | 81(.189) | 85(.053) | 87(.033) | 89(.012) | |
| 4 | 6 | 5(3),1(1) | 72(.165) | 74(.106) | 76(.048) | 78(.032) | |
| 4 | 6 | 4(3),2(2) | 74(.146) | 76(.062) | 78(.049) | 80(.013) | 82(.009) |
| 4 | 6 | 4(3),1(2),1(1) | 65(.182) | 67(.096) | 69(.051) | 71(.021) | 75(.006) |
| 4 | 6 | 4(3),2(1) | 58(.190) | 60(.073) | 62(.032) | 66(.009) | |
| 4 | 6 | 3(3),3(2) | 67(.122) | 69(.078) | 71(.036) | 75(.010) | 77(.009) |
| 4 | 6 | 3(3),2(2),1(1) | 60(.105) | 62(.048) | 66(.020) | 68(.005) | |
| 4 | 6 | 3(3),1(2),2(1) | 51(.191) | 53(.078) | 55(.055) | 57(.029) | 59(.008) |
| 4 | 6 | 3(3),3(1) | 44(.173) | 46(.103) | 48(.050) | 50(.038) | |
| 4 | 6 | 2(3),4(2) | 60(.139) | 62(.069) | 66(.032) | 68(.009) | 70(.006) |
| 4 | 6 | 2(3),3(2),1(1) | 53(.109) | 55(.082) | 57(.044) | 59(.015) | 61(.009) |
| 4 | 6 | 2(3),2(2),2(1) | 46(.146) | 48(.068) | 50(.055) | 52(.013) | 54(.010) |
| 4 | 6 | 2(3),1(2),3(1) | 39(.191) | 41(.078) | 43(.055) | 45(.029) | 47(.008) |
| 4 | 6 | 2(3),4(1) | 34(.190) | 36(.073) | 38(.032) | 42(.009) | |
| 4 | 6 | 1(3),5(2) | 53(.142) | 55(.111) | 57(.062) | 61(.016) | 63(.008) |
| 4 | 6 | 1(3),4(2),1(1) | 46(.186) | 48(.090) | 52(.025) | 54(.020) | 56(.006) |
| 4 | 6 | 1(3),3(2),2(1) | 41(.109) | 43(.082) | 45(.044) | 47(.015) | 49(.009) |
| 4 | 6 | 1(3),2(2),3(1) | 36(.105) | 38(.048) | 42(.020) | 44(.005) | |
| 4 | 6 | 1(3),1(2),4(1) | 29(.182) | 31(.096) | 33(.051) | 35(.021) | 39(.006) |
| 4 | 6 | 1(3),5(1) | 24(.165) | 26(.106) | 28(.048) | 30(.032) | |
| 4 | 6 | 6(2) | 48(.117) | 50(.102) | 52(.040) | 54(.032) | 56(.011) |
| 4 | 6 | 5(2),1(1) | 41(.142) | 43(.111) | 45(.062) | 49(.016) | 51(.008) |
| 4 | 6 | 4(2),2(1) | 36(.139) | 38(.069) | 42(.032) | 44(.009) | 46(.006) |
| 4 | 6 | 3(2),3(1) | 31(.122) | 33(.078) | 35(.036) | 39(.010) | 41(.009) |
| 4 | 6 | 2(2),4(1) | 26(.146) | 28(.062) | 30(.049) | 32(.013) | 34(.009) |
| 4 | 6 | 1(2),5(1) | 21(.189) | 25(.053) | 27(.033) | 29(.012) | |
| 4 | 6 | 6(1) | 18(.180) | 20(.062) | 26(.019) | | |
| 4 | 7 | 7(3) | 123(.077) | 125(.052) | 127(.021) | 135(.005) | |
| 4 | 7 | 6(3),1(2) | 110(.182) | 112(.089) | 114(.072) | 118(.017) | 122(.010) |
| 4 | 7 | 6(3),1(1) | 101(.132) | 103(.073) | 105(.036) | 109(.014) | 111(.010) |
| 4 | 7 | 5(3),2(2) | 101(.159) | 103(.103) | 105(.053) | 109(.021) | 111(.017) |
| 4 | 7 | 5(3),1(2),1(1) | 92(.135) | 94(.069) | 98(.035) | 100(.012) | 102(.006) |
| 4 | 7 | 5(3),2(1) | 83(.103) | 85(.081) | 87(.051) | 89(.017) | 91(.010) |
| 4 | 7 | 4(3),3(2) | 92(.170) | 94(.093) | 98(.050) | 100(.018) | 102(.012) |
| 4 | 7 | 4(3),2(2),1(1) | 83(.135) | 85(.109) | 87(.068) | 91(.017) | 95(.006) |
| 4 | 7 | 4(3),1(2),2(1) | 74(.182) | 76(.098) | 78(.080) | 82(.020) | 84(.008) |
| 4 | 7 | 4(3),3(1) | 67(.119) | 69(.079) | 71(.043) | 75(.012) | 77(.005) |
| 4 | 7 | 3(3),4(2) | 83(.170) | 87(.088) | 89(.039) | 93(.014) | 95(.010) |
| 4 | 7 | 3(3),3(2),1(1) | 76(.122) | 78(.104) | 80(.037) | 84(.014) | 86(.007) |
| 4 | 7 | 3(3),2(2),2(1) | 67(.149) | 69(.107) | 71(.062) | 75(.020) | 77(.011) |
| 4 | 7 | 3(3),1(2),3(1) | 60(.138) | 62(.073) | 66(.034) | 68(.014) | 70(.007) |
| 4 | 7 | 3(3),4(1) | 53(.119) | 55(.079) | 57(.043) | 61(.012) | 63(.005) |
| 4 | 7 | 2(3),5(2) | 76(.149) | 78(.130) | 82(.046) | 86(.011) | 90(.006) |
| 4 | 7 | 2(3),4(2),1(1) | 67(.180) | 71(.083) | 73(.037) | 77(.019) | 79(.008) |
| 4 | 7 | 2(3),3(2),2(1) | 60(.171) | 62(.095) | 66(.051) | 68(.020) | 70(.013) |
| 4 | 7 | 2(3),2(2),3(1) | 53(.149) | 55(.107) | 57(.062) | 61(.020) | 63(.011) |
| 4 | 7 | 2(3),1(2),4(1) | 46(.182) | 48(.098) | 50(.080) | 54(.020) | 56(.008) |
| 4 | 7 | 2(3),5(1) | 41(.103) | 43(.081) | 45(.052) | 47(.017) | 49(.010) |

TABLE A (Con.)

| No. of Columns | No. of Rows | Row Totals | Sums of Squares and (Probabilities) | | | | |
|---|---|---|---|---|---|---|---|
| 4 | 7 | 1(3),6(2) | 69(.167) | 71(.106) | 73(.052) | 79(.013) | 81(.007) |
| 4 | 7 | 1(3),5(2),1(1) | 60(.204) | 62(.120) | 66(.069) | 70(.020) | 74(.006) |
| 4 | 7 | 1(3),4(2),2(1) | 53(.180) | 57(.083) | 59(.037) | 63(.019) | 65(.008) |
| 4 | 7 | 1(3),3(2),3(1) | 48(.122) | 50(.104) | 52(.037) | 56(.014) | 58(.007) |
| 4 | 7 | 1(3),2(2),4(1) | 41(.135) | 43(.109) | 45(.068) | 49(.017) | 53(.006) |
| 4 | 7 | 1(3),1(2),5(1) | 36(.135) | 38(.069) | 42(.035) | 44(.012) | 46(.006) |
| 4 | 7 | 1(3),6(1) | 31(.132) | 33(.073) | 35(.036) | 39(.014) | 41(.010) |
| 4 | 7 | 7(2) | 62(.147) | 66(.090) | 68(.041) | 74(.011) | 76(.006) |
| 4 | 7 | 6(2),1(1) | 55(.167) | 57(.106) | 59(.052) | 65(.013) | 67(.007) |
| 4 | 7 | 5(2),2(1) | 48(.149) | 50(.130) | 54(.046) | 58(.011) | 62(.006) |
| 4 | 7 | 4(2),3(1) | 41(.170) | 45(.088) | 47(.039) | 51(.014) | 53(.010) |
| 4 | 7 | 3(2),4(1) | 36(.170) | 38(.093) | 42(.050) | 44(.018) | 46(.012) |
| 4 | 7 | 2(2),5(1) | 31(.159) | 33(.103) | 35(.053) | 39(.021) | 41(.017) |
| 4 | 7 | 1(2),6(1) | 26(.182) | 28(.089) | 30(.072) | 34(.017) | 38(.010) |
| 4 | 7 | 7(1) | 25(.077) | 27(.052) | 29(.021) | 37(.005) | |
| 4 | 8 | 8(3) | 156(.116) | 158(.095) | 160(.034) | 166(.017) | 168(.007) |
| 4 | 8 | 7(3),1(2) | 143(.193) | 147(.072) | 151(.033) | 153(.027) | 157(.005) |
| 4 | 8 | 7(3),1(1) | 132(.166) | 134(.091) | 138(.053) | 140(.019) | 142(.012) |
| 4 | 8 | 6(3),2(2) | 132(.200) | 134(.117) | 138(.069) | 142(.019) | 148(.006) |
| 4 | 8 | 6(3),1(2),1(1) | 121(.163) | 125(.091) | 127(.041) | 133(.011) | 135(.007) |
| 4 | 8 | 6(3),2(1) | 112(.126) | 114(.105) | 116(.035) | 120(.016) | 122(.011) |
| 4 | 8 | 5(3),3(2) | 121(.198) | 125(.112) | 127(.055) | 135(.010) | 137(.006) |
| 4 | 8 | 5(3),2(2),1(1) | 112(.152) | 114(.130) | 116(.051) | 124(.009) | 126(.006) |
| 4 | 8 | 5(3),1(2),2(1) | 101(.186) | 105(.082) | 107(.039) | 111(.021) | 113(.007) |
| 4 | 8 | 5(3),3(1) | 92(.169) | 94(.098) | 98(.052) | 100(.023) | 102(.013) |
| 4 | 8 | 4(3),4(2) | 112(.180) | 116(.069) | 118(.061) | 124(.014) | 126(.010) |
| 4 | 8 | 4(3),3(2),1(1) | 103(.164) | 105(.104) | 107(.052) | 113(.013) | 115(.008) |
| 4 | 8 | 4(3),2(2),2(1) | 92(.202) | 94(.121) | 100(.032) | 102(.020) | 106(.007) |
| 4 | 8 | 4(3),1(2),3(1) | 83(.176) | 87(.088) | 89(.041) | 95(.009) | 97(.006) |
| 4 | 8 | 4(3),4(1) | 76(.129) | 78(.108) | 80(.035) | 86(.010) | 88(.005) |
| 4 | 8 | 3(3),5(2) | 103(.194) | 105(.128) | 111(.041) | 115(.013) | 117(.007) |
| 4 | 8 | 3(3),4(2),1(1) | 94(.146) | 98(.090) | 100(.043) | 106(.011) | 108(.007) |
| 4 | 8 | 3(3),3(2),2(1) | 85(.166) | 87(.110) | 89(.055) | 95(.015) | 97(.009) |
| 4 | 8 | 3(3),2(2),3(1) | 76(.154) | 78(.132) | 80(.052) | 88(.009) | 90(.006) |
| 4 | 8 | 3(3),1(2),4(1) | 67(.176) | 71(.088) | 73(.041) | 79(.009) | 81(.006) |
| 4 | 8 | 3(3),5(1) | 60(.169) | 62(.098) | 66(.052) | 68(.023) | 70(.013) |
| 4 | 8 | 2(3),6(2) | 94(.173) | 98(.111) | 100(.056) | 108(.011) | 110(.006) |
| 4 | 8 | 2(3),5(2),1(1) | 85(.196) | 89(.070) | 91(.056) | 97(.014) | 99(.008) |
| 4 | 8 | 2(3),4(2),2(1) | 76(.181) | 80(.070) | 82(.062) | 88(.015) | 90(.011) |
| 4 | 8 | 2(3),3(2),3(1) | 69(.166) | 71(.110) | 73(.055) | 79(.015) | 81(.009) |
| 4 | 8 | 2(3),2(2),4(1) | 60(.202) | 62(.121) | 68(.032) | 70(.020) | 74(.007) |
| 4 | 8 | 2(3),1(2),5(1) | 53(.186) | 57(.082) | 59(.039) | 63(.021) | 65(.007) |
| 4 | 8 | 2(3),6(1) | 48(.126) | 50(.105) | 52(.035) | 56(.016) | 58(.011) |
| 4 | 8 | 1(3),7(2) | 87(.156) | 89(.088) | 93(.050) | 99(.012) | 103(.005) |
| 4 | 8 | 1(3),6(2),1(1) | 78(.186) | 80(.089) | 84(.044) | 88(.022) | 90(.017) |
| 4 | 8 | 1(3),5(2),2(1) | 69(.196) | 73(.070) | 75(.056) | 81(.014) | 83(.008) |
| 4 | 8 | 1(3),4(2),3(1) | 62(.146) | 66(.090) | 68(.043) | 74(.011) | 76(.007) |
| 4 | 8 | 1(3),3(2),4(1) | 55(.164) | 57(.104) | 59(.052) | 65(.013) | 67(.008) |
| 4 | 8 | 1(3),2(2),5(1) | 48(.152) | 50(.130) | 52(.051) | 60(.009) | 62(.006) |
| 4 | 8 | 1(3),1(2),6(1) | 41(.163) | 45(.091) | 47(.041) | 53(.011) | 55(.007) |
| 4 | 8 | 1(3),7(1) | 36(.166) | 38(.091) | 42(.053) | 44(.019) | 46(.012) |

TABLE A (Con.)

| No. of Columns | No. of Rows | Row Totals | Sums of Squares and (Probabilities) | | | | |
|---|---|---|---|---|---|---|---|
| 4 | 8 | 1(3),7(1) | 36(.166) | 38(.091) | 42(.053) | 44(.019) | 46(.012) |
| 4 | 8 | 8(2) | 80(.109) | 82(.097) | 84(.056) | 90(.024) | 94(.008) |
| 4 | 8 | 7(2),1(1) | 71(.156) | 73(.088) | 77(.050) | 83(.012) | 87(.005) |
| 4 | 8 | 6(2),2(1) | 62(.173) | 66(.111) | 68(.056) | 76(.011) | 78(.006) |
| 4 | 8 | 5(2),3(1) | 55(.194) | 57(.128) | 63(.041) | 67(.013) | 69(.007) |
| 4 | 8 | 4(2),4(1) | 48(.180) | 52(.069) | 54(.061) | 60(.014) | 62(.010) |
| 4 | 8 | 3(2),5(1) | 41(.198) | 45(.112) | 47(.055) | 55(.010) | 57(.006) |
| 4 | 8 | 2(2),6(1) | 36(.200) | 38(.117) | 42(.069) | 46(.019) | 52(.006) |
| 4 | 8 | 1(2),7(1) | 31(.193) | 35(.072) | 39(.033) | 41(.027) | 45(.005) |
| 4 | 8 | 8(1) | 28(.116) | 30(.095) | 32(.034) | 38(.017) | 40(.007) |
| 5 | 2 | 2(4) | 16(.200) | | | | |
| 5 | 2 | 2(3) | 12(.100) | | | | |
| 5 | 2 | 2(2) | 8(.100) | | | | |
| 5 | 2 | 2(1) | 4(.200) | | | | |
| 5 | 3 | 3(4) | 36(.040) | | | | |
| 5 | 3 | 2(4),1(3) | 29(.160) | 31(.080) | | | |
| 5 | 3 | 2(4),1(2) | 26(.120) | | | | |
| 5 | 3 | 2(4),1(1) | 21(.160) | | | | |
| 5 | 3 | 1(4),2(3) | 26(.160) | 28(.040) | | | |
| 5 | 3 | 1(4),1(3),1(2) | 23(.120) | | | | |
| 5 | 3 | 1(4),2(2) | 20(.060) | | | | |
| 5 | 3 | 1(4),2(1) | 12(.160) | | | | |
| 5 | 3 | 3(3) | 23(.190) | 27(.010) | | | |
| 5 | 3 | 2(3),1(2) | 20(.090) | 22(.030) | | | |
| 5 | 3 | 2(3),1(1) | 17(.060) | | | | |
| 5 | 3 | 1(3),2(2) | 17(.090) | 19(.030) | | | |
| 5 | 3 | 1(3),2(1) | 11(.120) | | | | |
| 5 | 3 | 1(3),1(2),1(1) | 14(.120) | | | | |
| 5 | 3 | 3(2) | 14(.190) | 18(.010) | | | |
| 5 | 3 | 2(2),1(1) | 11(.160) | 13(.040) | | | |
| 5 | 3 | 1(2),1(1) | 8(.160) | 10(.080) | | | |
| 5 | 3 | 3(1) | 9(.040) | | | | |
| 5 | 4 | 4(4) | 58(.136) | 64(.008) | | | |
| 5 | 4 | 3(4),1(3) | 53(.064) | 57(.016) | | | |
| 5 | 4 | 3(4),1(2) | 46(.168) | 50(.024) | | | |
| 5 | 4 | 3(4),1(1) | 43(.032) | | | | |
| 5 | 4 | 2(4),2(3) | 50(.040) | 52(.008) | | | |
| 5 | 4 | 2(4),1(3),1(2) | 41(.132) | 43(.072) | 45(.024) | | |
| 5 | 4 | 2(4),1(3),1(1) | 36(.112) | 38(.048) | | | |
| 5 | 4 | 2(4),2(2) | 36(.144) | 38(.084) | 40(.012) | | |
| 5 | 4 | 2(4),1(2),1(1) | 31(.192) | 33(.048) | | | |
| 5 | 4 | 2(4),2(1), | 28(.032) | | | | |
| 5 | 4 | 1(4),3(3) | 41(.178) | 43(.106) | 45(.040) | | |
| 5 | 4 | 1(4),2(3),1(2) | 36(.198) | 38(.114) | 40(.024) | 42(.012) | |
| 5 | 4 | 1(4),2(3),1(1) | 33(.072) | 35(.024) | | | |
| 5 | 4 | 1(4),1(3),2(2) | 33(.102) | 35(.042) | 37(.012) | | |
| 5 | 4 | 1(4),1(3),2(1) | 23(.192) | 25(.048) | | | |
| 5 | 4 | 1(4),1(3),1(2),1(1) | 28(.120) | 30(.048) | | | |
| 5 | 4 | 1(4),3(2) | 28(.166) | 30(.078) | 34(.006) | | |
| 5 | 4 | 1(4),2(2),1(1) | 25(.072) | 27(.024) | | | |

TABLE A (Con.)

| No. of Columns | No. of Rows | Row Totals | Sums of Squares and (Probabilities) | | | | |
|---|---|---|---|---|---|---|---|
| 5 | 4 | 1(4),1(2),2(1) | 20(.112) | 22(.048) | | | |
| 5 | 4 | 1(4),3(1) | 19(.032) | | | | |
| 5 | 4 | 4(3) | 38(.151) | 40(.043) | 42(.025) | | |
| 5 | 4 | 3(3),1(2) | 33(.141) | 35(.069) | 37(.021) | | |
| 5 | 4 | 3(3),1(1) | 28(.136) | 30(.078) | 34(.006) | | |
| 5 | 4 | 2(3),2(2) | 30(.111) | 34(.015) | | | |
| 5 | 4 | 2(3),1(2),1(1) | 25(.102) | 27(.042) | 29(.012) | | |
| 5 | 4 | 2(3),2(1) | 20(.144) | 22(.084) | 24(.012) | | |
| 5 | 4 | 1(3),3(2) | 25(.141) | 27(.069) | 29(.021) | | |
| 5 | 4 | 1(3),2(2),1(1) | 20(.198) | 22(.114) | 24(.024) | 26(.012) | |
| 5 | 4 | 1(3),1(2),2(1) | 17(.132) | 19(.072) | 21(.024) | | |
| 5 | 4 | 1(3),3(1) | 14(.163) | 18(.024) | | | |
| 5 | 4 | 4(2) | 22(.151) | 24(.043) | 26(.025) | | |
| 5 | 4 | 3(2),1(1) | 17(.173) | 19(.106) | 21(.040) | | |
| 5 | 4 | 2(2),2(1) | 18(.040) | 20(.008) | | | |
| 5 | 4 | 1(2),3(1) | 13(.064) | 17(.016) | | | |
| 5 | 4 | 4(1) | 10(.136) | 16(.008) | | | |
| 5 | 5 | 5(4) | 88(.098) | 92(.034) | | | |
| 5 | 5 | 4(4),1(3) | 83(.069) | 85(.016) | | | |
| 5 | 5 | 4(4),1(2) | 72(.171) | 74(.114) | 76(.043) | | |
| 5 | 5 | 4(4),1(1) | 65(.168) | 67(.083) | 73(.006) | | |
| 5 | 5 | 3(4),2(3) | 74(.146) | 76(.064) | 80(.011) | 82(.006) | |
| 5 | 5 | 3(4),1(3),1(2) | 67(.156) | 69(.041) | 71(.024) | 73(.010) | |
| 5 | 5 | 3(4),1(3),1(1) | 60(.109) | 62(.042) | 64(.013) | 66(.010) | |
| 5 | 5 | 3(4),2(2) | 60(.144) | 62(.062) | 64(.029) | 66(.012) | |
| 5 | 5 | 3(4),1(2),1(1) | 53(.157) | 55(.077) | 57(.019) | 59(.010) | |
| 5 | 5 | 3(4),2(1) | 48(.083) | 50(.026) | 52(.006) | | |
| 5 | 5 | 2(4),3(3) | 67(.192) | 69(.062) | 71(.040) | 73(.014) | 75(.009) |
| 5 | 5 | 2(4),2(3),1(2) | 60(.179) | 62(.092) | 64(.041) | 66(.019) | 68(.007) |
| 5 | 5 | 2(4),2(3),1(1) | 53(.203) | 55(.107) | 57(.030) | 59(.019) | |
| 5 | 5 | 2(4),1(3),2(2) | 55(.143) | 57(.042) | 59(.032) | 61(.010) | |
| 5 | 5 | 2(4),1(3),1(2),1(1) | 48(.139) | 50(.077) | 52(.029) | 54(.010) | |
| 5 | 5 | 2(4),1(3),2(1) | 41(.186) | 43(.128) | 45(.038) | 47(.010) | |
| 5 | 5 | 2(4),3(2) | 48(.178) | 50(.106) | 52(.042) | 54(.018) | |
| 5 | 5 | 2(4),2(2),1(1) | 43(.158) | 45(.058) | 47(.024) | | |
| 5 | 5 | 2(4),1(2),2(1) | 36(.186) | 38(.128) | 40(.038) | 42(.010) | |
| 5 | 5 | 2(4),3(1) | 33(.083) | 35(.026) | 37(.006) | | |
| 5 | 5 | 1(4),4(3) | 62(.122) | 64(.057) | 66(.029) | 68(.014) | 70(.005) |
| 5 | 5 | 1(4),3(3),1(2) | 55(.177) | 57(.059) | 59(.049) | 61(.017) | |
| 5 | 5 | 1(4),3(3),1(1) | 48(.171) | 50(.106) | 52(.046) | 54(.017) | |
| 5 | 5 | 1(4),2(3),2(2) | 50(.135) | 52(.063) | 54(.027) | 56(.008) | |
| 5 | 5 | 1(4),2(3),1(2),1(1) | 43(.198) | 45(.082) | 47(.036) | 49(.012) | |
| 5 | 5 | 1(4),2(3),2(1) | 38(.158) | 40(.058) | 42(.024) | | |
| 5 | 5 | 1(4),1(3),3(2) | 45(.106) | 47(.056) | 49(.020) | 51(.009) | |
| 5 | 5 | 1(4),1(3),2(2),1(1) | 38(.198) | 40(.082) | 42(.036) | 44(.012) | |
| 5 | 5 | 1(4),1(3),1(2),2(1) | 33(.139) | 35(.077) | 37(.029) | 39(.010) | |
| 5 | 5 | 1(4),1(3),3(1) | 28(.157) | 30(.077) | 32(.019) | 34(.010) | |
| 5 | 5 | 1(4),4(2) | 40(.109) | 42(.051) | 44(.023) | 46(.010) | |
| 5 | 5 | 1(4),3(2),1(1) | 33(.171) | 35(.106) | 37(.046) | 39(.017) | |
| 5 | 5 | 1(4),2(2),2(1) | 28(.203) | 30(.107) | 32(.030) | 34(.019) | |
| 5 | 5 | 1(4),1(2),3(1) | 25(.109) | 27(.042) | 29(.013) | 31(.010) | |

TABLE A (Con.)

| No. of Columns | No. of Rows | Row Totals | Sums of Squares and (Probabilities) | | | | |
|---|---|---|---|---|---|---|---|
| 5 | 5 | 1(4),4(1) | 20(.168) | 22(.083) | 28(.006) | | |
| 5 | 5 | 5(3) | 57(.080) | 59(.068) | 61(.027) | 63(.009) | |
| 5 | 5 | 4(3),1(2) | 50(.167) | 52(.086) | 54(.040) | 56(.014) | 58(.010) |
| 5 | 5 | 4(3),1(1) | 45(.109) | 47(.051) | 49(.023) | 51(.010) | |
| 5 | 5 | 3(3),2(2) | 45(.135) | 47(.074) | 49(.032) | 51(.016) | |
| 5 | 5 | 3(3),1(2),1(1) | 40(.106) | 42(.056) | 44(.020) | 46(.009) | |
| 5 | 5 | 3(3),2(1) | 33(.178) | 35(.106) | 37(.042) | 39(.018) | |
| 5 | 5 | 2(3),3(2) | 40(.135) | 42(.074) | 44(.032) | 46(.016) | |
| 5 | 5 | 2(3),2(2),1(1) | 35(.135) | 37(.063) | 39(.027) | 41(.008) | |
| 5 | 5 | 2(3),1(2),2(1) | 30(.143) | 32(.042) | 34(.032) | 36(.010) | |
| 5 | 5 | 2(3),3(1) | 25(.144) | 27(.062) | 29(.029) | 31(.012) | |
| 5 | 5 | 1(3),4(2) | 35(.167) | 37(.086) | 39(.040) | 41(.014) | 43(.010) |
| 5 | 5 | 1(3),3(2),1(1) | 30(.177) | 32(.059) | 34(.049) | 36(.017) | |
| 5 | 5 | 1(3),2(2),2(1) | 25(.179) | 27(.092) | 29(.041) | 31(.019) | 33(.007) |
| 5 | 5 | 1(3),1(2),3(1) | 22(.156) | 24(.041) | 26(.024) | 28(.010) | |
| 5 | 5 | 1(3),4(1) | 17(.171) | 19(.114) | 21(.043) | | |
| 5 | 5 | 5(2) | 32(.080) | 34(.068) | 36(.027) | 38(.009) | |
| 5 | 5 | 4(2),1(1) | 27(.122) | 29(.057) | 31(.029) | 33(.014) | 35(.005) |
| 5 | 5 | 3(2),2(1) | 22(.192) | 24(.062) | 26(.040) | 28(.014) | 30(.009) |
| 5 | 5 | 2(2),3(1) | 19(.146) | 21(.064) | 25(.011) | 27(.006) | |
| 5 | 5 | 1(2),4(1) | 18(.069) | 20(.016) | | | |
| 5 | 5 | 5(1) | 13(.098) | 17(.034) | | | |
| 6 | 2 | 2(5) | 20(.167) | | | | |
| 6 | 2 | 2(4) | 16(.067) | | | | |
| 6 | 2 | 1(4),1(3) | 13(.200) | | | | |
| 6 | 2 | 2(3) | 12(.050) | | | | |
| 6 | 2 | 1(3),1(2) | 9(.200) | | | | |
| 6 | 2 | 2(2) | 8(.067) | | | | |
| 6 | 2 | 2(1) | 4(.167) | | | | |
| 6 | 3 | 3(5) | 45(.028) | | | | |
| 6 | 3 | 2(5),1(4) | 38(.111) | 40(.056) | | | |
| 6 | 3 | 2(5),1(3) | 35(.083) | | | | |
| 6 | 3 | 2(5),1(2) | 30(.111) | | | | |
| 6 | 3 | 2(5),1(1) | 25(.139) | | | | |
| 6 | 3 | 1(5),2(4) | 35(.111) | 37(.022) | | | |
| 6 | 3 | 1(5),2(3) | 27(.175) | 29(.025) | | | |
| 6 | 3 | 1(5),2(2) | 21(.044) | | | | |
| 6 | 3 | 1(5),2(1) | 13(.139) | | | | |
| 6 | 3 | 1(5),1(4),1(3) | 30(.200) | 32(.067) | | | |
| 6 | 3 | 1(5),1(4),1(2) | 27(.133) | | | | |
| 6 | 3 | 1(5),1(3),1(2) | 24(.100) | | | | |
| 6 | 3 | 3(4) | 32(.111) | | | | |
| 6 | 3 | 2(4),1(3) | 29(.040) | 31(.013) | | | |
| 6 | 3 | 2(4),1(2) | 24(.133) | 26(.027) | | | |
| 6 | 3 | 2(4),1(1) | 21(.044) | | | | |
| 6 | 3 | 1(4),2(3) | 24(.190) | 26(.040) | 28(.010) | | |
| 6 | 3 | 1(4),2(2) | 18(.133) | 20(.027) | | | |
| 6 | 3 | 1(4),2(1) | 12(.111) | | | | |
| 6 | 3 | 1(4),1(3),1(2) | 21(.120) | 23(.040) | | | |
| 6 | 3 | 1(4),1(3),1(1) | 18(.100) | | | | |

TABLE A (Con.)

| No. of Columns | No. of Rows | Row Totals | Sums of Squares and (Probabilities) | | | |
|---|---|---|---|---|---|---|
| 6 | 3 | 1(4),1(2),1(1) | 15(.133) | | | |
| 6 | 3 | 3(3) | 21(.160) | 23(.070) | | |
| 6 | 3 | 2(3),1(2) | 18(.190) | 20(.040) | 22(.010) | |
| 6 | 3 | 2(3),1(1) | 15(.175) | 17(.025) | | |
| 6 | 3 | 1(3),2(2) | 17(.040) | 19(.013) | | |
| 6 | 3 | 1(3),2(1) | 11(.083) | | | |
| 6 | 3 | 1(3),1(2),1(1) | 12(.200) | 14(.067) | | |
| 6 | 3 | 3(2) | 14(.111) | | | |
| 6 | 3 | 2(2),1(1) | 11(.111) | 13(.022) | | |
| 6 | 3 | 1(2),2(1) | 8(.111) | 10(.056) | | |
| 6 | 3 | 3(1) | 9(.028) | | | |
| 6 | 4 | 4(5) | 72(.167) | 74(.097) | | |
| 6 | 4 | 3(5),1(4) | 67(.167) | 69(.037) | 73(.009) | |
| 6 | 4 | 3(5),1(3) | 62(.097) | 66(.014) | | |
| 6 | 4 | 3(5),1(2) | 55(.185) | 59(.019) | | |
| 6 | 4 | 3(5),1(1) | 52(.023) | | | |
| 6 | 4 | 2(5),2(4) | 62(.141) | 66(.022) | | |
| 6 | 4 | 2(5),2(3) | 52(.075) | 54(.038) | | |
| 6 | 4 | 2(5),2(2) | 40(.185) | 42(.074) | 44(.007) | |
| 6 | 4 | 2(5),2(1) | 32(.023) | | | |
| 6 | 4 | 2(5),1(4),1(3) | 57(.067) | 59(.039) | 61(.011) | |
| 6 | 4 | 2(5),1(4),1(2) | 50(.170) | 52(.059) | 54(.022) | |
| 6 | 4 | 2(5),1(4),1(1) | 45(.083) | 47(.037) | | |
| 6 | 4 | 2(5),1(3),1(2) | 45(.161) | 47(.083) | 49(.017) | |
| 6 | 4 | 2(5),1(3),1(1) | 40(.153) | 42(.042) | | |
| 6 | 4 | 2(5),1(2),1(1) | 35(.204) | 37(.037) | | |
| 6 | 4 | 1(5),3(4) | 57(.093) | 59(.058) | 61(.019) | |
| 6 | 4 | 1(5),2(4),1(3) | 52(.107) | 54(.053) | 56(.009) | |
| 6 | 4 | 1(5),2(4),1(2) | 45(.204) | 47(.113) | 49(.027) | 51(.009) |
| 6 | 4 | 1(5),2(4),1(1) | 40(.204) | 42(.059) | 44(.015) | |
| 6 | 4 | 1(5),1(4),2(3) | 47(.145) | 49(.040) | 51(.015) | |
| 6 | 4 | 1(5),1(4),2(2) | 37(.121) | 39(.053) | 41(.009) | |
| 6 | 4 | 1(5),1(4),2(1) | 27(.204) | 29(.037) | | |
| 6 | 4 | 1(5),1(4),1(3),1(2) | 42(.120) | 44(.040) | 46(.013) | |
| 6 | 4 | 1(5),1(4),1(3),1(1) | 37(.094) | 39(.033) | | |
| 6 | 4 | 1(5),1(4),1(2),1(1) | 32(.148) | 34(.044) | | |
| 6 | 4 | 1(5),3(3) | 42(.149) | 44(.059) | 46(.024) | |
| 6 | 4 | 1(5),2(3),1(2) | 37(.155) | 39(.077) | 41(.015) | 43(.005) |
| 6 | 4 | 1(5),2(3),1(1) | 32(.192) | 34(.062) | 36(.012) | |
| 6 | 4 | 1(5),1(3),2(2) | 34(.089) | 36(.022) | 38(.007) | |
| 6 | 4 | 1(5),1(3),2(1) | 24(.153) | 26(.042) | | |
| 6 | 4 | 1(5),3(2) | 29(.129) | 31(.056) | | |
| 6 | 4 | 1(5),2(2),1(1) | 24(.204) | 26(.059) | 28(.015) | |
| 6 | 4 | 1(5),1(2),2(1) | 21(.083) | 23(.037) | | |
| 6 | 4 | 1(5),1(3),1(2),1(1) | 29(.094) | 31(.033) | | |
| 6 | 4 | 1(5),3(1) | 20(.023) | | | |
| 6 | 4 | 4(4) | 52(.138) | 54(.074) | 56(.017) | 58(.010) |
| 6 | 4 | 3(4),1(3) | 47(.179) | 49(.059) | 51(.027) | 53(.006) |
| 6 | 4 | 3(4),1(2) | 42(.152) | 44(.061) | 46(.023) | |
| 6 | 4 | 3(4),1(1) | 37(.129) | 39(.056) | | |
| 6 | 4 | 2(4),2(3) | 42(.182) | 44(.082) | 46(.036) | |

TABLE A (Con.)

| No. of Columns | No. of Rows | Row Totals | Sums of Squares and (Probabilities) | | | | |
|---|---|---|---|---|---|---|---|
| 6 | 4 | 2(4),2(3) | 42(.182) | 44(.082) | 46(.036) | | |
| 6 | 4 | 2(4),1(3),1(2) | 37(.194) | 39(.104) | 41(.024) | 43(.011) | |
| 6 | 4 | 2(4),1(3),1(1) | 34(.089) | 36(.022) | 38(.007) | | |
| 6 | 4 | 2(4),2(2) | 34(.119) | 36(.033) | 38(.015) | | |
| 6 | 4 | 2(4),1(2),1(1) | 29(.121) | 31(.053) | 33(.009) | | |
| 6 | 4 | 2(4),2(1) | 24(.185) | 26(.074) | 28(.007) | | |
| 6 | 4 | 1(4),3(3) | 39(.131) | 41(.037) | 43(.019) | 45(.005) | |
| 6 | 4 | 1(4),2(3),1(2) | 34(.148) | 36(.052) | 38(.024) | | |
| 6 | 4 | 1(4),2(3),1(1) | 29(.155) | 31(.077) | 33(.015) | 35(.005) | |
| 6 | 4 | 1(4),1(3),2(2) | 29(.194) | 31(.104) | 33(.024) | 35(.011) | |
| 6 | 4 | 1(4),1(3),1(2),1(1) | 26(.120) | 28(.040) | 30(.013) | | |
| 6 | 4 | 1(4),1(3),2(1) | 21(.161) | 23(.083) | 25(.017) | | |
| 6 | 4 | 1(4),3(2) | 26(.152) | 28(.061) | 30(.023) | | |
| 6 | 4 | 1(4),2(2),1(1) | 21(.204) | 23(.113) | 25(.027) | 27(.009) | |
| 6 | 4 | 1(4),1(2),2(1) | 18(.170) | 20(.059) | 22(.022) | | |
| 6 | 4 | 1(4),3(1) | 15(.185) | 19(.019) | | | |
| 6 | 4 | 4(3) | 34(.179) | 36(.071) | 38(.035) | 40(.008) | |
| 6 | 4 | 3(3),1(2) | 31(.131) | 33(.037) | 35(.019) | 37(.005) | |
| 6 | 4 | 3(3),1(1) | 26(.149) | 28(.059) | 30(.023) | | |
| 6 | 4 | 2(3),2(2) | 26(.184) | 28(.082) | 30(.036) | | |
| 6 | 4 | 2(3),1(2),1(1) | 23(.145) | 25(.040) | 27(.015) | | |
| 6 | 4 | 2(3),2(1) | 20(.075) | 22(.038) | | | |
| 6 | 4 | 1(3),3(2) | 23(.179) | 25(.059) | 27(.027) | 29(.006) | |
| 6 | 4 | 1(3),2(2),1(1) | 20(.107) | 22(.053) | 24(.009) | | |
| 6 | 4 | 1(3),1(2),2(1) | 17(.067) | 19(.039) | 21(.011) | | |
| 6 | 4 | 1(3),3(1) | 14(.097) | 18(.014) | | | |
| 6 | 4 | 4(2) | 20(.138) | 22(.074) | 24(.017) | 26(.010) | |
| 6 | 4 | 3(2),1(1) | 17(.093) | 19(.058) | 21(.019) | | |
| 6 | 4 | 2(2),2(1) | 14(.141) | 18(.022) | | | |
| 6 | 4 | 1(2),3(1) | 11(.167) | 13(.037) | 17(.009) | | |
| 6 | 4 | 4(1) | 8(.167) | 10(.0[illegible]) | | | |
| 6 | 5 | 5(5) | 113(.059) | 117(.020) | | | |
| 6 | 5 | 4(5),1(4) | 104(.176) | 108(.040) | 110(.008) | | |
| 6 | 5 | 4(5),1(3) | 97(.093) | 99(.065) | 101(.021) | | |
| 6 | 5 | 4(5),1(2) | 90(.094) | 92(.040) | | | |
| 6 | 5 | 4(5),1(1) | 81(.128) | 83(.066) | | | |
| 6 | 5 | 3(5),2(4) | 97(.122) | 99(.084) | 101(.032) | | |
| 6 | 5 | 3(5),1(4),1(3) | 90(.141) | 92(.078) | 94(.017) | 96(.010) | |
| 6 | 5 | 3(5),1(4),1(2) | 83(.149) | 85(.043) | 87(.017) | 89(.006) | |
| 6 | 5 | 3(5),1(4),1(1) | 76(.091) | 78(.026) | 80(.008) | 82(.006) | |
| 6 | 5 | 3(5),2(3) | 83(.181) | 85(.060) | 87(.025) | 89(.011) | |
| 6 | 5 | 3(5),1(3),1(2) | 76(.157) | 78(.057) | 80(.026) | 82(.008) | |
| 6 | 5 | 3(5),1(3),1(1) | 69(.127) | 71(.053) | 73(.012) | 75(.007) | |
| 6 | 5 | 3(5),2(2) | 69(.158) | 71(.075) | 73(.023) | 75(.009) | |
| 6 | 5 | 3(5),1(2),1(1) | 62(.184) | 64(.071) | 66(.015) | 68(.006) | |
| 6 | 5 | 3(5),2(1) | 57(.066) | 59(.019) | | | |
| 6 | 5 | 2(5),3(4) | 90(.170) | 92(.099) | 94(.026) | 96(.017) | 98(.006) |
| 6 | 5 | 2(5),2(4),1(3) | 85(.079) | 87(.039) | 89(.016) | 91(.008) | |
| 6 | 5 | 2(5),2(4),1(2) | 76(.186) | 78(.080) | 80(.038) | 82(.013) | 84(.006) |
| 6 | 5 | 2(5),2(4),1(1) | 69(.162) | 71(.077) | 73(.018) | 75(.012) | |

TABLE A (Con.)

| No. of Columns | No. of Rows | Row Totals | Sums of Squares and (Probabilities) | | | | |
|---|---|---|---|---|---|---|---|
| 6 | 5 | 2(5),1(4),2(3) | 78(.102) | 80(.052) | 82(.017) | 84(.010) | |
| 6 | 5 | 2(5),1(4),1(3),1(2) | 71(.129) | 73(.043) | 75(.024) | 77(.008) | |
| 6 | 5 | 2(5),1(4),1(3),1(1) | 64(.117) | 66(.044) | 68(.019) | 70(.006) | |
| 6 | 5 | 2(5),1(4),2(2) | 64(.149) | 66(.057) | 68(.031) | 70(.009) | |
| 6 | 5 | 2(5),1(4),1(2),1(1) | 57(.153) | 59(.083) | 61(.025) | 63(.007) | |
| 6 | 5 | 2(5),1(4),2(1) | 52(.110) | 54(.031) | 56(.006) | | |
| 6 | 5 | 2(5),3(3) | 71(.156) | 73(.057) | 75(.034) | 77(.012) | |
| 6 | 5 | 2(5),2(3),1(2) | 64(.177) | 66(.077) | 68(.041) | 70(.014) | |
| 6 | 5 | 2(5),2(3),1(1) | 57(.187) | 59(.107) | 61(.035) | 63(.015) | |
| 6 | 5 | 2(5),1(3),2(2) | 59(.135) | 61(.046) | 63(.024) | 65(.006) | |
| 6 | 5 | 2(5),1(3),1(2),1(1) | 52(.156) | 54(.071) | 56(.025) | 58(.006) | |
| 6 | 5 | 2(5),1(3),2(1) | 45(.201) | 47(.127) | 49(.035) | 51(.007) | |
| 6 | 5 | 2(5),3(2) | 52(.188) | 54(.094) | 56(.038) | 58(.011) | |
| 6 | 5 | 2(5),2(2),1(1) | 47(.157) | 49(.051) | 51(.017) | | |
| 6 | 5 | 2(5),1(2),2(1) | 42(.110) | 44(.031) | 46(.006) | | |
| 6 | 5 | 2(5),3(1) | 37(.066) | 39(.019) | | | |
| 6 | 5 | 1(5),4(4) | 85(.100) | 87(.054) | 89(.023) | 91(.012) | |
| 6 | 5 | 1(5),3(4),1(3) | 78(.127) | 80(.068) | 82(.024) | 84(.016) | 86(.005) |
| 6 | 5 | 1(5),3(4),1(2) | 71(.157) | 73(.056) | 75(.036) | 77(.013) | |
| 6 | 5 | 1(5),3(4),1(1) | 64(.143) | 66(.063) | 68(.029) | 70(.010) | |
| 6 | 5 | 1(5),2(4),2(3) | 71(.185) | 73(.073) | 75(.047) | 77(.019) | 79(.007) |
| 6 | 5 | 1(5),2(4),1(3),1(2) | 66(.098) | 68(.056) | 70(.021) | 72(.007) | |
| 6 | 5 | 1(5),2(4),1(3),1(1) | 59(.134) | 61(.051) | 63(.022) | 65(.005) | |
| 6 | 5 | 1(5),2(4),2(2) | 59(.163) | 61(.064) | 63(.033) | 65(.010) | |
| 6 | 5 | 1(5),2(4),1(2),1(1) | 52(.192) | 54(.092) | 56(.037) | 58(.012) | |
| 6 | 5 | 1(5),2(4),2(1) | 47(.157) | 49(.051) | 51(.017) | | |
| 6 | 5 | 1(5),1(4),3(3) | 66(.119) | 68(.071) | 70(.029) | 72(.012) | |
| 6 | 5 | 1(5),1(4),2(3),1(2) | 59(.191) | 61(.081) | 63(.045) | 65(.016) | 67(.007) |
| 6 | 5 | 1(5),1(4),2(3),1(1) | 54(.117) | 56(.051) | 58(.018) | 60(.006) | |
| 6 | 5 | 1(5),1(4),1(3),2(2) | 54(.142) | 56(.068) | 58(.026) | 60(.011) | |
| 6 | 5 | 1(5)1(4)1(3)1(2)1(1) | 49(.088) | 51(.041) | 53(.013) | | |
| 6 | 5 | 1(5),1(4),1(3),2(1) | 42(.156) | 44(.071) | 46(.025) | 48(.006) | |
| 6 | 5 | 1(5),1(4),3(2) | 49(.111) | 51(.057) | 53(.021) | 55(.008) | |
| 6 | 5 | 1(5),1(4),2(2),1(1) | 42(.192) | 44(.092) | 46(.037) | 48(.012) | |
| 6 | 5 | 1(5),1(4),1(2),2(1) | 37(.153) | 39(.083) | 41(.025) | 43(.007) | |
| 6 | 5 | 1(5),1(4),3(1) | 32(.184) | 34(.071) | 36(.015) | 38(.006) | |
| 6 | 5 | 1(5),4(3) | 61(.099) | 63(.059) | 65(.023) | 67(.010) | |
| 6 | 5 | 1(5),3(3),1(2) | 54(.167) | 56(.087) | 58(.035) | 60(.017) | |
| 6 | 5 | 1(5),3(3),1(1) | 49(.108) | 51(.056) | 53(.021) | 55(.008) | |
| 6 | 5 | 1(5),2(3),2(2) | 49(.133) | 51(.073) | 53(.030) | 55(.013) | |
| 6 | 5 | 1(5),2(3),1(2),1(1) | 44(.117) | 46(.051) | 48(.018) | 50(.006) | |
| 6 | 5 | 1(5),2(3),2(1) | 37(.187) | 39(.107) | 41(.035) | 43(.015) | |
| 6 | 5 | 1(5),1(3),3(2) | 44(.143) | 46(.067) | 48(.028) | 50(.010) | |
| 6 | 5 | 1(5),1(3),2(2),1(1) | 39(.134) | 41(.051) | 43(.022) | 45(.005) | |
| 6 | 5 | 1(5),1(3),1(2),2(1) | 34(.117) | 36(.044) | 38(.019) | 40(.006) | |
| 6 | 5 | 1(5),1(3),3(1) | 29(.127) | 31(.053) | 33(.012) | 35(.007) | |
| 6 | 5 | 1(5),4(2) | 39(.163) | 41(.069) | 43(.032) | 45(.010) | |
| 6 | 5 | 1(5),3(2),1(1) | 34(.143) | 36(.063) | 38(.029) | 40(.010) | |
| 6 | 5 | 1(5),2(2),2(1) | 29(.162) | 31(.077) | 33(.018) | 35(.012) | |
| 6 | 5 | 1(5),1(2),3(1) | 26(.091) | 28(.026) | 30(.008) | 32(.006) | |
| 6 | 5 | 1(5),4(1) | 21(.128) | 23(.066) | | | |

TABLE A (Con.)

| No. of Columns | No. of Rows | Row Totals | Sums of Squares and (Probabilities) | | | | |
|---|---|---|---|---|---|---|---|
| 6 | 5 | 5(4) | 78(.152) | 80(.086) | 82(.033) | 84(.024) | 86(.009) |
| 6 | 5 | 4(4),1(3) | 73(.091) | 75(.062) | 77(.027) | 79(.011) | |
| 6 | 5 | 4(4),1(2) | 66(.120) | 68(.073) | 70(.031) | 72(.011) | 74(.005) |
| 6 | 5 | 4(4),1(1) | 59(.163) | 61(.069) | 63(.032) | 65(.010) | |
| 6 | 5 | 3(4),2(3) | 66(.142) | 68(.090) | 70(.040) | 72(.017) | 74(.008) |
| 6 | 5 | 3(4),1(3),1(2) | 61(.102) | 63(.058) | 65(.023) | 67(.012) | |
| 6 | 5 | 3(4),1(3),1(1) | 54(.143) | 56(.067) | 58(.028) | 60(.010) | |
| 6 | 5 | 3(4),2(2) | 54(.168) | 56(.086) | 58(.038) | 60(.017) | 62(.006) |
| 6 | 5 | 3(4),1(2),1(1) | 49(.111) | 51(.057) | 53(.021) | 55(.008) | |
| 6 | 5 | 3(4),2(1) | 42(.188) | 44(.094) | 46(.038) | 48(.011) | |
| 6 | 5 | 2(4),3(3) | 61(.121) | 63(.074) | 65(.031) | 67(.016) | 69(.005) |
| 6 | 5 | 2(4),2(3),1(2) | 54(.195) | 56(.106) | 58(.049) | 60(.024) | 62(.010) |
| 6 | 5 | 2(4),2(3),1(1) | 49(.133) | 51(.073) | 53(.030) | 55(.013) | |
| 6 | 5 | 2(4),1(3),2(2) | 49(.159) | 51(.093) | 53(.041) | 55(.020) | |
| 6 | 5 | 2(4),1(3),1(2),1(1) | 44(.142) | 46(.068) | 48(.026) | 50(.011) | |
| 6 | 5 | 2(4),1(3),2(1) | 39(.135) | 41(.046) | 43(.024) | 45(.006) | |
| 6 | 5 | 2(4),3(2) | 44(.168) | 46(.086) | 48(.038) | 50(.017) | 52(.006) |
| 6 | 5 | 2(4),2(2),1(1) | 39(.163) | 41(.064) | 43(.033) | 45(.010) | |
| 6 | 5 | 2(4),1(2),2(1) | 34(.149) | 36(.057) | 38(.031) | 40(.009) | |
| 6 | 5 | 2(4),3(1) | 29(.158) | 31(.075) | 33(.023) | 35(.009) | |
| 6 | 5 | 1(4),4(3) | 56(.126) | 58(.061) | 60(.032) | 62(.015) | 64(.006) |
| 6 | 5 | 1(4),3(3),1(2) | 49(.182) | 51(.112) | 53(.054) | 57(.008) | 59(.006) |
| 6 | 5 | 1(4),3(3),1(1) | 44(.167) | 46(.087) | 48(.035) | 50(.017) | |
| 6 | 5 | 1(4),2(3),2(2) | 44(.195) | 46(.106) | 48(.049) | 50(.024) | 52(.010) |
| 6 | 5 | 1(4),2(3),1(2),1(1) | 39(.191) | 41(.081) | 43(.045) | 45(.016) | 47(.007) |
| 6 | 5 | 1(4),2(3),2(1) | 34(.177) | 36(.077) | 38(.041) | 40(.014) | |
| 6 | 5 | 1(4),1(3),3(2) | 41(.102) | 43(.058) | 45(.023) | 47(.012) | |
| 6 | 5 | 1(4),1(3),2(2),1(1) | 36(.098) | 38(.056) | 40(.021) | 42(.007) | |
| 6 | 5 | 1(4),1(3),1(2),2(1) | 31(.129) | 33(.043) | 35(.024) | 37(.008) | |
| 6 | 5 | 1(4),1(3),3(1) | 26(.157) | 28(.057) | 30(.026) | 32(.008) | |
| 6 | 5 | 1(4),4(2) | 36(.120) | 38(.073) | 40(.031) | 42(.011) | 44(.005) |
| 6 | 5 | 1(4),3(2),1(1) | 31(.157) | 33(.056) | 35(.036) | 37(.013) | |
| 6 | 5 | 1(4),2(2),2(1) | 26(.186) | 28(.080) | 30(.038) | 32(.013) | 34(.006) |
| 6 | 5 | 1(4),1(2),3(1) | 23(.149) | 25(.043) | 27(.017) | 29(.006) | |
| 6 | 5 | 1(4),4(1) | 20(.094) | 22(.040) | | | |
| 6 | 5 | 5(3) | 51(.132) | 53(.068) | 55(.036) | 57(.011) | 59(.009) |
| 6 | 5 | 4(3),1(2) | 46(.126) | 48(.061) | 50(.032) | 52(.015) | 54(.006) |
| 6 | 5 | 4(3),1(1) | 41(.099) | 43(.059) | 45(.023) | 47(.010) | |
| 6 | 5 | 3(3),2(2) | 41(.121) | 43(.074) | 45(.031) | 47(.016) | 49(.005) |
| 6 | 5 | 3(3),1(2),1(1) | 36(.119) | 38(.071) | 40(.029) | 82(.012) | |
| 6 | 5 | 3(3),2(1) | 31(.156) | 33(.057) | 35(.034) | 37(.012) | |
| 6 | 5 | 2(3),3(2) | 36(.142) | 38(.090) | 40(.040) | 42(.017) | 44(.008) |
| 6 | 5 | 2(3),2(2),1(1) | 31(.185) | 33(.073) | 35(.047) | 37(.019) | 39(.007) |
| 6 | 5 | 2(3),1(2),2(1) | 28(.102) | 30(.052) | 32(.017) | 34(.010) | |
| 6 | 5 | 2(3),3(1) | 23(.181) | 25(.060) | 27(.025) | 29(.011) | |
| 6 | 5 | 1(3),4(2) | 33(.091) | 35(.062) | 37(.027) | 39(.011) | |
| 6 | 5 | 1(3),3(2),1(1) | 28(.127) | 30(.068) | 32(.024) | 34(.016) | 36(.005) |
| 6 | 5 | 1(3),2(2),2(1) | 25(.079) | 27(.039) | 29(.016) | 31(.008) | |
| 6 | 5 | 1(3),1(2),3(1) | 20(.141) | 22(.078) | 24(.017) | 26(.010) | |
| 6 | 5 | 1(3),4(1) | 17(.093) | 19(.065) | 21(.021) | | |
| 6 | 5 | 5(2) | 28(.152) | 30(.086) | 32(.033) | 34(.024) | 36(.009) |

TABLE A (Con.)

| No. of Columns | No. of Rows | Row Totals | Sums of Squares and (Probabilities) | | | | |
|---|---|---|---|---|---|---|---|
| 6 | 5 | 4(2),1(1) | 25(.100) | 27(.054) | 29(.023) | 31(.012) | |
| 6 | 5 | 3(2),2(1) | 20(.170) | 22(.099) | 24(.026) | 26(.017) | 28(.006) |
| 6 | 5 | 2(2),3(1) | 17(.122) | 19(.084) | 21(.032) | | |
| 6 | 5 | 1(2),4(1) | 14(.176) | 18(.040) | 20(.008) | | |
| 6 | 5 | 5(1) | 13(.059) | 17(.020) | | | |

* Use of the table is discussed on page 7.

## TABLE B*

### SUMS OF SQUARES OF COLUMN TOTALS AND CORRESPONDING PROBABILITIES FOR THE MEDIAN TEST FOR COLUMN EFFECTS

| No. of Columns | No. of Rows | Row Totals | Sums of Squares and (Probabilities) | | | | |
|---|---|---|---|---|---|---|---|
| 3 | 3 | 3(1) | 9(.111) | | | | |
| 3 | 4 | 4(1) | 16(.037) | | | | |
| 3 | 5 | 5(1) | 17(.136) | 25(.012) | | | |
| 3 | 6 | 6(1) | 20(.177) | 26(.053) | | | |
| 3 | 7 | 7(1) | 27(.136) | 29(.078) | 37(.021) | | |
| 3 | 8 | 8(1) | 32(.142) | 34(.110) | 38(.059) | 40(.033) | 50(.008) |
| 3 | 9 | 9(1) | 41(.166) | 45(.050) | 51(.025) | 53(.014) | |
| 3 | 10 | 10(1) | 46(.178) | 50(.093) | 54(.059) | 66(.010) | 68(.006) |
| 3 | 11 | 11(1) | 57(.132) | 59(.098) | 61(.053) | 69(.026) | 73(.010) |
| 3 | 12 | 12(1) | 66(.115) | 72(.070) | 74(.048) | 80(.017) | 86(.012) |
| 4 | 2 | 2(2) | 8(.167) | | | | |
| 4 | 3 | 3(2) | 18(.028) | | | | |
| 4 | 4 | 4(2) | 24(.134) | 26(.079) | | | |
| 4 | 5 | 5(2) | 36(.109) | 38(.047) | 42(.016) | | |
| 4 | 6 | 6(2) | 48(.117) | 50(.102) | 52(.040) | 54(.032) | 56(.011) |
| 4 | 7 | 7(2) | 62(.147) | 66(.090) | 68(.041) | 74(.011) | 76(.006) |
| 4 | 8 | 8(2) | 80(.109) | 82(.097) | 84(.056) | 90(.024) | 94(.008) |
| 4 | 9 | 9(2) | 98(.178) | 100(.104) | 106(.043) | 114(.010) | 116(.006) |
| 4 | 10 | 10(2) | 116(.187) | 122(.091) | 126(.062) | 136(.011) | 138(.009) |
| 5 | 2 | 2(2) | 8(.100) | | | | |
| 5 | 3 | 3(2) | 14(.190) | 18(.010) | | | |
| 5 | 4 | 4(2) | 22(.151) | 24(.043) | 26(.025) | | |
| 5 | 5 | 5(2) | 32(.080) | 34(.068) | 36(.027) | 38(.009) | |
| 5 | 6 | 6(2) | 42(.138) | 44(.080) | 46(.053) | 50(.017) | 52(.007) |
| 5 | 7 | 7(2) | 54(.163) | 56(.093) | 60(.037) | 64(.015) | 66(.011) |
| 5 | 8 | 8(2) | 68(.139) | 70(.122) | 74(.048) | 82(.010) | 84(.005) |
| 6 | 2 | 2(3) | 12(.050) | | | | |
| 6 | 3 | 3(3) | 21(.160) | 23(.070) | | | |
| 6 | 4 | 4(3) | 34(.179) | 36(.071) | 38(.035) | 40(.008) | |
| 6 | 5 | 5(3) | 51(.132) | 53(.068) | 55(.036) | 57(.011) | 59(.009) |
| 6 | 6 | 6(3) | 70(.131) | 72(.075) | 74(.050) | 78(.013) | 80(.008) |
| 7 | 2 | 2(3) | 12(.029) | | | | |
| 7 | 3 | 3(3) | 21(.079) | 23(.030) | | | |
| 7 | 4 | 4(3) | 32(.162) | 34(.065) | 36(.024) | 38(.010) | |
| 7 | 5 | 5(3) | 45(.203) | 47(.131) | 49(.063) | 53(.016) | 55(.008) |
| 7 | 6 | 6(3) | 62(.188) | 66(.074) | 68(.048) | 72(.014) | 74(.008) |
| 8 | 2 | 2(4) | 16(.014) | | | | |
| 8 | 3 | 3(4) | 28(.140) | 30(.030) | 32(.010) | | |
| 8 | 4 | 4(4) | 46(.091) | 48(.047) | 50(.016) | 52(.006) | |
| 8 | 5 | 5(4) | 66(.146) | 68(.087) | 70(.045) | 74(.010) | 76(.005) |

* Use of the table is discussed on page 7.

TABLE B (Con.)

| No. of Columns | No. of Rows | Row Totals | Sums of Squares and (Probabilities) | | | | |
|---|---|---|---|---|---|---|---|
| 9 | 2 | 2(4) | 14(.167) | 16(.008) | | | |
| 9 | 3 | 3(4) | 26(.183) | 28(.069) | 30(.013) | | |
| 9 | 4 | 4(4) | 42(.165) | 44(.085) | 46(.033) | 48(.016) | |
| 10 | 2 | 2(5) | 18(.103) | | | | |
| 10 | 3 | 3(5) | 35(.085) | 37(.028) | | | |
| 10 | 4 | 4(5) | 56(.117) | 58(.055) | 60(.025) | 62(.008) | |
| 11 | 2 | 2(5) | 18(.067) | | | | |
| 11 | 3 | 3(5) | 33(.135) | 35(.042) | 37(.012) | | |
| 11 | 4 | 4(5) | 52(.182) | 54(.098) | 56(.046) | 58(.020) | 60(.008) |
| 12 | 2 | 2(6) | 22(.040) | | | | |
| 12 | 3 | 3(6) | 40(.165) | 42(.064) | 44(.017) | | |
| 12 | 4 | 4(6) | 66(.135) | 68(.067) | 70(.032) | 72(.013) | 74(.005) |
| 13 | 2 | 2(6) | 22(.025) | | | | |
| 13 | 3 | 3(6) | 40(.091) | 42(.031) | 44(.008) | | |
| 14 | 2 | 2(7) | 24(.143) | 26(.015) | | | |
| 14 | 3 | 3(7) | 47(.121) | 49(.043) | 51(.013) | | |
| 15 | 2 | 2(7) | 24(.100) | 26(.009) | | | |
| 15 | 3 | 3(7) | 45(.161) | 47(.066) | 49(.021) | 51(.006) | |
| 16 | 2 | 2(8) | 28(.061) | 30(.005) | | | |
| 16 | 3 | 3(8) | 52(.197) | 54(.086) | 56(.031) | 58(.009) | |