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THE FORMATION OF ADDITION AND SUBTRACTION CONCEPTS BY PUPILS IN GRADES ONE AND TWO. FINAL REPORT.

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DESCRIPTORS- NUMBER CONCEPTS, ARITHMETIC, SUBTRACTION, *MATHEMATICAL CONCEPTS, *STUDENT ATTITUDES, *ACHIEVEMENT, LONGITUDINAL STUDIES, *METHODS RESEARCH, GRADE 1, GRADE 2, CONCEPT FORMATION, STATISTICAL ANALYSIS, *SYMBOLIC LEARNING, YPSILANTI PUBLIC SCHOOLS,

TO DETERMINE THE EFFECT OF INSTRUCTIONAL APPROACHES FOR SUBTRACTION AND/OR TIME OF SYMBOLIZATION UPON (1) ACHIEVEMENT IN ARITHMETIC, (2) ATTAINMENT OF NUMBER CONCEPTS, (3) ATTITUDES TOWARD ARITHMETIC IN GRADES ONE AND TWO, THIS REPORT ASSESSES FOUR EXPERIMENTAL CLASSES, THERE WERE TWO FIRST GRADE CLASSES OF 20 PUPILS EACH AND TWO SECOND GRADE CLASSES OF 17 PUPILS EACH. THE INSTRUCTIONAL APPROACH USED FOUR TREATMENTS (1) PPW-E (PART-PART-WHOLE WITH EARLY SYMBOLIZATION), (2) PPW-L (LATE SYMBOLIZATION), (3) TA-E (TAKE-AWAY WITH EARLY SYMBOLIZATION), (4) TA-L (LATE SYMBOLIZATION). A CONCEPT ATTAINMENT TEST WAS DESIGNED FOR USE AT THE BEGINNING AND END OF GRADE ONE. OTHER TESTS WERE ALSO ADMINISTERED BEFORE, DURING, AND AFTER GRADES ONE AND TWO. THE TESTS USED WERE THE CONCEPT ATTAINMENT TEST (BEGINNING AND END OF GRADE ONE), THE STANFORD ACHIEVEMENT TEST (END OF GRADE TWO), A SUBTRACTION APPLICATIONS AND TRANSFER TEST (END OF GRADES ONE AND TWO), LORGE-THORNDIKE INTELLIGENCE TEST (BEGINNING OF GRADE ONE), AN ARITHMETIC ATTITUDE SCALE (THREE TIMES IN EACH GRADE), SUBTRACTION FACTS TEST (END OF GRADE TWO), AND A LOGIC TEST (END OF THE STUDY). THERE WERE NINE CONCLUSIONS. (1) GRADES ONE AND TWO CAN LEARN USING PPW. (2) TA WAS MORE EFFECTIVE FOR TEACHING SUBTRACTION SKILLS AFTER GRADE ONE BUT, AFTER GRADE TWO, THE EFFECT OF TA AND PPW EQUALIZED. (3) AFTER GRADE TWO, PPW WAS SUPERIOR IN TEACHING APPLICATIONS OF SUBTRACTION. (4) AFTER GRADE TWO, PPW WAS SUPERIOR IN PROBLEMS REQUIRING TRANSFER. (5) PPW WAS SUPERIOR IN RELATING ADDITION AND SUBTRACTION. PPW FOSTERED PARTITIONING. (6) TA WAS INITIALLY EASIER TO TEACH. (7) HIGHER ACHIEVEMENT FOLLOWED EARLY SYMBOLIZATION. (8) CHANGES IN ATTITUDE WERE NOT PRODUCED BY ANY OF THE FOUR TREATMENTS. (9) NO DIFFERENCES WERE ATTRIBUTABLE TO SEX. FUTURE RESEARCH USING HIGHER GRADES, LARGER NUMBERS OF STUDENTS, AND OTHER MATHEMATICAL TOPICS SHOULD BE DONE. (LG)

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THE FORMATION OF ADDITION AND SUBTRACTION CONCEPTS BY PUPILS IN GRADES ONE AND TWO

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Joseph N. Payne

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The University of Michigan

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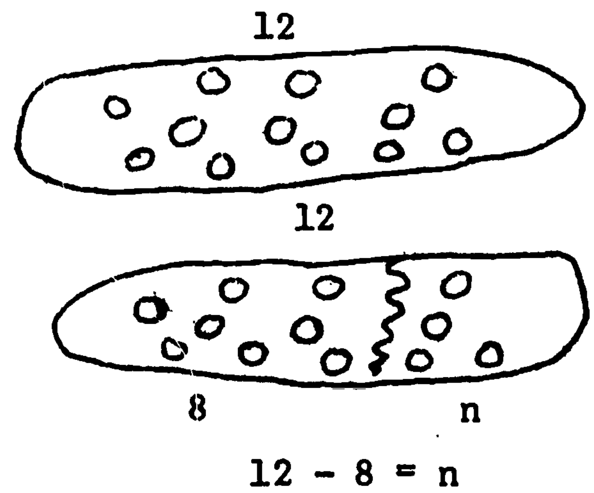
INTRODUCTION

The Problem

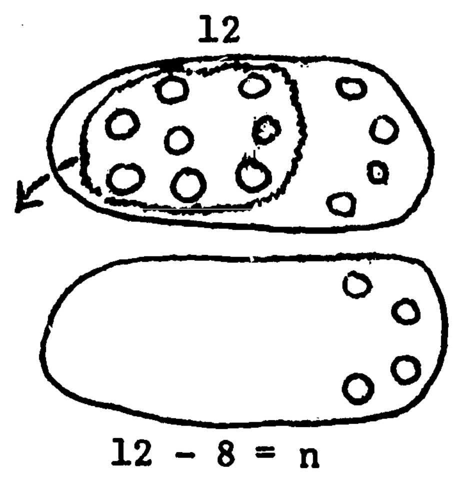
The purpose of this study was to investigate the effect of two independent variables on arithmetic achievement and attitudes for pupils in grades one and two. The two independent variables were 1) the approach to subtraction and 2) the time addition and subtraction were symbolized in writing.

Two approaches to subtraction were used. One approach used a more general model than usually used in introducing subtraction. This model was named "Part-Part-Whole (PPW)." In the PPW model, subtraction was presented as finding the number of one part or subset when the number of the whole set and the number of a given subset were known. A drawing for the PPW model is shown at the right. The total, 12, is known; the number of one subset, 8, is known; finding the number, <u>n</u>, in the other subset is subtraction. The physical action involves splitting a set but leaving the entire set and the two subsets visible at all times.



PPW

The second model was the "Take-Away (TA)" model. The physical model begins with a set of 12. A set of 8 is taken away. Finding the number in the remaining set is subtraction. After the set removal, only the set whose number is to be found remains.



TA

The second independent variable, time of symbolization of addition and subtraction, involved early (E) and late (L) symbolization. E symbolization meant concurrent introduction of symbolic statements at the time the concepts of addition and subtraction were introduced. L symbolization meant a six-week delay in the symbolization after the concepts had been introduced. For the PPW-L classes, "split" and "is" replaced "-" and

"="; for the TA-L classes, "take-away" and "is" were used for "-" and "=". For PPW-L and TA-L classes the words "and" and "is" replaced "+" and "=". The two independent variables resulting in four treatments are summarized in this drawing:

		Instructional Approach	
		Part-Part-Whole (PPW)	Take-Away (TA)
Time of Symbolization	Early (E)	PPW-E	TA-E
	Late (L)	PPW-L	TA-L

The central problem investigated in this study was the following: How do the approach to subtraction and the time of symbolization affect achievement in arithmetic, attitude toward arithmetic, and the formation of basic concepts relating to number for pupils in grades one and two?

Background and Related Literature

Among the major mathematical concepts in the elementary school, the operation of subtraction has been one of the most difficult for children to use and to explain. Brownell found that ". . . explanations for solutions were much more difficult for subtraction than for addition problems, especially for subtraction problem 5 which requires a comparison . . ." (3:80). Gibb's study showed that subtraction as usually taught is not used readily in application problems requiring comparison and missing addend formulations (7).

One of the difficulties in generalizing subtraction could be that the original introduction of the concept itself may lack the generality needed to accommodate later applications. Inhelder (11) has suggested that young children should be taught general mathematical concepts in order to facilitate learning varied applications of these concepts. Ausubel (1) agreed with Inhelder's thesis but suggested that the time spent in doing it might be prohibitive at the primary level.

Recognizing the inadequacy of the usual approach to subtraction, many experimental materials and new textbooks utilize the "missing addend" approach to subtraction. This model has the same disadvantage because it is not easily generalized to "take-away" applications. Furthermore, teachers encounter difficulty with the initial presentation using this model.

The PPW model is general enough to accommodate the three major types of applications, take-away, comparison, and missing addend. Furthermore, it is relatively easy to show the mathematical connection between addition and subtraction because the same physical model is used for both operations.

A primary question is whether or not children can learn the more general PPW model at all. To understand this model, the child must have an understanding of the conservation of the whole upon subdivision. The work of Piaget (5&14) suggests that this conservation does not occur until age 7 or 8, later than the usual time subtraction is introduced.

Another factor that could affect the difficulty encountered with subtraction is the time the learner symbolizes the operation. Hendrix (10) suggested that immediate symbolization of a concept could interfere with its formulation as measured by utilization. She found that children who did not symbolize their generalizations were more able to apply the concepts in unfamiliar stiuations than were the children who made an immediate symbolization of the concept. These results were corroborated by Haslerud and Meyers (9).

The problem of this study grew from the current emphasis in the mathematics curriculum on mathematical concepts, the fact that children encounter difficulty with subtraction as taught, the suggestion that the generality of the concept taught could facilitate later learning, and the suggestion that the time a concept is symbolized could affect attainment of that concept.

Objectives

The specific goals of this study were as follows:

In grades one and two, how do the instructional approaches for subtraction and/or time of symbolization affect the child's

1. achievement in arithmetic including addition and subtraction computation, subtraction applications, unfamiliar subtraction examples (transfer), general problem solving, and total arithmetic achievement?
2. attainment of number concepts?
3. attitudes toward arithmetic?

METHOD

The experiment was conducted in the Ypsilanti Public Schools during the school years 1964-65 (pupils were in grade one) and 1965-66 (same pupils were in grade two). The contract with HEW was to be negotiated for the '64-'65 school year but delays in Washington caused the contract to be negotiated late in the spring of 1965 after most of the experimental work was completed in grade one. Nevertheless, data are included for the '64-'65 school year even though HEW funds supported only the processing of data for that year. The lateness of the grant did enable the study to be extended in grade two for 1965-66. This resulted in the more long-range study encompassing both grades.

In 1964-65, Arthur F. Coxford, then a doctoral student in mathematics education at The University of Michigan, taught six first grade classes for the entire sch year to eliminate the variable of different teachers. Four classes were PE (Part-Part-Whole Early), TE (Take-Away Early), PL (Part-Part-Whole Late) and TL (Take-Away Late). Two other classes in a low socio-economic school were taught using Part-Part-Whole with early and late symbolization but the extremely low ability made comparisons with the four classes impossible. These two classes were dropped from the second grade study in 1965-66. Data from the two classes for the grade one study are not included in this report but are included in Coxford's dissertation (4). Coxford taught the usual first grade content to all classes and varied only the approach to subtraction and time of symbolization. A detailed description of the content taught and the amount of time devoted to each topic is contained in Coxford's dissertation.

In 1965-66 Alan R. Osborne, another doctoral student in mathematics education at The University of Michigan, taught the same four classes as Coxford when the students were in grade two. Osborne varied only the approach to subtraction because all students had symbolized addition and subtraction at the end of grade one. A complete description of content taught is contained in Osborne's dissertation (13).

From the pupils in the experimental program in grades one and two, a random selection was made to equalize the number in the four treatments with the result of twenty pupils in each treatment for grade one and seventeen in grade two.

In grade one, a concept attainment test was designed and administered using an interview technique at the beginning and end of the year. The major areas assessed by the concept attainment test were (1) ability to count, (2) one-to-one correspondence including its conservation and order relationship between a set and a proper subset, and (3) three types of subtraction applications. The interviewer observed the responses to each item on the test in an attempt to assess the way the child thought. Three forms of the test were used with the form used by each interviewer randomly assigned. A chi-square analysis revealed that there were no differences in the results obtained by the various interviewers. The reliability of the three forms was established.

The Stanford Achievement Test, Primary I Battery, Form W, was administered in May, 1966 (end of grade two). Various sub-sections of this test were used in assessing final results of the experimental treatments.

A Subtraction Applications and Transfer Test was constructed and administered at the end of grade one. A similar test was constructed and administered at the end of grade two.

The Lorge-Thorndike Intelligence Test, Primary Battery, Form A, was administered in October, 1964 (beginning of grade one).

An Arithmetic Attitude Scale developed by Harshman (8) was administered three times in grade one and three times in grade two. The attitude score for a given pupil for a given grade was the mean of the three scores.

At the end of grade two, a timed six-minute test on subtraction facts was administered.

The effect of the approaches to subtraction on pupils' ability in logic as measured by an interview was assessed at the end of grade two. Results from the logic test are not included in this report but are included in Osborne's dissertation (13).

The primary statistical methods used in the study were one-way and two-way analysis of covariance. In grade one, covariates included the concepts pretest, sub-sections I and II of the concept pretest, and I.Q. In grade two, covariates included concept post-test from grade one, Stanford post-test from grade one, and various sub-sections of these tests.

RESULTS

Grade One

Covariate measures are reported in Table 1.

TABLE 1

CLASS MEANS AND STANDARD DEVIATIONS OF COVARIATE MEASURES, FALL, 1964

Covariate		Class PE n = 20	TE n = 20	PL n = 20	TL n = 20
Concept Pretest - Part I	$\bar{X}$	168.50	161.00	169.50	154.00
	S.D.	38.70	48.00	59.34	38.17
Concept Pretest - Part II	$\bar{X}$	105.75	86.25	96.00	71.55
	S.D.	33.89	38.57	39.02	35.13
Total Concept Pretest	$\bar{X}$	317.75	281.75	304.50	261.55
	S.D.	72.50	100.94	105.61	83.07
I.Q.	$\bar{X}$	116.95	118.10	96.80	101.10
	S.D.	11.02	14.84	11.13	12.32

Achievement data obtained at the end of grade one are summarized in Table 2.

TABLE 2

CLASS MEANS AND STANDARD DEVIATIONS OF ACHIEVEMENT SCORES SPRING, 1965

Achievement Measure		Class PE	TE	PL	TL
Subtraction Application	$\bar{X}$	4.60	3.85	3.40	3.65
	S.D.	0.75	2.01	2.11	1.63
Non-Take-Away Application	$\bar{X}$	4.70	3.75	3.30	3.45
	S.D.	0.92	1.96	2.27	1.85
Subtraction Transfer	$\bar{X}$	4.65	4.65	3.35	3.90
	S.D.	2.52	2.91	3.13	2.40
Addition Computation	$\bar{X}$	6.45	4.75	4.15	4.45
	S.D.	2.26	2.40	3.01	3.05
Subtraction Computation	$\bar{X}$	6.25	6.10	4.10	5.55
	S.D.	2.83	3.68	3.66	3.50
Problem Solving	$\bar{X}$	12.60	10.65	7.85	9.70
	S.D.	2.62	4.39	4.03	4.67
Stanford Achievement	$\bar{X}$	30.45	26.60	20.20	24.90
	S.D.	7.84	11.74	11.95	12.95

The unadjusted achievement scores are consistently highest for PE and lowest for PL, as shown in Table 2.

Achievement data were analyzed using a two-way analysis of covariance using each covariate in Table 1.

Summarized in Table 3 are the adjusted mean achievement scores where significant differences occurred. Other means are omitted from this report for brevity but are included in Coxford's dissertation. Significant interaction results are reported later.

TABLE 3

CLASS MEANS FOR SIGNIFICANTLY DIFFERENT ACHIEVEMENT SCORES

	Approach			Time of Symbolization		
	PPW	TA	p	E	L	p
Covariate - Concept Pre-Test, Part I						
Non-Take-Away Applications				4.20	3.40	.05
Addition Computation				5.55	4.35	.05
Problem Solving				11.56	8.84	.01
Stanford Achievement				28.31	22.77	.05
Covariate - Concept Pre-Test, Part II						
Subtraction Computation	4.68	6.32	.05			
Problem Solving				11.29	9.11	.01
Covariate - Total Concept Pre-Test						
Addition Computation				5.45	4.45	.05
Subtraction Computation	4.75	6.25	.05			
Problem Solving				11.40	9.00	.01
Stanford Achievement	23.65	27.43	.05	27.81	23.26	.05

When significant differences occurred, the early symbolization group was superior. On subtraction computation, the TA approach was superior as it was on Stanford Achievement. However, these results are tempered by the lack of significant differences on either approach or time of symbolization using I.Q. as a covariate.

In no case were there reliable differences on Subtraction Application and Subtraction Transfer although the trend suggested that TA was slightly more effective than PPW. When adjusted by I.Q., there was a slight trend for PPW to be more effective for Subtraction Applications.

Significant interaction, $p < .05$, for Addition Computation and for Stanford Achievement was found for each covariate except I.Q., for Problem Solving using all covariates, and for Non-Take-Away Applications using Concept Pretest, Part II, as covariate. In each case the combination of PPW and L was least effective. In all but one case the combination of PPW and E was most effective. On Non-Take-Away Applications, significant interaction occurred using Concept Pretest, Part II, as covariate. On the Non-Take-Away Applications the PPW approach was slightly more effective than TA, and especially so when I.Q. was the covariate.

From the statistical analyses using sex, there appeared to be no reliable differences associated with either the approach variable or the time of symbolization variable.

Gain scores on the Concept Attainment Test and on sub-sections of the test were analyzed using I.Q. and Total Concept Pretest as covariates. The dominant trend was for the PPW-L to be consistently smaller than any other class. In each case where significant differences occurred, the most effective combination was PPW and E while the least effective was PPW and L.

Using gain scores, the PPW group had consistently greater scores on the item measuring conservation of the whole upon subdivision, but the results were not statistically reliable. There were no reliable differences in attitude toward arithmetic attributable to the independent variables although the trend showed TA scores higher.

Discussion of Grade One Results

While the significant results favored early symbolization for the four classes reported in this study, there was conflicting evidence from the two very low ability groups (data in Coxford's thesis). The late symbolization class had greater mean scores for both Subtraction Application and Subtraction Transfer with I.Q. as covariate. Also, it was Coxford's observation that delayed symbolization facilitated learning. Furthermore, the differences in I.Q. were particularly large for the E and L groups included in this report; although E generally performed better, the adjustment by I.Q. reversed the trend.

Coxford noted the relative difficulty in teaching subtraction using PPW. All classes encountered difficulty when asked to name the missing part. Such difficulty could be expected from the research reported by Bourne (2), Fredrick (6), and Kates and Yudin (12). This difficulty in teaching probably accounts for the observed decrease in attitude scores for one PPW class although the other PPW class seemed to master the approach and reflected an improved attitude toward arithmetic.

On Non-Take-Away Applications with adjustment for I.Q. and Concept Pretest, Part I, the PPW had the greater mean. The relatively large differences in favor of the PPW group suggested that there could be a significant relationship between the generality of the subtraction concept introduced and the applicability of that concept. Further study was recommended by Coxford.

While the PPW approach was very successful with children of higher I.Q., it was less successful for children of lower ability. For the TA approach, the high ability children in many achievement areas were surpassed by children with lower ability. The TA approach seemed to suit the average ability children better than children of higher ability.

In summary, the PPW approach did not lead to immediate subtraction computational skills, but it appeared to facilitate solutions of subtraction applications. Whether computational skills will improve and whether the generality of PPW will be of growing value is best determined by examining results at the end of grade 2.

Grade Two

Covariate measures are reported in Table 4.

TABLE 4

CLASS MEANS AND STANDARD DEVIATIONS OF COVARIATE MEASURES, FALL, 1965

Covariate		PE n = 17	TE n = 17	PL n = 17	TL n = 17
I.Q.	$\bar{X}$	117.71	117.29	99.29	102.47
	S.D.	11.30	15.67	11.04	12.26
Concept Post Test[a]	$\bar{X}$	445.18	361.65	376.47	395.06
	S.D.	43.51	83.54	83.21	64.30
Stanford Achievement[a]	$\bar{X}$	38.29	29.71	26.88	31.77
	S.D.	7.85	14.15	13.55	16.64
Problem Solving[a]	$\bar{X}$	12.53	9.00	8.29	10.00
	S.D.	2.70	4.74	4.06	4.98
Addition Computation[a]	$\bar{X}$	6.65	4.35	4.00	4.53
	S.D.	2.23	2.50	2.72	3.26
Subtraction Computation[a]	$\bar{X}$	6.47	5.41	3.53	5.24
	S.D.	2.74	3.66	3.79	3.68
Non-Take-Away Applications[a]	$\bar{X}$	3.06	2.12	2.24	2.41
	S.D.	.66	1.50	1.44	1.28
Subtraction Applications[a]	$\bar{X}$	4.65	3.47	3.41	3.71
	S.D.	.79	2.04	1.94	1.76
Subtraction Transfer[a]	$\bar{X}$	4.71	3.94	3.06	4.24
	S.D.	2.69	3.01	3.09	2.39

[a]These measures were criterion measures in Table 2. They are reported here because the number of cases in each cell decreased from twenty to seventeen.

The major achievement measures obtained at the end of the second year are reported in Table 5.

TABLE 5

CLASS MEANS AND STANDARD DEVIATIONS OF ACHIEVEMENT MEASURES, SPRING, 1966

Criterion		PE n = 17	TE n = 17	PL n = 17	TL n = 17
		Class			
Subtraction Facts	X̄	64.18	35.29	26.29	27.47
	S.D.	21.24	25.99	20.82	22.81
Stanford Achievement	X̄	57.06	47.41	49.06	47.35
	S.D.	4.53	12.19	9.73	12.51
Problem Solving	X̄	15.71	12.59	13.00	13.00
	S.D.	1.96	4.08	3.45	4.58
Addition Computation	X̄	9.71	8.35	8.77	8.18
	S.D.	.77	2.50	2.22	1.94
Subtraction Computation	X̄	9.47	9.00	8.65	8.59
	S.D.	.51	1.54	1.94	2.06
Non-Take-Away	X̄	3.41	2.71	2.35	2.35
Applications	S.D.	.87	1.26	1.27	1.27
Subtraction	X̄	5.24	4.12	3.82	3.59
Applications	S.D.	.97	1.80	1.78	2.00
Total Transfer	X̄	6.12	3.94	5.00	5.35
	S.D.	1.50	2.30	1.73	2.32
Subtraction Transfer	X̄	2.41	1.82	2.24	2.29
	S.D.	1.12	1.29	1.25	1.40
Total Interview	X̄	20.18	16.24	16.24	14.59
	S.D.	3.28	3.90	3.80	2.30
Interview Applications	X̄	5.00	3.41	3.53	3.35
	S.D.	1.17	1.17	1.63	1.37
Total Subtraction	X̄	15.47	11.35	11.94	11.35
Applications	S.D.	2.42	3.89	2.79	3.94

In Table 5 the unadjusted means for PE are consistently the greatest. Also the unadjusted means for PL are now greater relative to the other three classes than they were in Spring, 1965. (See Table 4 and Table 2.)

The analysis of these data was carried out by use of two-way analysis of covariance using the covariates in Table 4. Each covariate was not used with each criterion measure but only those covariates which correlated well with the criterion measure and exhibited large differences.

Summarized in Table 6 are the adjusted mean achievement scores which were significantly different. Limited space requires the ommission of the other means which may be found in Osborne's dissertation. Significant interaction results are reported later.

TABLE 6

ADJUSTED CLASS MEANS FOR SIGNIFICANTLY DIFFERENT ACHIEVEMENT SCORES

Covariate	Achievement Measure	Approach PPW	Approach TA	Approach p	Time of Symbolization E	Time of Symbolization L	Time of Symbolization p
I.Q.	Stanford Achievement	53.36	47.05	.005			
Stanford Achievement (1965)	Stanford Achievement	52.52	47.92	.01			
I.Q.	Problem Solving	14.44	12.71	.05			
I.Q.	Subtraction Facts	45.81	30.81	.005	42.80	33.82	.01
Stanford Achievement	Subtraction Facts	44.05	32.57	.005			
I.Q.	Total Subtraction Applications	13.79	11.27	.005			
Concept Post Test	Total Subtraction Applications	13.36	11.70	.05			
Stanford Achievement	Total Subtraction Applications	13.56	11.50	.005			
Subtraction Applications (1965)	Total Subtraction Applications	13.45	11.61	.01			
Concept Post Test	Subtraction Applications				4.62	3.77	.05
Subtraction Applications (1965)	Subtraction Applications				4.58	3.81	.05
Concept Post Test	Non-Take-Away Applications				3.02	2.39	.05
Stanford Achievement	Non-Take-Away Applications				2.97	2.44	.05
Non-Take-Away Applications (1965)	Non-Take-Away Applications				3.02	2.40	.05
I.Q.	Total Transfer	5.62	4.58	.025	4.27	5.93	.005
Subtraction Transfer (1965)	Total Transfer	5.59	4.61	.05			
I.Q.	Total Interview	18.31	15.31	.005			
Concept Post Test	Total Interview	17.72	15.90	.05	17.94	15.68	.01
Stanford Achievement	Total Interview	18.02	15.60	.005	17.93	15.89	.01

When significant differences occurred, the PPW approach was superior in every case. The level of significance of the differences is particularly noteworthy.

The evidence of the effect of time of symbolization is not as strong. On one achievement measure L was superior and in the other cases E was significantly higher. Time of symbolization seemed to have no effect on the scores obtained from the standardized test for any covariate.

There were only four significant interactions. Three occurred when I.Q. was the covariate: Subtraction Facts ($p < .01$), PE greatest mean others nearly equal; Total Subtraction Applications ($p < .05$), PE greatest mean, TE least; and Total Transfer ($p < .01$), TE least mean, others nearly equal. The final significant interaction, $p < .05$, occurred in the criterion measure Total Transfer with covariate Subtraction Transfer. Again TE obtained the least mean and the others had approximately equal means.

The Total Subtraction Applications included results from three tests, the eight problem Applications Test designed by Osborne, the six subtraction applications from the Stanford Test, and two subtraction applications from the interview. The Total Subtraction Applications strongly favored PPW. While there were not significant differences on Non-Take-Away Applications, the means for the PPW group were consistently higher for all covariates used.

An interview was designed and administered to ascertain the child's perception of the deductive-structural aspect of arithmetic, his conceptual basis of subtraction, and the success and method he used in solving applications. The interview and the complete results are in Osborne's thesis.

Item 4 on the interview was the problem: "If $154 + 73 = 227$, then $227 - 73 = \square$ " to assess the inverse relation between addition and subtraction. The PPW mean was significantly higher, $p < .05$, using I.Q. as covariate.

Items 2 and 6 required manipulation of physical objects to solve subtraction problems. Significantly more PPW students, $p < .005$, used a group removal method as opposed to a one by one removal for all covariates used.

Item 10 on the interview was designed to assess the child's conservation of the whole upon subdivision. For the questions with two rows of chips with one row subsequently partitioned, no differences were attributable to the independent variables. For a single row of chips using the grade one results on the same question as covariate, the PPW approach was superior, $p < .025$.

The results of the interview showed that there was minimal contamination of either TA or PPW students. These results support the consistency of the approach treatments.

There were no reliable differences in attitude towards arithmetic, attitude towards addition, or attitude towards subtraction.

Discussion of Grade Two

The effect of time of symbolization was less discernable at the end of grade two. Since the difference of six weeks in time of symbolization occurred at the beginning of grade one, almost two years earlier, little effect should actually be expected. However, the evidence does not support Hendrix's suggestion that earlier symbolization had an inhibiting effect. The most plausible point of view seems to be the one expressed by Coxford at the end of grade one, namely that delay of symbolization facilitated initial learning particularly for low ability pupils.

The superiority of PPW for problem solving was enhanced by the end of grade two. This lends strong support to Inhelder's contention that children should be taught general mathematical concepts to facilitate learning various applications of the concept. Further support is gained from the Transfer Test results favoring PPW. That significant results did not accrue on Non-Take-Away Applications even though PPW means were consistently higher may be due, in part, to the small number of test items.

At the end of grade one, TA produced significantly better subtraction computation results. This superiority was not maintained in grade two. Scores for PPW and TA were comparable. It was surprising that PPW did produce significantly better results on the Subtraction Facts Test at the end of grade two.

Clearly, the PPW approach helped students make the connection between addition and subtraction more effectively. Furthermore the

PPW approach fostered group removal or partitioning while TA did not.

The superiority of the PE class was maintained in grade two. There was some feeling that this could have been caused by the sociology and homogeneity of the group and, perhaps, even the regular classroom teacher. However, the great improvement in PL by the end of grade two suggests that the approach variable did operate to produce the significant results favoring PPW.

The results showing the connection between approach to subtraction and conservation of one-to-one correspondence are for the most part inconclusive. This may have been caused by the structure of the Concepts Test in grade one and the Interview in grade two. Further research is needed to establish the connection. Furthermore, it would be helpful to ascertain the causal connection, i.e. whether conservation of one-to-one correspondence is needed for a PPW approach or whether a PPW approach facilitates conservation.

CONCLUSIONS

1. Students in grades one and two can learn subtraction using a Part-Part-Whole (PPW) model.
2. At the end of grade one, the Take-Away (TA) approach to subtraction was more effective than the PPW approach for teaching subtraction skills. At the end of grade two, the approaches were equally effective with the PPW more effective for teaching subtraction facts.
3. At the end of grade one, the PPW approach was slightly but not significantly superior to TA in teaching applications of subtraction. By the end of grade two the PPW approach was significantly better than TA.
4. By the end of grade two, the PPW approach was more effective for solving subtraction problems requiring transfer.
5. The PPW approach was more effective in teaching the connection between addition and subtraction. Furthermore, the PPW approach fostered group removal or partitioning as opposed to one-by-one removal while TA did not.
6. TA was easier to teach in the initial presentation in grade one than was PPW.
7. A delay of six weeks in the time of symbolization of addition and subtraction after introduction of the concepts did not enhance achievement. In some cases, the earlier symbolization was followed by higher achievement. The difficulty with early symbolization for low ability students was noted by the teachers.
8. Neither the approach nor time of symbolization produced differences in attitudes towards arithmetic.
9. No differences were attributable to sex for approach or time of symbolization.

IMPLICATIONS AND RECOMMENDATIONS

1. This study should lead to modifications of the treatment of subtraction in grades one and two. Greater confidence could be placed in this recommendation if the study were validated with larger numbers of students and classes.
2. The problem should be extended to grades three and four to ascertain the effect of initial approach to subtraction on the more complicated computational algorithms.
3. Perhaps more time using the Part-Part-Whole model in kindergarten and the beginning of first grade prior to the introduction of the concepts of addition and subtraction and then symbolization would make their initial presentation easier. Such a plan should be investigated.
4. It seems clear that if the TA model and the accompanying language are used, the three sets (whole and two parts) should be visible to the learner. Leaving only the set whose number is to be ascertained in subtraction likely is nothing more than a counting exercise rather than a subtraction problem.
5. The effect of generality of initial presentation should be investigated for other mathematical topics such as division and rational numbers.

SUMMARY

The purpose of this study was to investigate the relative effectiveness of two approaches to subtraction and two different times of symbolization of addition and subtraction for pupils in grades one and two. The two approaches were Part-Part-Whole (PPW) and Take-Away (TA). The time of symbolization variable involved early (E) and late (L) symbolization. E symbolization meant concurrent introduction of symbolic statements at the time the concepts of addition and subtraction were introduced. L symbolization meant a six-week delay in the time of symbolization.

Four grade one classes were taught mathematics by Arthur F. Coxford and the same classes taught mathematics by Alan R. Osborne in grade two.

Data were analyzed primarily using one-way and two-way analyses of covariance. For grades one and two covariates included Lorge-Thorndike Intelligence scores and a concepts attainment test designed by Coxford. In grade two additional covariates include Stanford Achievement Test scores, Primary Battery, from grade one.

Achievement data included Stanford Achievement Test scores, applications, interview results, addition and subtraction computation scores, and scores from a transfer test. The attitude scale developed by Harshman was used.

In grade one, the four classes were equalized at twenty for statistical analysis and equalized at seventeen for grade two.

Conclusions and recommendations are contained at the end of the previous section.

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