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SECONDARY SCHOOL MATHEMATICS CURRICULUM IMPROVEMENT STUDY. FINAL REPORT.

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THIS SECONDARY SCHOOL MATHEMATICS CURRICULUM IMPROVEMENT STUDY GROUP (SSMCIS), COMPOSED OF BOTH AMERICAN AND EUROPEAN EDUCATORS, WAS GUIDED BY TWO MAIN OBJECTIVES--(1) TO CONSTRUCT AND EVALUATE A UNIFIED SECONDARY SCHOOL MATHEMATICS PROGRAM FOR GRADES 7-12 THAT WOULD TAKE THE CAPABLE STUDENT WELL INTO CURRENT COLLEGE MATHEMATICS, AND (2) DETERMINE EDUCATIONAL REQUIREMENTS FOR TEACHERS OF SUCH A PROGRAM. AIMS AND PROCEDURES WERE ESTABLISHED, AND A FLOW CHART OF SCOPE AND SEQUENCE WAS USED AS A GUIDE FOR THE PREPARATION OF THE COURSE I (SEVENTH GRADE) SYLLABUS. A TEAM OF EIGHT MATHEMATICIANS WROTE THE TEXTBOOK FOR COURSE I. TEACHER GUIDES WERE WRITTEN AND DISTRIBUTED FOR USE IN PILOT CLASSES. NINE METROPOLITAN NEW YORK AREA JUNIOR HIGH SCHOOLS PARTICIPATING IN THE EXPERIMENTAL PROGRAM NAMED TWO TEACHERS EACH TO BE GIVEN 100 HOURS OF SPECIAL TRAINING, 50 HOURS IN FUNDAMENTAL MATHEMATICS, AND 50 HOURS IN MODERN METHODS OF TEACHING. THE COURSE WAS THEN TAUGHT BY EACH TEAM OF TEACHERS. EVALUATION WAS ACCOMPLISHED IN THREE WAYS--(1) BY DIRECT OBSERVATION THROUGH CLASSROOM VISITATION BY THE PROJECT DIRECTOR AND STAFF MEMBERS, (2) THROUGH DISCUSSIONS WITH COURSE TEACHERS AND PROJECT PERSONNEL DURING FOUR FULL DAY CONFERENCES, AND (3) BY TESTING, USING THE SEQUENTIAL TEST OF EDUCATIONAL PROGRESS, AND THREE DIFFERENT SSMCIS TESTS. RESULTS OF THE PILOT CLASSES INDICATED THAT THE NEW COURSE, BASED ON FUNDAMENTAL CONCEPTS AND STRUCTURES, WAS STIMULATING TO TEACHERS, CHALLENGING AND INTERESTING TO THE STUDENTS, AND GAVE PROMISE AS A FEASIBLE ONE YEAR COURSE FOR THE SEVENTH GRADE. (DH)

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October, 1967

U.S. DEPARTMENT OF
HEALTH, EDUCATION, AND WELFARE

Office of Education
Bureau of Research

FINAL REPORT
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SECONDARY SCHOOL MATHEMATICS CURRICULUM IMPROVEMENT STUDY

Howard F. Fehr

Teachers College, Columbia University

New York, New York

October 1967

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U.S. DEPARTMENT OF
HEALTH, EDUCATION, AND WELFARE

Office of Education
Bureau of Research

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SUMMARY

The Secondary School Mathematics Curriculum Improvement Study (SSMCIS) has two main objectives:

1) To formulate and test a unified secondary school mathematics program (7-12) that will take capable students well into current collegiate mathematics;

2) To determine the education required by teachers who will implement such a program.

To inaugurate the study, leading United States and European mathematicians and educators met to formulate a position paper stating the aims and procedures of the study and to construct a flow charted analysis of the proposed 7-12 mathematics course. Using the scope and sequence flow chart as a guide, a detailed syllabus was prepared for Course I (seventh grade). This syllabus was expanded in papers describing methods of writing and teaching each specific topic.

Using the detailed syllabus as a guide, a team of eight mathematical educators wrote the textbook for Course I. Each chapter was written by one writer, reviewed by the other writers and a consulting mathematician, and then revised before preparation for printing. Teachers guides and solutions to exercises were written for each chapter and mimeographed for distribution to the teachers of pilot classes.

Nine junior high schools in the metropolitan New York area were selected to participate in the experimental teaching of Course I. Each of these schools designated two capable and interested teachers who were given summer instruction in preparation for teaching Course I. The instruction included 50 hours in the fundamental mathematical concepts underlying the unified mathematics program and 50 hours in contemporary methods of teaching. During the following academic year each team of two teachers taught a single pilot class using the SSMCIS textbook.

The experimental teaching was evaluated in three ways. The director and project staff members made frequent visits to the classes for direct observation. The students were tested by three examinations--prepared by the project staff--designed specifically to measure learning of important new concepts such as operational system, mapping, and geometric transformation as well as standard topics. Teachers, staff, and consultants met at four full day conferences to discuss progress and problems in the experimental teaching.

Results of the experimental teaching showed that the new mathematics course, based on fundamental concepts and structures, gave promise of meeting the expectations of the proposed six year program. It was stimulating for the teachers to teach, challenging and interesting for the students, and, with several revisions, a feasible one year course for the seventh grade.

Introduction

During the past decade the United States has been engaged in revising the elementary and secondary school mathematics curriculum--primarily by up-dating the existing traditional curriculum. Modest recommendations of the Commission on Mathematics have been largely accepted by curriculum and syllabus bodies and by writers of commercially produced textbooks. Implementation of this program by the SMSG has had wide acceptance and massive experimental use throughout the country.

Throughout all of our reform movements the traditional division of mathematics instruction into separate years of arithmetic, algebra, and geometry has been maintained. Beyond introduction of new concepts, little has been gained in bringing more advanced study into the high school through more efficient methods of organizing the subject matter. Bolder and more radical recommendations for the improvement of secondary school education in mathematics have been made both in this country, notably by the UICSM, and in Europe, notably in Belgium, Switzerland, and Denmark.

What has been called for is reconstruction of the entire curriculum from a global point of view--one which eliminates the barriers separating the several branches of mathematics and unifies the subject through its general concepts (sets, operations, mappings, and relations) and builds the fundamental structures of the number systems, algebra, and geometry (groups, rings, fields, and vector spaces). The efficiency gained by such organization should permit introduction into the high school program of much that was previously considered undergraduate mathematics.

In September 1965, the Commissioner of Education, Department of Health, Education, and Welfare, Office of Education, approved for support for a period of 18 months the Secondary School Mathematics Curriculum Improvement Study (SSMCIS), an experimental study whose objective would be the construction of a unified school mathematics curriculum for grades seven through twelve. This is a report of the activities and findings of the SSMCIS from its inception to the end of the first year of experimental teaching in June 1967.

Planning the 7-12 Program

Long range planning of the proposed six year study was begun with a two day meeting of chief consultants in November 1965. The participants at this meeting outlined procedures for subsequent syllabus conferences, writing of the experimental textbooks, and evaluation of teaching in pilot classes.

In June 1966 a group of eighteen leading United States and European mathematicians and educators met for 20 days to outline the scope and sequence of a six year unified secondary school mathematics program. The first half of the conference was devoted to producing a complete flow charted analysis of the proposed course. Then topics planned for the seventh grade were expanded in working papers which outlined the mathematical content of each textbook chapter and made specific suggestions for writing and teaching these ideas.

Writing of Course I

During July and August 1966, a team of eight mathematical educators wrote the textbook for Course I, using the syllabus produced in June as a guide. Each textbook chapter was written by one writer, reproduced for review by the other writers and consulting mathematicians, and then rewritten, incorporating the reviewers' suggestions. Teachers guides and solutions to exercises were written for each chapter. These notes, mimeographed and distributed to teachers of experimental classes, included discussions of fundamental mathematical ideas underlying each chapter, hints for possible class activity to accompany reading of the text, and suggested time allotment to the various topics.

The Course I textbook (Experimental Edition) contained 16 chapters and was published in three volumes. Chapter titles and brief descriptions appear below. A more detailed description, including chapter subheadings, is given as Appendix A of this report.

Chapter	Title	Description
0	Planning a Mathematical Process	Introduction to flow charting algorithms.
1	Finite Number Systems	Study of properties of modular arithmetic systems.
2	Sets and Operations	Introduction to binary operations.
3	Mathematical Mappings	Introduction to concept of mapping.

Chapter	Title	Description
4	The Integers	Addition of Integers.
5	Probability and Statistics	First concepts of probability and descriptive statistics.
6	Multiplication of Integers	Definition and properties of $(Z,\cdot)$.
7	Lattice Points in the Plane and Mappings on $Z \times Z$	Lattice representation of pairs of integers.
8	Sets and Relations	Set notation and properties of relations.
9	Transformations of the Plane and Orientations in the Plane	Line reflections, point symmetries, rotations, and translations.
10	Segments, Angles, and Isometries	Measure of segments and angles and preservation under certain mappings.
11	Elementary Number Theory	Divisibility, primes, and the Euclidean algorithm.
12	The Rational Numbers	Addition, multiplication, and order of rational numbers.
13	Mass Point Geometry	A small deductive system involving mass points.
14	Some Applications of the Rational Numbers	Dilations, ratio and proportion, percent, and translations.
15	Incidence Geometry	A small axiomatic affine geometry.

Education of Teachers

During the June 1966 conference a special program of study was arranged to prepare all the teachers of experimental classes to teach Course I. Then each of the experimental teachers studied four hours daily for thirty days during the Teachers College 1966 Summer Session. The instruction covered fundamental mathematical concepts underlying the unified mathematics program and contemporary methods of teaching standard and new mathematical topics.

The program included the courses:

TX 4351--Modern Mathematical Structures

Theory of sets; groups, rings, ordered fields, isomorphism; affine space; euclidean space; real numbers; vector spaces; numerical calculus; statistics.

taught by Mr. Burt Kaufman of Southern Illinois University, and

TX 4406--Teaching Contemporary Junior High School Mathematics

Emphasis on teaching mathematics as a unified branch of knowledge. Teaching: set theory, mapping, relations, and functions; structure of number systems; groups, rings, and field properties; algebraic structure; vectors; translations, reflections, rotations; symmetries; plane geometry. Experimental programs and evaluation of mathematical learning.

taught by Dr. Julius H. Hlavaty, a chief consultant to the project.

The following is a list of the teachers and the schools in which they taught the experimental classes during the 1966-67 school year:

Carbondale, Illinois, University High School
Mr. Dave Masters

Elmont, New York, Alva Stanforth Junior High School
Mr. Alexander Imre
Mr. Edward Keenan

Fort Washington, Pennsylvania, Germantown Academy
Mr. Ronald Craig
Mr. Wirt Thompson

Glen Rock, New Jersey, Glen Rock Junior High School
Mr. James Law
Mr. Neil McDermott

Leonia, New Jersey, Leonia High School
Miss Christine McGoey
Mr. Kenneth McGown

New York, N.Y., Hunter College High School
Mr. Richard Klutch
Miss Ruth Morgan

Port Chester, New York, Ridge Street School
Miss Riva Machlin
Mr. Thomas Reistetter

Teaneck, New Jersey, Benjamin Franklin Junior High School
Mrs. Annabelle Cohen
Mr. Otto Krupp

Thomas Jefferson Junior High School
Mr. Franklin Armour
Miss Louise Fischer

Westport, Connecticut, Bedford Junior High School
Mr. James Detweiler
Mr. Ray Walch

Each teacher was taken through selected chapters of the following books:

1) T. J. Fletcher, ed., Some Lessons in Mathematics, Cambridge Press.

2) D. E. Mansfield and M. Bruckheimer, Major Topics in Modern Mathematics, Harcourt, Brace, World.

3) G. Papy, Mathématiques Moderne I, Didier.

4) Mimeographed version of the experimental Course I textbook.

All teachers showed intense interest and cooperated splendidly in acquiring the spirit and content of the proposed new curriculum.

Teaching Course I

Nine junior high schools in the Metropolitan New York area and one in Carbondale, Illinois, were selected to participate in the experimental teaching of Course I. In each school two teachers who had received special summer instruction were assigned to teach a single pilot class. Eight of these classes were at the seventh grade level and two at the eighth grade level--involving a total of 350 students.

The SSMCIS program is at present designed for capable students roughly those in the top 20% of their class with respect to mathematical ability. With this population in mind, pilot classes were selected by the participating schools using prior mathematics achievement and scores on aptitude tests as the main criteria.

For the first year large classes (35-40 students) were encouraged so that a gradual drop out would enable a class of sufficient size to complete the sixth year of the program. However, at the end of the first year very few pupils are leaving the program and all classes, except that at Glen Rock, where administrative problems make it impossible to continue, will move ahead into Course II during the 1967-68 school year. Teachers and students alike have found the mathematics intellectually stimulating, interesting, and enjoyable. In fact, several students who have been forced to leave the program because of family change of residence have asked to be allowed to continue studying the SSMCIS textbooks on their own.

Because the teachers of pilot classes were working as a team in the experimental class, they were often able to help each other with difficulties that arose in understanding or teaching the new material. Having had this year of team teaching experience, the teachers are now prepared to teach Course I individually. The revised Course I text will therefore be used in approximately 20 new seventh grade classes during the 1967-68 school year.

During the School year, the director and project staff members made frequent personal visits to observe the experimental teaching. Each class was observed at least four times. Visits to these schools included discussion with the teachers and administrators concerning progress and problems with the experimental course.

The teachers were further assisted by four full day Saturday meetings at Teachers College where teaching problems were reviewed with selected consultants and the project director. At these meetings many teaching difficulties were resolved and valuable criticisms of the textbook were gathered.

Evaluation of Course I

The six year mathematics program, of which Course I is only the first part, introduces many new concepts into the secondary school mathematics curriculum and integrates both standard and new topics in a global organization not characteristic of existing programs. Student achievement in such a program cannot adequately be measured using conventional standardized tests. For this reason, student learning was tested by three extramural examinations, constructed by the project staff.

To guide construction of these and future measurement instruments, the Course I textbook was analyzed to produce a taxonomy of cognitive objectives. This taxonomy delineated goals of instruction in terms of subject matter and related behaviors. The framework for this analysis is illustrated in the following tables.

TABLE I

SCHEME FOR TAXONOMY OF OBJECTIVES

Mathematical Objectives

Structures: Arithmetic and Algebra Geometry Probability

Fundamental Concepts: Sets Operations Relations Mappings Logic

Behavioral Objectives

I. Ability to recall definitions, notations, operations, concepts.

II. Ability to manipulate and calculate efficiently.

III. Ability to interpret symbolic data or processes.

IV. Ability to communicate mathematical ideas.

V. Ability to apply concept to a purely mathematical situation--solve problems.

VI. Ability to apply concept to problems in other situations--solve word problems.

VII. Ability to transfer learning to a new situation in mathematics.

VIII. Ability to construct or follow a mathematical argument.

Of course not all these categories apply to each subject matter topic, but the goals were checked against subject matter in a two-way cellular chart similar to the following:

CONTENT	I	II	III	IV	V	VI	VII	VIII
	Ability to recall definitions, notations, operations concepts.	Ability to manipulate and calculate efficiently.	Ability to interpret symbolic data or process.	Ability to communicate mathematical ideas.	Ability to apply concept to a purely mathematical situation. Solve problems.	Ability to apply concept to problemsin other situations. Solve word problems.	Ability to transfer learning to a new situation in mathematics.	Ability to construct or follow a mathematical argument. (Proof)
Sets	X	X	X	X	X	X	X	X
A. Notation								
1. Inclusion ⊂	X		X	X	X			
2. Membership ∈	X		X	X	X			
3. Empty ∅	X		X	X				
4. Ven Diagrams	X		X	X	X			
B. Cardinality								
1. Finite	X							
2. Infinite	X							
C. Set Equality								
1. Subsets	X	X						
2. Disjoint Sets	X							
D. Universal Sets · Partitions	X	X	X		X	X		
E. Set Operation	X							
1. Union	X	X	X	X	X		X	X
2 Intersection	X	X	X	X	X		X	X
3 Complement		X	X	X	X		X	X

TABLE II
SAMPLE PAGE FROM TAXONOMY
OF OBJECTIVES

The three examinations, constructed from this taxonomy were designed to measure learning of the important new concepts in Course I such as operational system, mapping, and geometric transformation as well as standard seventh and eighth grade topics which remain a part of the new course. Copies of these tests, administered in November, February, and May, appear as Appendix B at the end of this report.

Although achievement on standardized traditional mathematics tests was not accepted as a measure of the success of the experimental program, it was of interest to determine whether or not study in the experimental Course I affected learning of traditional topics. To accomplish this objective all students were administered the Sequential Test of Educational Progress--Mathematics, Form 3A--in September 1966 and again in September 1967. The results of this testing are shown in Table III following, along with the scores on the three project designed tests.

These test results clearly show that students in the project classes suffered no decline in mathematical skills when compared with students studying more traditional programs. Moreover, the achievement of these students on the project tests shows that they were learning to work with many new and powerful mathematical tools not a part of the traditional mathematics fare of seventh graders.

Item error analysis of the project test papers provided insight into the success of particular aspects of Course I instruction. The complete data available from these and other measures of student aptitude and achievement will be analyzed statistically during the year 1967-68.

Revision of Course I

Throughout the 1966-67 school year the project staff gathered detailed reactions to Course I from consultants and teachers, test results, and observations of the experimental teaching. These findings suggested the following revision of Course I.

1. The chapter on flow charting mathematical processes should be rewritten to emphasize graphic representation of algorithms rather than review and examination of the computational procedures of whole number arithmetic. In revised form this chapter would then be more appropriate as a summarizing chapter than an introductory one.

2. The chapter on addition of integers should be rewritten from a different point of view since the isomorphism concept will be removed from Course I and placed in Course II.

TABLE III

TEST SCORES SSMCIS PILOT CLASSES 1966-67

Class	STEP Class Mean Percentile 9/66	STEP Class Mean Percentile 9/67	SSMCIS Test I Mean % Correct 11/66	SSMCIS Test II Mean % Correct 2/67	SSMCIS Test III Mean % Correct 5/67 I	II
1.	98+	98+	77%	63%	79%	74%
2.	98+	#	83%	59%	66%	50%
3.	98+	98+	80%	57%	74%	75%
4.	98+	98+	82%	66%	72%	69%
5.	#	#	82%	55%	60%	50%
6.	#	#	#	59%	72%	66%
7.	98+	98+	67%	45%	62%	50%
8.	98	98	67%	56%	#	#
9.	97	#	78%	54%	60%	63%
10.	98+	98+	82%	60%	63%	55%

#Scores unavailable

3. The chapter on mappings should be simplified and shortened by omitting the work with central and parallel projections.

4. The chapter on rational numbers should be rewritten in a style consistent with the new approach to the integers.

5. The concept of orientation should be transferred from the chapter on transformation in the plane to Course II.

6. The chapter on mass points and affine geometry should be expanded and made part of Course II.

Some of these revisions began in the spring of 1967; the remainder were left for the summer writing group.

This revised Course I textbook will be pilot tested again in the same schools during the 1967-68 school year. However, this year each teacher will teach a class of seventh graders individually. Therefore, the new seventh grade experimental population will include over 20 classes and 700 students. The work of this year will be carefully evaluated and in July 1968 a final revised edition of Course I will be placed in the public domain.

Planning for Course II

The June 1966 syllabus conference prepared a tentative list of chapters for Course II. However, the order of these chapters, details for writing the textbook, and modifications due to the experimental teaching of Course I was determined in the spring of 1967.

To initiate the Summer 1967 writing, a pre-planning session was held on Saturday and Sunday, March 11-12. The persons attending were Howard F. Fehr (Director), Hans-Georg Steiner, Julius H. Hlavaty, Edgar Ray Lorch, Marshall H. Stone, and Thomas J. Hill (consultants) and H. Laverne Thomas and James Fey (research assistants).

The following items were discussed:

1. Review of the experimental study to March 1, 1967.

2. Revision of Course I in light of the experimental results.

3. Improvement of Teachers Manuals for the course.

4. Testing program and its results.

5. Preparations for the Office of Education Site Visiting Committee on April 10, 1967.

6. Organization of the June syllabus conference.

7. Organization of the summer writing.

8. Teacher education programs.

9. Extension of pilot classes to the eighth year.

In particular, the group outlined the responsibilities of the June 1967 conference for planning Course II. Copies of this report were mailed to all participants in the June conference one month in advance of that meeting. A copy of this report appears as Appendix C.

On April 10, the Office of Education sent a Site Team of experts to examine the work of the project, its accomplishments, and its projected plans for the future. The O.E. Team consisted of:

> Miss Veryl Schult, Office of Education
> Dr. Andrew Molnar, Office of Education
> Professor Gail Young, Tulane University
> Dr. Leon Cohen, Executive Officer, Mathematics Division, National Academy of Sciences
> Dr. Robert Davis, Syracuse University and Webster College.

The persons representing the Project were:

> Professor Howard F. Fehr, Director
> Professor Thomas J. Hill, Coordinator of Writing Team
> Dr. Julius H. Hlavaty, Consultant
> H. Laverne Thomas, Research Assistant
> James Fey, Research Assistant.

Persons representing the cooperating schools were:

> Dr. Bernard Miller, Principal, Hunter High School
> Mr. James Winn, Secondary Curriculum Coordinator, Teaneck, N.J. Public Schools
> Mr. Franklin Armour, Teacher, Thomas Jefferson Junior High School, Teaneck, N.J.
> Mr. Edward Keenan, Teacher, Alva Stanforth Junior High School, Elmont, New York
> Mr. Ray Walch, Mathematics Coordinator, Westport, Conn.

The report of this visit is in the files of the Office of Education.

A proposal for support of the continuation of the experimental curriculum study for a period of 18 months was presented to the Office of Education in January 1967. On June 14, 1967, the proposal was approved, and initial financing of $112,000 of Federal Funds allocated for the period June 15, 1967 to January 31, 1968, a period of 7 months. The financing of the remaining 11 months of approved project activity is to be negotiated in early Fall 1967.

Immediate Future Activity

The activities as presented in the new proposal are:

1. To develop the complete syllabus and write the text and teachers manuals for Course II, experimental edition.

2. To revise Course I, for a second year of controlled teaching. At the end of the school year 1967-68, Course I will be examined for minor revisions and then placed in the public domain.

3. To teach, under controlled conditions, the Experimental Course II during the year 1967-68.

4. To hold a June conference in 1968, followed by Summer writing, to make revisions of Course II, to prepare a syllabus and write Course III, to train the teachers to teach Course III experimentally, and to initiate the classroom teaching.

Summary of Accomplishments to June 30, 1967

The first phase of SSMCIS has produced a flow chart of a global integrated curriculum in mathematics for grades 7 through 12. This chart breaks down the traditional barriers among the subjects arithmetic, algebra, geometry, and analysis, and reorganizes their development using the more general and unifying concepts of sets, relations, mappings, functions, and the structures of group, field, and vector spaces. From the flow charts, and extended notes prepared by the high-ranking mathematicians of Europe and the USA, a group of mathematical educators wrote a complete experimental textbook Course I for grade 7. The Course was tested in ten classes involving 350 children and the results of this teaching provided sufficient information to revise the textbook into a sound and teachable program for grade 7.

The experimental teachers pursued an intensive teacher education program of 100 hours of study before initiating the experiment and by critical evaluation helped to evolve the revised seventh year course. The training program gave a comprehensive knowledge of the background a junior high school teacher must have to administer the course successfully. The study program was bolstered by Teachers Manuals, one for each chapter, produced by the writers and mathematicians. These manuals listed goals, supplementary materials, and solutions (or answers) to all problems in the textbooks.

The Study has planned for the continuation of the project by writing material and initiating the teaching of Course II (eighth grade) and Course III (ninth grade). It has also initiated means of releasing the results of all the experiment with respect to grade 7 to the public domain after the Summer of 1968.

Concomitantly it has initiated and furthered two doctoral researches, one on the hierarchy of learning of the function concept, the other on oral communication in the mathematics classroom.

APPENDIX A

COURSE I CONTENT

Chapter

0 PLANNING A MATHEMATICAL PROCESS
- Flow Charts
- Branching and Looping in Flow Charts
- Flow Charting Addition
- Flow Chart for Subtraction
- Operations and Non-operations
- Flow Charting Multiplication

1 FINITE NUMBER SYSTEMS
- Jane Anderson's Arithmetic
- Clock Numbers and Whole Numbers
- Clock Arithmetic
- Calender Arithmetic
- Open Sentences
- New Clocks
- Rotations
- Subtraction in Clock Arithmetic
- Multiplication in Clock Arithmetic
- Division in Clock Arithmetic
- Properties of Clock Arithmetic
- The Associative and Distributive Properties

2 SETS AND OPERATIONS
- Ordered Pairs of Numbers and Assignments
- What is an Operation?
- Computations with Operations
- Open Sentences
- Properties of Operations
- Cancellation Laws
- Two Operational Systems
- What is a Group?

3 MATHEMATICAL MAPPINGS
- What is a Mathematical Mapping?
- Arrow Diagrams and Mappings
- Mapping of Dial Numbers
- Sequences
- Composition of Mappings
- Inverse and Identity Mappings
- Translations Along a Line
- Mappings from W to W on Parallel Lines
- More on Mappings from W to W on Parallel Lines
- Parallel Projections

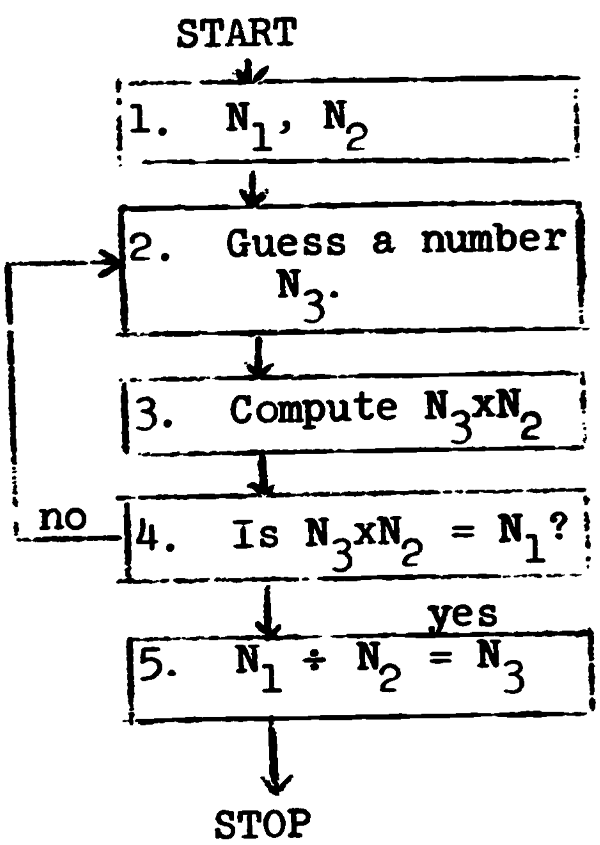
APPENDIX B

INTERIM AND FINAL TESTS OF

SSMCIS

Test I November 1966

I. At the right is a flow chart for one method of dividing a number N_1 by another number N_2. Match each phrase below with the box number or numbers which it describes.



__________	decision box	2,3,4
__________	loop	3
__________	operation box	1
__________	output box	5
__________	input box	4

II. Given below are tables for addition and multiplication in Z_6.

+	0	1	2	3	4	5
0	0	1	2	3	4	5
1	1	2	3	4	5	0
2	2	3	4	5	0	1
3	3	4	5	0	1	2
4	4	5	0	1	2	3
5	5	0	1	2	3	4

·	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	1	2	3	4	5
2	0	2	4	0	2	4
3	0	3	0	3	0	3
4	0	4	2	0	4	2
5	0	5	4	3	2	1

A. Compute each of the following in $(Z_6,+,\cdot)$

1. $2 + (3 + 5)$ ________
2. $2 \cdot (3 \cdot 5)$ ________
3. $4 + ({}^-3)$ ________
4. $5 - ({}^-2)$ ________
5. $4 \cdot (2 + 5)$ ________

B. Find the solution set for each of thefollowing open sentences in $(Z_6,+,\cdot)$

1. $x + 3 = 1$ _____
2. $3 \cdot y = 2$ _____
3. $5z + {}^-2 = 3$ _____
4. ${}^-3 \cdot p = 0$ _____

C. Give an example which shows why each of the following is false.

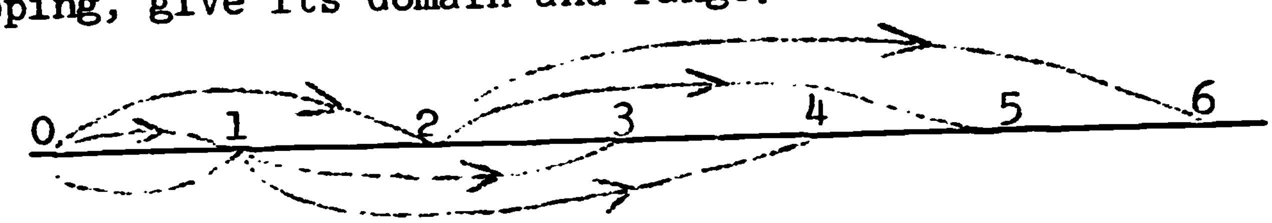
1. Every element of $(Z_6,+,\cdot)$ has a multiplicative inverse.

_______________________.

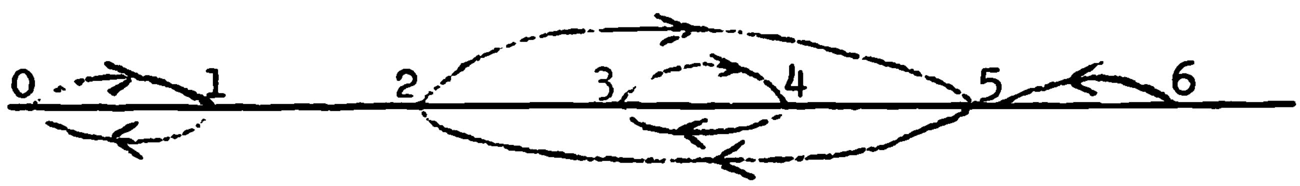
2. Subtractionis commutative in Z_6 ______________________

III. For each of the following diagrams state whether or not the diagram represents a mapping. If it does represent a mapping, give its domain and range.

a)



No _____ Yes _____ Domain: { } Range: { }

b) 

No _____ Yes _____ Domain: { } Range: { }

IV. Two operations, o and # are defined on the set of whole numbers by a o b = a and a # b = 2a + b

A. Compute the following:

1. 5 o 11 _____

2. 5 # 11 _____

3. 8 # (12 o 1007) _____

4. (6 o 5) # 10 _____

B. Identify each of the following as true or false:

__________1. In (W, o), o is commutative.

__________2. In (W, #), # is commutative.

__________3. In (W, o), o is associative.

__________4. a # b = a # c implies b = c.

C. Find the solution sets of the following open sentences in (W, o, #).

1. 7 # y = 15 _____

2. x o 7 = 10 _____

3. 8 o (r # y) = 9 _____

V. A mapping between sets of whole numbers is to have the rule n ———> 2n - 5.

a) What set of whole numbers cannot belong to the domain of this mapping? {__________}

b) What set of whole numbers can be in the range of this mapping? {__________}

VI. Let the mapping f from W to W have the rule $n \longrightarrow n + 3$, and let the mapping g from W to W have the rule $n \longrightarrow 2n$.

a) What is the image of 5 under f? ____

b) What is the image of 5 under the composition of f with g? ____________

VII. Consider the mapping from Z_5 to Z_5 given by the rule $n \longrightarrow (n \cdot n) + 2$.

a) On the figure at the right construct an arrow diagram for the mapping.

0

4 1

3 2

b) Is the mapping a one-to-one mapping? Yes ___ No___

Why? ________________________________

c) Is the mapping an onto mapping? Yes ___ No___

Why? ________________________________

Test II February 1967

Given at the right are multiplication and addition tables for Z_4.

+	0	1	2	3
0	0	1	2	3
1	[illegible]	2	3	0
2	2	3	0	1
3	3	0	1	2

·	0	1	2	3
0	0	0	0	0
1	0	1	2	3
2	0	2	0	2
3	0	3	2	1

1. Determine which of the following are or are not properties of $(Z_4, +, \cdot)$. In either case, illustrate your answer with a numerical example. (For example: "ab = ba" is true -- $2 \cdot 3 = 2$ and $3 \cdot 2 = 2$.)

a) $x(yz) = (yx)z$ for any x, y, and z in Z_4. _____

example: ______________________

b) If $xy = xz$ then $y = z$ for any x, y and z in Z_4. ______

example: ______________________

c) $0 \cdot x = x$ for any x in Z_4. ______

example: ______________________

d) If $xy = x + y$, then $x = y = 0$. ______

example: ______________________

2. Find the solution set of each of the following open sentences in $(Z_4, +, \cdot)$.

a) $2x = 3$ ______

b) $3x + {}^{-}2 = 0$ ______

c) $2(x + 1) = 2$ ______

3. For each of the following questions write the letter corresponding to your choice of the correct response in the space at the right of the question.

1) f is a mapping of Z_6 to Z_6 with rule $n \longrightarrow 3n + 2$ and g is a mapping of Z_6 to Z_6 with rule $n \longrightarrow 3n - 4$.

Thus

a) f and g are the same mapping.

b) f and g are different mappings.

c) f and g are inverse mappings.

d) f and g are onto mappings. ______

2)

0 1 2 3 4 5 6 7 8 ——> b

——————————> a
0 1 2 3 4 5 6 7 8

Lines a and b are parallel and scaled with the same unit and in the same direction.

2.1) f is a translation from W (on a) to W (on b) which maps 3 to 14. Then f must have the rule

a) $n \longrightarrow 5n - 1$

b) $n \longrightarrow 17 - n$

c) $n \longrightarrow n + 11$

d) $n \longrightarrow 3n + 5$ ______

2.2) h is a mapping of the subset {1,2,3,...,20} of W (on a) to W (on b) with rule $n \longrightarrow 20 - n$. In this mapping

a) h is parallel projection

b) the image of 17 is to the right of the image of 15.

c) h is a translation.

d) the image of 17 is to the left of the image of 15.

____

2.3) If g is a central projection of W (on a) to W (on b) where the center of the projection is not between lines a and b, and g: $3 \longrightarrow 15$, then g has the rule

a) $n \longrightarrow 5n - 3$

b) $n \longrightarrow 18 - n$

c) $n \longrightarrow 5n$

d) $n \longrightarrow 6n - 3$

____

2.4) If j is a translation of W (on a) to W (on b), A, B, C are points on line a such that j: $A \longrightarrow A'$ j: $B \longrightarrow B'$, j: $C \longrightarrow C'$, and $AC = \frac{3}{4}AB$, then

a) $A'C' = \frac{1}{4}A'B'$

b) $\overleftrightarrow{AB'}$ is parallel to $\overleftrightarrow{A'B}$

c) $C'B' = \frac{3}{4}A'B'$

d) $A'C' = \frac{3}{4}A'B'$

____

4. Three operations $\#_1$, $\#_2$, $\#_3$ are defined on the set of whole numbers by $a \#_1 b$ = minimum of a and b, $a \#_2 b = 0$, $a \#_3 b = 2a \#_1 b$.

4.1) Compute the following:

a) $16 \#_1 33$ ______

b) $(4 \#_2 13) \#_3 12$ ______

c) $5 \#_1 (7 \#_2 11)$ ______

4.2) Identify each of the following as true or false.

____ a) In $(W,\#_1)$, $\#_1$ is commutative.

____ b) In $(W,\#_2)$, if $a \#_2 b = a \#_2 c$, then $b = c$.

____ c) In $(W,\#_1,\#_2)$ $a \#_3 b = 2(a \#_1 b)$

____ d) In $(W,\#_1,\#_2)$, $a \#_1 (b \#_2 c) = (a \#_1 b) \#_2 (a \#_1 c)$

4.3) Find the solution set of each of the following open sentences.

a) $y \#_2 (3 \#_1 (4 \#_3 1)) = 5.$

b) $10 \#_1 (y \#_3 5) = 7$

c) $y \#_1 7 = 7 \#_3 1$

5. Compute each of the following in $(Z,+)$, the integers.

a) $36 + {}^-54$ ______

b) ${}^-36 + {}^+57$ ______

c) $({}^-21 - {}^-30) - {}^-12$ ______

d) $|{}^-17 + {}^-3|$ ______

e) $|{}^-17| + |{}^-3|$ ______

6. Fifteen balls numbered 1-15 are placed in a box, If the balls are well mixed and one ball is drawn out of the box, what is the probability that a ball is drawn which has

a) the number 3 ______

b) an odd number ______

c) a number greater than 4 ______

7. For each of the following insert the correct symbol ($<$, $>$, $=$) in the blank.

   a) $^{-}97$ ______ $^{-}32$

   b) $|^{-}13 + 10|$ ______ $|^{-}73 - {}^{-}76|$

   c) $14 - {}^{-}79$ ______ $79 + {}^{-}14$

   d) $^{-}(^{-}(^{-}6))$ ______ $^{+}6$

   d) $|^{-}x|$ ______ $|x|$ for x any integer.

8. A standard thumbtack is tossed 15 times. On each toss it can land up or down. Determine which of the following statements are true and which are false.

   a) It is impossible for the tack to land "up" all 15 times.

   b) The tack will either land "up" seven times and "down" eight times or "up" eight times and "down" seven times.

   c) The relative frequency of occurrence of "up" plus the relative frequency of occurrence of "down" is 1.

   d) If in an earlier toss of 15 the tack had turned "up" eight times and "down" seven times, then this result is certain to occur in each following group of 15 tosses

9. Compute each of the following in the indicated operational system.

a)	$5 + (3 \cdot 6)$	In $(Z_7, +, \cdot)$	______
b)	$(4 - {}^{-}2) \cdot 3$	In $(Z_5, +, \cdot)$	______
c)	$20(6 + {}^{-}4)$	In $(Z_{24}, +, \cdot)$	______
d)	$(3 + 4)^2$	In $(Z_6, +, \cdot)$	______

10. Indicate by writing "True" or "False" which of the following statements are true and which false.

   a) In $(Z,+)$ each element has an additive inverse ______

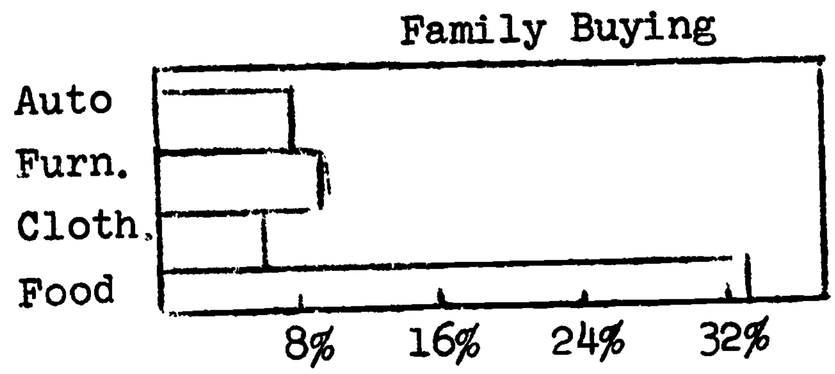
   b) In $(Z,+)$ $a - b = b - a$ for any integers a and b ______

   c) The equation $a + x = b$ has an integer solution for every pair of integers a and b ______

d) If $a + b = c + a$, then $b = c$ for any integers a, b, c. ______

e) $|a - b| < |a + b|$ for any integers a and b ______

11. Referring to the bar graph at the right, answer each of the following questions:



a) What does each unit on the horizontal scale represent? ______

b) The amount spent for food is about how many times as much as that spent for funniture? ______

c) If the horizontal scale were 2% to each unit shown, how would the length of each bar be changed? ______

12. Find the solution set in $(Z,+)$ for each of the following open sentences.

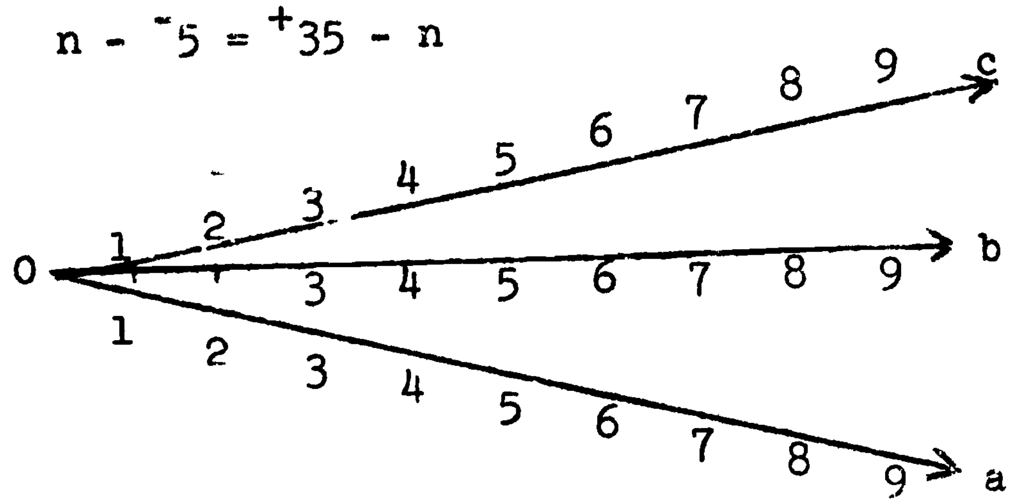
a) $n + {}^-8 = {}^+5$ ______

b) $n \} {}^-396 = {}^-1732 - {}^+21$ ______

c) $|n| + |n - {}^+4| = 24$ ______

d) $n - {}^-5 = {}^+35 - n$

13.



Lines a, b, c intersect at the same point and are scaled with the same unit, with zero at the point of intersection. For each of the following record the letter of your choice for the correct response in the space provided at the right.

13.1 f is a mapping from W (on a) to W(on b) with rule $n \longrightarrow 2n$. If straight lines are drawn connecting points on line a to their images on line b, then

a) the lines drawn are parallel.

b) the lines drawn are not parallel and do not intersect at a common point.

c) the lines drawn intersect at a common point.

d) f is a central projection.

____

13.2 If g: W (on a) ⟶ W (on b) and h: W(on b) ⟶ W (on c) are parallel projections then h <u>with</u> g: W (on a) ⟶ W (on c)

a) is a central projection.

b) has rule of the form $n \longrightarrow n + a$.

c) is neither a parallel nor a central projection.

d) is a parallel projection.

____

14. A large corporation sponsors 20 television programs each week in the New York Area. A sample of 20 people were selected and questioned as to the number of these programs they watched during the preceding week. The results were then recorded in a grouped frequency table as shown below.

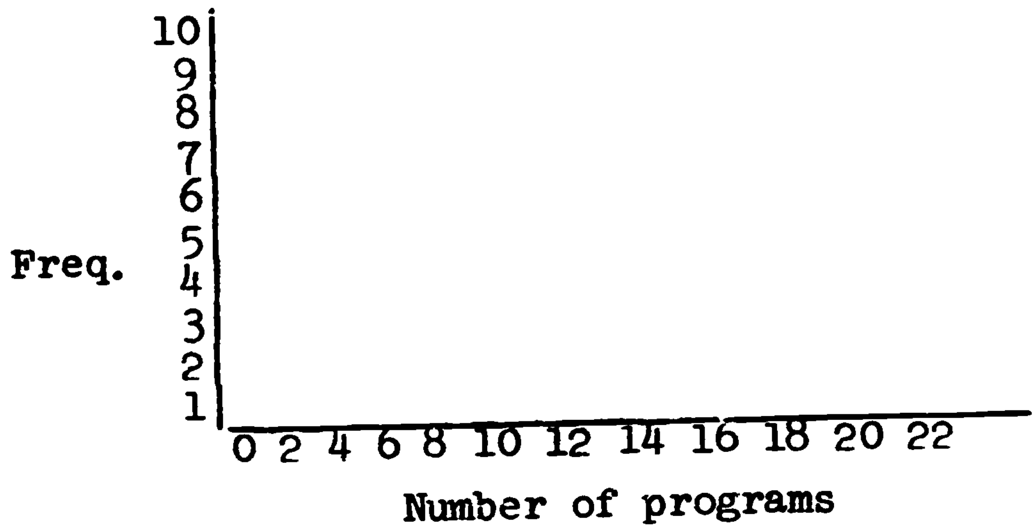
Number of programs watched (class interval)	Frequency
0-4	5
4-8	8
8-12	2
12-16	4
16-20	1

Each interval includes its upper limit.

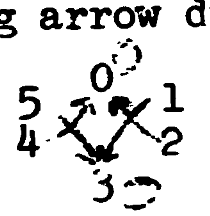
a) How many people watched no more than 8 programs? ____

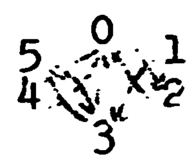
b) How many people watched more than 12 programs? ____

c) On the diagram complete the frequency histogram using the data given in the table.



15. The following arrow diagrams represent mappings of Z_6 to Z_6.





15.1 Draw an arrow diagram for the mapping g with f.

15.2 Find a rule of the form $n \longrightarrow an$ for the mapping $f: Z_6 \longrightarrow Z_6$. ____________

15.3 Find the set of all elements of Z_6 whose image under f with g is

a) 2 ____________

b) 5 ____________

16. Given three mappings f, g, and h of W to W with the rules $f: n \longrightarrow 5n$, $g: n \longrightarrow n + 7$ and $h: n \longrightarrow 3n + 2$,

a) Find the image of 7 under h with (f with g). ______

b) Find all values of x such that h with f: $x \longrightarrow 47$.

______

FINAL TEST

PART I

1) Compute in the system of integers — (Z, +, •).

a) (-13) - (-17) = _____

b) 11 + (-5) = _____

c) (-13)(5) = _____

d) (-13)(-5) = _____

e) (20 x (-9)) + (20 x (-16)) = _____

f) -50(15 - 23) = _____

2) Solve each of the following open sentences in the system of integers -- (Z, +, •).

a) $x + 10 = 2$ _____

b) $3x = 20 - x$ _____

c) $|x - 5| = 5$ _____

d) $x + 3(x - 2) = 2$ _____

e) $x\,(x + 2) = 0$ _____

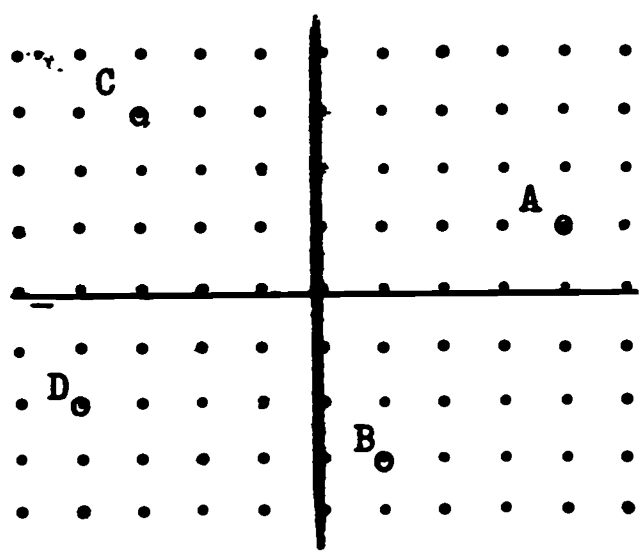
3) For each point labeled on the lattice at the right, give the ordered pair of integers which are its coordinates.

A = _____

B = _____

C = _____

D = _____

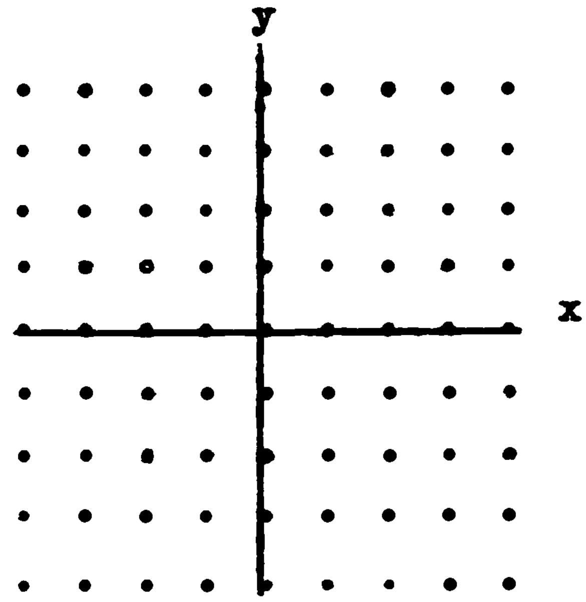


4) On the lattice at the right, draw a closed curve encircling the set of points with coordinates (x,y) satisfying:

a) $y = -x$.

b) $y = x + 1$.

Indicate which set of points is a) and which is b).



5) Answer true (T) or false (F).

____ a) $|-20 + 36| < |20 - (-36)|$

____ b) $-232 > -752$

____ c) $(Z, +)$ is isomorphic to $(Q', +)$ where Q' is the set of all rational numbers which can be written in the form $\frac{a}{1}$ for some integer a.

____ d) $(Z^+, \cdot)$ is isomorphic to $(Q, \cdot)$ where Z^+ is the set of positive integers and zero.

____ e) $-5 \in \{ w : w \text{ is a whole number} \}$.

____ f) $\{1, 3, 5, 7, \ldots\} \cap \{0, 2, 4, 6, \ldots\} = W$.

____ g) $\{a, b\} \times \{c, d\} = \{(a,c), (b,d)\}$.

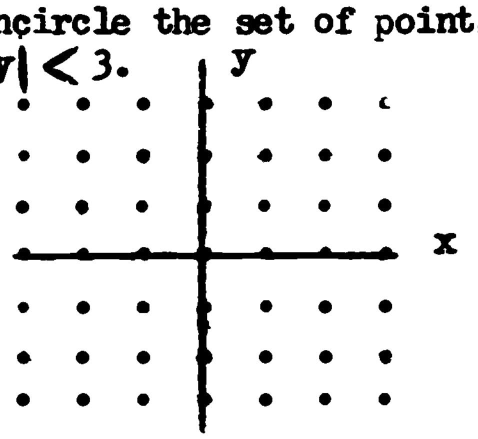
____ h) For whole numbers x, y, and z, if $x < y$ and $y < z$, then $x < z$.

____ i) If E is an event with probability $\frac{3}{8}$, then the probability of the complementary event $\bar{E}$ is $\frac{3}{8}$.

____ j) The additive inverse of $\frac{-12}{21}$ is $\frac{12}{21}$.

____ k) The multiplicative inverse of $\frac{-12}{21}$ is $\frac{21}{12}$.

6) On the lattice below, encircle the set of points with coordinates meeting the condition $|x| + |y| < 3$.



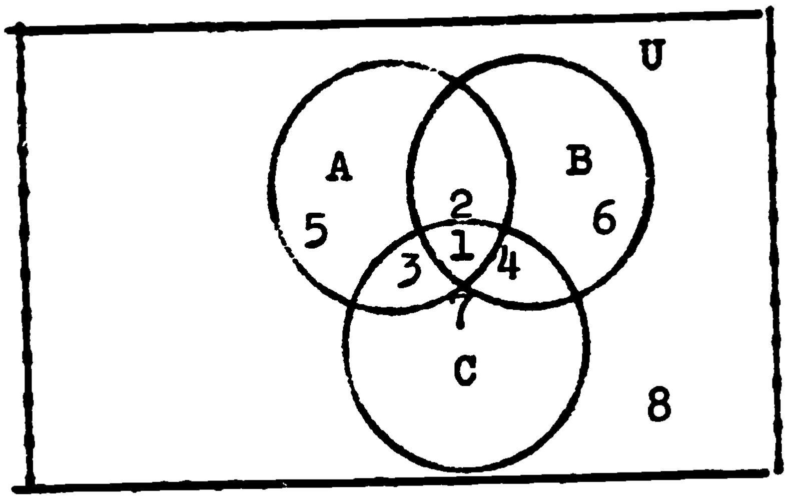
7) Each region of the Venn diagram below is numbered. Next to each set given at the left, list all the regions of the diagram which make up the set. The answer to the first part is given as an example.

A <u>1, 2, 3, 5</u>

$A \cup C$ ____________

$B \cap C$ ____________

$\bar{B}$ ____________



8) The pointer on the dial at the right is spun. Match the event below with its probability.

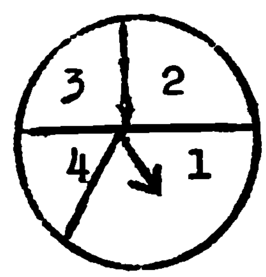
_____ a) 1 $\frac{1}{6}$

_____ b) 1 or 2 or 4 $\frac{1}{2}$

_____ c) 4 $\frac{1}{3}$

_____ d) 1 or 4 $\frac{2}{3}$

$\frac{3}{4}$



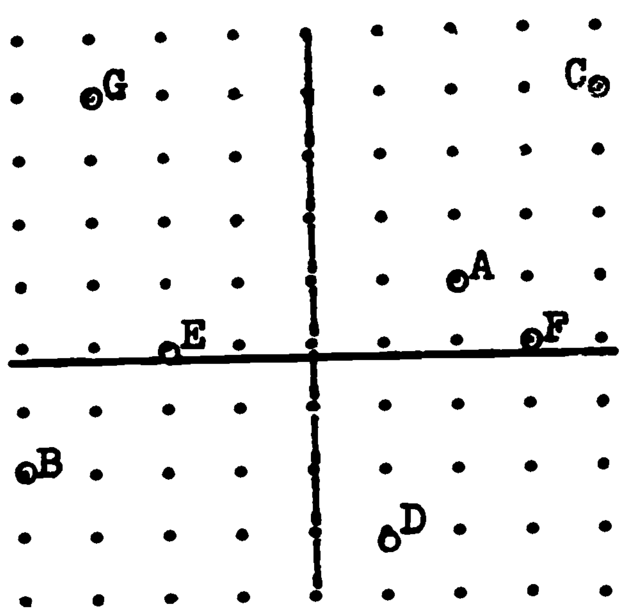
9) Listed below are the rules for four mappings of Z x Z. Each mapping takes point A = (2,1) into a differant image. Select the image of A for each mapping from the set of labeled points on the lattice.

____ $T_{2,3}$: $(x,y) \dashrightarrow (x+2,y+3)$

____ $T_{-1,-4}$: $(x,y) \dashrightarrow (x-1,y-4)$

____ D_{-2}: $(x,y) \dashrightarrow (-2x, -2y)$

____ $T_{-1,-4} \circ T_{2,3}$: $(x,y) \dashrightarrow$?



10) From a school of 900 students, one student is chosen at random to represent the school at a state convention.

_______ a) What is each student's probability of being chosen?

_______ b) If the school has 40% boys, what is the probability that a girl will be selected?

_______ c) If the school has 250 eighth graders, what is the probability that one of them is chosen?

11) Mr. Green finds that on a long trip he averages 200 miles in 4 hours of driving. He is planning a trip of 900 miles.

__________ a) Using t for the number of hours needed to travel 900 miles, write a correct proportion involving t.

__________ b) Find t.

12) What is the actual cost of a tennis racket marked $19.95 if it is sold at a discount of 15% ?

____

13) John has had $500 in his savings account from January 1 to July 1. If the bank pays him interest at the rate of $2\frac{1}{2}$% for 6 months, how much interest will he receive on July 1?

____

PART II

1) Match each fraction on the left with its equivalent on the right.

_______ a) $\frac{1}{2}$ $\qquad\qquad$ $\frac{14}{16}$

_______ b) $\frac{7}{8}$ $\qquad\qquad$ $\frac{25}{28}$

_______ c) $\frac{27}{15}$ $\qquad\qquad$ $\frac{36}{72}$

_______ d) $\frac{112}{56}$ $\qquad\qquad$ $\frac{9}{5}$

$\frac{8}{4}$

2) Compute in the system of rational numbers -- $(Q,+,\cdot)$.

_______ a) $\frac{-2}{3} \cdot \frac{5}{-12}$

_______ b) $\frac{14}{5} \div \frac{3}{7}$

_______ c) $\frac{-7}{10} + \frac{2}{5}$

_______ d) $\frac{-5}{6} - \frac{-21}{8}$

_______ e) $\frac{2}{7} \cdot \frac{1}{3} + \frac{2}{7} \cdot \frac{2}{3}$

_______ f) 17.375 + 43.625

_______ g) 14.7 - 32.3

_______ h) 4.31 x 7.5

_______ i) 5.544 ÷ 1.32

3) Solve these open sentences in the system of rational numbers -- $(Q, +, \cdot)$.

_________ a) $\frac{5}{3} = \frac{35}{x}$

_________ b) $-7x = 5$

_________ c) $\frac{7}{9} + x = \frac{5}{18}$

_________ d) $1.62 + y = 3.18$

_________ e) $\frac{y}{3} = \frac{12}{5}$

_________ f) $x^2 = \frac{4}{9}$

_________ g) $\frac{8}{3} \div y = 16$

4) Compute in $(Q, \propto)$ where Q is the set of rational numbers and $a \propto b = \frac{a + b}{2}$.

_________ a) $10 \propto 27$

_________ b) $(5 \propto 11) \propto 7$

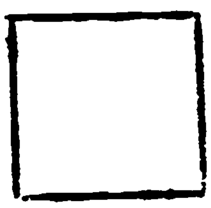
_________ c) $\frac{1}{3} \propto \frac{1}{2}$

5) In the following table, indicate with a T or an F whether the property listed on the left holds in the system listed at the top. One sample has been done for you; namely, $(a + b)c = ca + cb$ is true for all a, b, c in the set of integers -- $(Z, +, \cdot)$

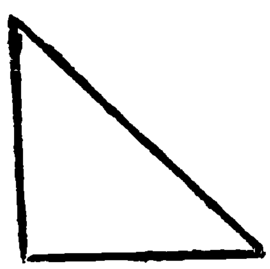
For all a, b, c	Whole nos. $(W,+,\cdot)$	Integers $(Z,+,\cdot)$	Rationals $(Q,+,\cdot)$
A. $(a + b) + c = a + (c + b)$			
B. $a - b = b - a$			
C. $a + x = 0$ has a unique solution.			
D. $a \leq b$ then $ac \leq bc$			
E. $(a + b)c = ca + cb$		T	
F. $ax = 1$ has a unique solution if $a \neq 0$.			

6) Draw in all lines of symmetry for the following figures.

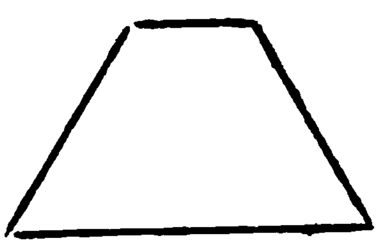
a)



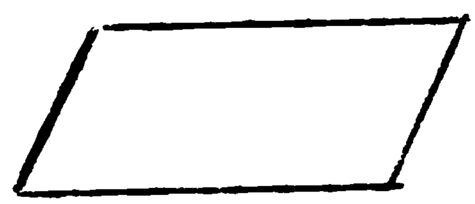
b)



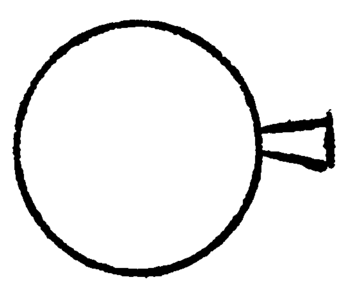
c)



d)



e)

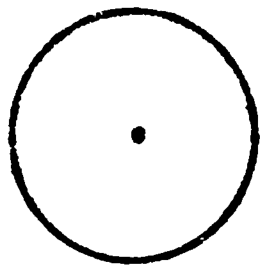


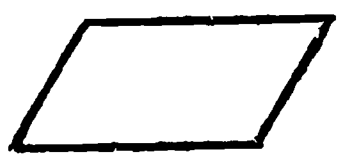
7) A plane geometric figure can have one or more of several types of symmetry: 1) Symmetry in a point, 2) Symmetry in a line, 3) Rotational symmetry. For each figure shown below, list the types of symmetry it has.

________ a) 

________ b) N

________ c) M

________ d) 

________ e) 

8) A salesman is allowed $.11 per mile for car expenses while driving his car in his work. If he received $149.27 in car expenses for one month, how many miles had he traveled that month in his work?

____

9) On a candy counter two chocolate bars of the same kind but different size are offered for sale. The smaller bar weighs 4½ ounces and costs 29¢. The larger bar weighs 6 3/4 ounces and costs 47¢. Which chocolate bar is the better buy?

____

APPENDIX C

Working Paper For June, 1967 Syllabus Conference

The 1966 June planning conference outlined the syllabus of a unified six year secondary school mathematics program. A special sub-committee made tentative partitions of the material by years and ordered those topics which were to be treated in the first year. In anticipation of the work of the forthcoming June conference, the following is a list of the topics planned for the second year. It will be the responsibility of the June conference to produce a detailed outline of this course and to make appropriate pedagogical suggestions. The topics are proposed here in an order suggested by the March planning committee. Following this list are explanatory notes indicating the specific content proposed for the chapters by the June, 1966 conference. The vocabulary in these notes is for mathematicians and not necessarily that to be used in the textbook for the teachers and pupils.

1. Mass point geometry.
2. An axiomatic treatment of affine plane geometry.
3. Statistics - central tendency and dispersion.
4. Sets and groups.
5. Fields and introduction to the real numbers.
6. Real functions.
7. Perpendicularity, scalar products, and the Pythagorean Theorem.
8. Combinatorics and probability.
9. Transformations in space.
10. Elementary trigonometry.
11. An axiomatic approach to the measure of plane sets.

The specific content of each of these proposed chapters is indicated by the following notes from the various June committee reports.

1. Mass point geometry -- This is to be Chapter 13 from the first year program. Experience of first year pilot classes has indicated that not all chapters of that book can be covered.

2. Axiomatic treatment of affine plane geometry -- This is to be an expansion of Chapter 15 also from the first year program. The chapter will encompass the following recommendations of the June 1966 geometry committee:

Grade 7: A small but precise axiomatic treatment of incidence and order axioms for a plane yielding notion of direction, system of axes, convex sets, halfplanes, and convex polygons.

Grade 8: Small axiomatic treatment of affine plane geometry, using incidence relations, order, and congruence relation on every line, with conservation of middle points by projection. Vectors as points of a plane with an origin, and as translations. Coordinates; equation of a line; inequations defining a half plane.

2. Statistics
<u>Descriptive Statistics</u>

Goal: To extend the treatment of topics presented in grade 7 in descriptive statistics to numerical treatment of central tendency and dispersion.

List of subjects:
1. Measures of central tendency
2. The summation symbol
3. Measures of dispersion
4. Scatter diagrams

Mathematics needed: Square roots

Time allotment: from one to two weeks

Commentary:

1. The following measures of central tendency are to be studied:
   a) the (arithmetic) mean
   b) the mode
   c) the median and the quartiles.

The above topics are to be presented, using the data compiled in the experiments at grade 7. The mean should be computed for grouped and ungrouped data. For data with an odd number of observations the median is defined as the middle observation and for data with an even number of observations the median is defined as the mean of the two middle observations. For grouped data the median and the quartiles are defined with the aid of the cumulative frequency polygon. Cases where the median is preferable to the mean as a measure of central tendency are discussed.

2. The use of the summation symbol is explained and illustrated.
   The following formulas are derived:

   a) $\sum\limits_{i=1}^{n} (a_i+b_i) = \sum\limits_{i=1}^{n} a_i + \sum\limits_{i=1}^{n} b_i$

   b) $\sum\limits_{i=1}^{n} (a_i-b_i) = \sum\limits_{i=1}^{n} a_i - \sum\limits_{i=1}^{n} b_i$

   c) $\sum\limits_{i=1}^{n} ca_i = c \sum\limits_{i=1}^{n} a_i$

   d) $\sum\limits_{i=1}^{n} c = nc$

   (Derive above intuitively, using $n = 4$)

3. The following measures of dispersion are to be studied:
   a) the range, b) the middle 90% range, c) the variance and the standard deviation. The use of the range in quality control in industry can be mentioned. The variance s^2 is defined as

$$s^2 = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2$$

   (Note that the above forula is a definition and is justified as a useful tool in the subsequent study of statistics. It is easy to show that it is indeed a measure of dispersion.)

   The formula

$$s^2 = \frac{1}{n-1}\left(\sum_{i=1}^{n} x_i^2 - n\bar{x}^2\right)$$

   is derived.

4. The students should make scatter diagrams with several sets of data involving pairs of observations. Some of these sets of data should be collected by the students themselves. The different sets of data should illustrate cases where:

   a) the scatter diagram suggests a line with positive slope,
   b) the same as a) with negative slope,
   c) no suggestions of a line.

   Some discussion of different uses of the fitted straight line should be made (see Mosteller, Rourke, Thomas, p. 364).

4. Sets and Groups

II. Group Theory and Field Theory

A. Notation and language of sets

1. Symbols: $\in$, $\cup$, $\cap$, $\subset$, $\subseteq$, complementation, Venn diagrams.

2. Subsets, power sets, measure of sets and unions of sets.

3. Conditions on sets and open sentences; formal implication and inclusion.

B. Groups

1. Definition of group $(G,*)$

2. Simple deductions from definition: uniqueness of identity and inverses, cancellation; introduce symbol $\square$ for inverse operation and deduce laws for connections between $*$ and $\square$. (Interpret this connection for rings and fields in terms of substraction and division); Solvability of linear equations.

3. Simple isomorphic groups; classifications of groups of given order: 2,3,4,5,6.

4. Subgroups, condition that a subset is a subgroup; diagram of subgroups of a group, related to geometry.

5. Mappings and permutations: the group of permutations S_3. (The 16-puzzle: see Kaufman notes).

5. Fields and introduction to the real numbers

1. Define a field $(F,+,\cdot)$ as a commutative group $(F,+)$ such that $(F - \{0\},\cdot)$ is a commutative group, $\cdot$ is distributive over +, and there are at least 2 elements in F.

2. Solutions of linear equations over fields in general.

3. Definition of an ordered field, and solutions of linear inequalities.

4. By divisibility argument show $\sqrt{2}$, $\sqrt{3}$, ... etc. are not rational.

5. Review the rational coordinatization of line and locate $\sqrt{2}$, $\sqrt{3}$, on line by Pythagorean relation. Define a sequence as a mapping from N into a set. Determine sequences of rationals approximating $\sqrt{2}$, $\sqrt{3}$, etc., and describe reals as numbers represented by infinite decimals. (See work by numerical group).

6. Real Functions -- Recommendations for the second year work on real functions come from two groups:

   A. Algebra --- Functions (on reals)

      1. Review of mappings on a set, arrow diagrams, ordered pairs, graphs

      2. Language of mappings: domain, range, 1-1, onto, image set: f is a function from its domain into its range and onto its image set; inverse function, composite function, with graphs.

      3. Algebra of functions: addition, multiplication (graphically) with examples; periodicity, monotonicity, symmetries.

      4. Polynomial functions in R.
         1. Linear functions and their graphs,
         2. Quadratic functions and their graphs,
         3. $x \longrightarrow x^n$, with transformation of its graphs,
         4. Step fcts., $[x]$, $|x|$ and their graphs.

   B. Analysis --- Precalculus (approximately grades 8-10)

   The list of topics in the precalculus treatment of functions is not strictly linearly ordered. Several of the concepts in II can be introduced in connection with some of the functions mentioned in III; and many of the functions mentioned in III will be used as examples very early.

      I. Concept of function

      1. General mappings. Concept of function, domain, and range. Different kinds of notation. Composite function, identity function.

      2. Functions given by formulae, tables, graphs, scales. Exercises in making tables and drawing graphs for functions given by formula (not necessarily very simple formulae).

3. Affine functions, piecewise linear functions. Useful examples throughout the course are functions of the form $\Sigma a_i|x - b_i|$, where a_i and b_i are given numbers.

4. The function Δf. Linear interpolation.

II. Description of real functions

1. Graphs and /or tables of f + g, f - g, fg, max [f,g], min [f,g], |f| where graphs or tables for f and g are given.

2. Monotone functions & piecewise monotone functions. Definition & simple properties.

3. Bounded functions. Sums, products, etc. of bounded functions. The set of bounded functions on a given domain forms a vectorspace. (This latter property should be noted when the concept of a vectorspace is at hand).

4. Maximum & Minimum (global & local)

5. Symmetry, even & odd functions. This concept can be introduced in connection with the study of the function $x \rightarrow x^2$.

6. Convexity of a function. Related to the definition of a convex point set.

7. Periodicity. Important examples: circular functions and x - [x].

8. Graphs of f(x) + a, f(x + a), af(x), f(ax) where the graph of f is given (this item requires knowledge of parallel displacements and some other simple affine mappings).

9. Special behavior of f(x) for large values of x. This item is of importance in connection with the functions x^n and $\frac{1}{x}$.

III. Special Functions

1. Step functions. Important example: [x]. Together with [x] it would be in order to investigate x - [x], and, perhaps x - [x] - 1/2.

2. $x \rightarrow x^2$, $x \rightarrow x^n$, $x \rightarrow \frac{1}{x}$, $x \rightarrow \sqrt{x}$, $x \rightarrow \sqrt[3]{x}$.

4. Systematic study of the polynomial functions of second degree with respect to the properties listed under II.

7. Perpendicularity, scalar products, and the Pythagorean Theorem --

Introduction of perpendicularity of directions; scale factor of two half lines.

Scalar product: law of cosines and Pythagorean Theorem. Study of $\sqrt{2}$. Equation of a circle in an orthonormal basis.

8. Combinatorics and Probability --

<u>Probability</u>

Goal: To develop on an intuitive level the background for the more formal study of probability in grades 9 - 12.

List of subjects:
Study of some experiments to develop the notions space, event, elementary probability, and uniform probability distributions.

Mathematics needed: Sets and subsets

Time allotment: 1 week

Commentary:

The following experiments can be used:
1) Tossing of a loaded die,
2) Tossing of a fair die,
3) Tossing of two thumbtacks,
4) Tossing of two dice,
5) Drawing of marbles from a box with replacement.

The experiment should be performed by the students and the observed data should be used to assign probabilities to the outcomes. By describing verbally various events in connection with these experiments, the students are led to associate the idea of an event with a subset of the outcome space. The advisability of assigning equal elementary probabilities of an outcome space should be discussed for each of the above experiments.

Combinatorics

Goal: To develop some fundamental notions of combinatorics.

List of subjects:
1. The fundamental principle of counting (the multiplication principle)
2. Permutations of n things taken r at a time
3. Number of subsets
4. The binomial theorem

Mathematics needed: Polynomial algebra

Time allotment: 1 week

Commentary:

1. The fundamental principle of counting (f.p.c.) is derived in connection with several practical examples, e.g.:
   a) number of paths resulting from the composition of two or more trips each of which has alternate routes;
   b) number of ways of choosing several courses from a menu with more than one choice for each course.
   As one application of the f.p.c. one can derive the total number of subsets including $\emptyset$ of a given set.

2. The number of permutations of n things taken r at a time, denoted by $(n)_r$, is derived as a consequence of the f.p.c. The case n = r is included.

3. The number of subsets with r elements from a set with n elements, denoted by $\binom{n}{r}$, is derived from 2) and the f.p.c. It is proved by combinatorics that
   $$\binom{n}{r} = \binom{n}{n-r} \quad \text{and} \quad \binom{n}{r} + \binom{n}{r+1} = \binom{n+1}{r+1}$$
   (For a somewhat different approach, see Engel, page 278).

4. The binomial theorem is proved by combinatorial reasoning.

9. Transformations in space --

Experimentation with shadows: Central projection from space to a plane, and, in particular, from a plane to a parallel plane. Enlargement of a photo; study of homotheties and similar sets. Shadow from the sun: parallel projection of a plane onto a plane; conservation of parallelism and middle point; use of two affine bases with different origins (transfer of drawings).

10. Elementary trigonometry --

Elementary trigonometry: Definition of sine, cosine, and tangent of an angle; law of sines. Concrete applications.

11. Axiomatic treatment of measure of plane sets --

Measure: A small axiomatic system on the measure of area of elementary plane sets ($o(x)$ is ≥ 0, increasing, additive for almost disjoint sets, invariant by isometries and $o(x) \geq 1$ when x is a unit square). We assume existence and uniqueness of o. Application to evaluation of usual areas; the formula $S' = k^2S$ for similar sets.

If we add the Archimedes-Cavalieri-Fubini principle, we can calculate more general areas.

Length of a curve using polygons and tube surrounding the curve. Semi-continuity of the length. The Von Koch curve. Relation between length of an arc of a circle and the area of corresponding sector. Approximate value of π.

To develop sense of measure, study variation of area of a rectangle with given perimeter (using a string); study various isoperimetric problems.

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ABSTRACT

The Secondary School Mathematics Curriculum Improvement Study (SSMCIS) has two main objectives: (1) To formulate and test a unified secondary mathematics program (7-12) that will take capable students well into current collegiate mathematics; (2) To determine the education required by teachers who will implement such a program.

Leading U.S. and European mathematicians and educators constructed a scope and sequence flow chart of the proposed 7-12 mathematics course. A detailed syllabus was prepared for Course I (seventh grade) and a textbook for Course I was written.

Nine junior high schools in the New York Area participated in the experimental teaching of Course I. Two capable and interested teachers from each school were given summer instruction in the mathematical concepts and methods of teaching required to teach the new course. During the following year each team of two teachers taught a single pilot class using the SSMCIS textbooks.

The experimental teaching was evaluated through personal observation by project staff, three tests designed to measure learning of important new concepts, and by four conferences involving project consultants and the teachers. The experimental teaching showed that the new mathematics course, based on fundamental concepts and structures was stimulating for the students and teachers, and, with several revisions, is a feasible one year course for the seventh grade.