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COST MODEL FOR LARGE URBAN SCHOOLS.

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THIS DOCUMENT CONTAINS A COST SUBMODEL OF AN URBAN EDUCATIONAL SYSTEM. THIS MODEL REQUIRES THAT PUPIL POPULATION AND PROPOSED SCHOOL BUILDING ARE KNOWN. THE COST ELEMENTS ARE--(1) CONSTRUCTION COSTS OF NEW PLANTS, (2) ACQUISITION AND DEVELOPMENT COSTS OF BUILDING SITES, (3) CURRENT OPERATING EXPENSES OF THE PROPOSED SCHOOL, (4) PUPIL TRANSPORTATION COSTS, (5) INSTRUCTIONAL EQUIPMENT COSTS, AND (6) DEBT SERVICE COSTS. VARIABLES CITED THAT DETERMINE CONSTRUCTION COSTS OF NEW SCHOOLS ARE ADMINISTRATION COSTS, SPACE PER PUPIL, TOTAL PUPILS, AND THE SQUARE FOOT COST. FROM EVIDENCE PRESENTED, THE ASSUMPTION THAT LARGER SCHOOL PLANTS COST LESS PER PUPIL CANNOT BE SUPPORTED. QUANTITY RATHER THAN QUALITY OF BUILDING WAS CONSIDERED. LAND COSTS ARE DETERMINED BY COST PER ACRE, LAND NEEDED FOR TYPE OF SCHOOL, LAND NEEDED PER PUPIL, AND TOTAL NUMBER OF PUPILS. CURRENT OPERATING COSTS ARE ESTIMATED FROM SALARY LEVEL AND NUMBER OF STAFF, EDUCATIONAL LEVEL AND NUMBER OF PUPILS, AND TEACHER-PUPIL RATIO. TRANSPORTATION EXPENSES ARE DETERMINED FROM EQUIPMENT COST, MAINTENANCE AND STORAGE COST, PUPILS TRANSPORTED, EFFECTIVE CAPACITY OF BUS PER MILE COST OF OPERATION, BUS SPEED, AND PUPIL COLLECTION TIME. INSTRUCTIONAL EQUIPMENT COSTS ARE DETERMINED FROM PURCHASE AND MAINTENANCE COSTS, AND NUMBER OF PUPILS USING EQUIPMENT. DEBT SERVICE COSTS ARE ESTIMATED FROM CONSTRUCTION, BUS, EQUIPMENT, LAND PURCHASE, INTEREST VARIABLES, AND AMORTIZATION SCHEDULE. (JZ)

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Division of Operations Analysis

COST MODEL FOR LARGE URBAN SCHOOLS

by

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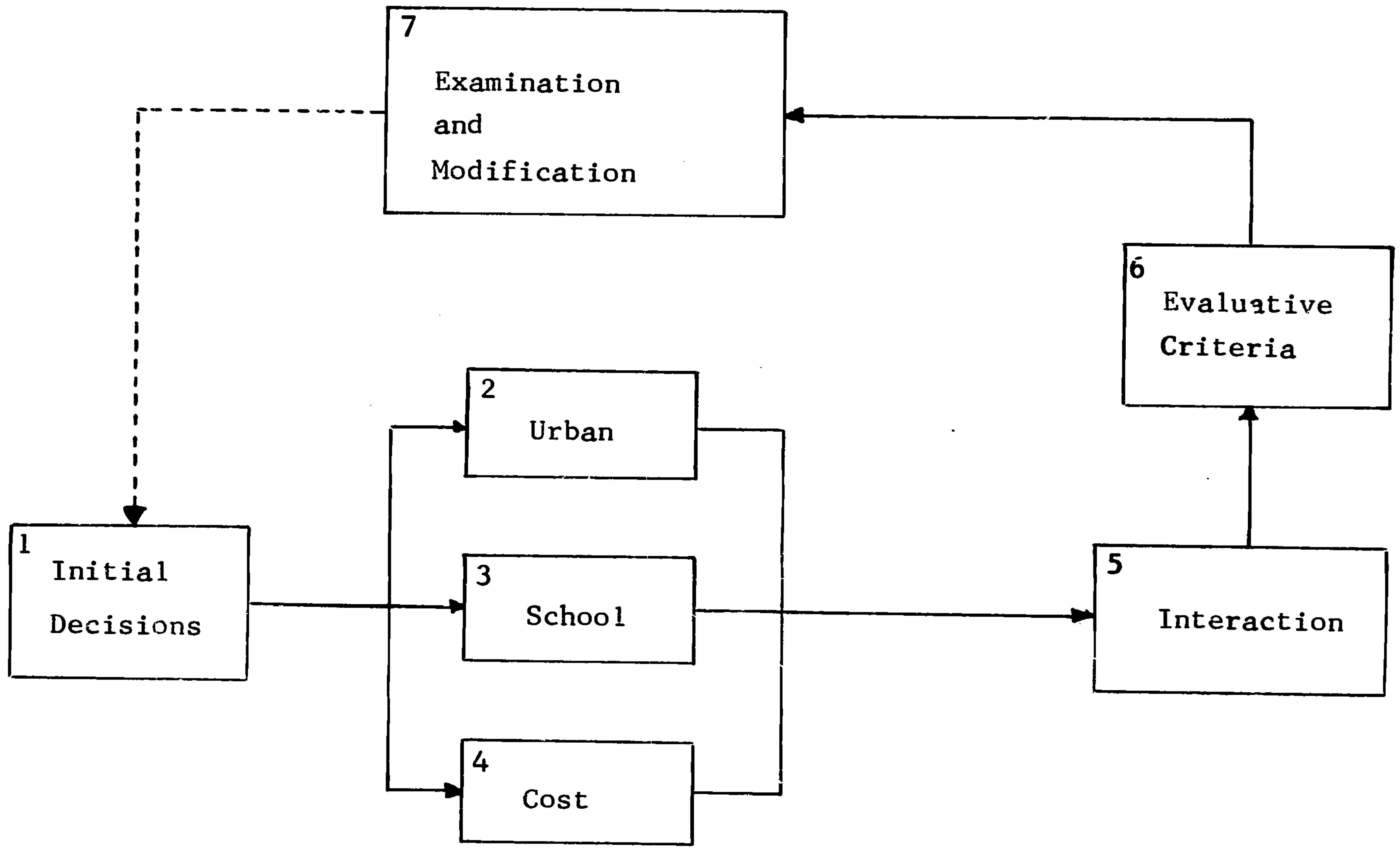
The framework for an urban educational system model was described in Urban Education Systems Analysis, Technical Note 24. The purpose of that note was to describe the factors that must be considered in the determination of educational policy relative to urban investment in school facilities. The investment policies considered are those concerned with the type of facilities and staff to be provided and their location and size. This policy is evaluated relative to the benefits and costs resulting from the investment decision. These benefits and costs in turn depend on the interaction of the school facilities and staff with the characteristics of the student population serviced and the characteristics of the urban setting. An overview of that analysis is shown in Figure 1. The logic and operation of the analysis is described in the above mentioned Technical Note 24.

Briefly restating some parts of that analysis several submodels are considered. An initial educational investment policy (Box 1 in Figure 1) is proposed. The Urban Submodel (Box 2) is concerned with the pupil population as characterized by their physical location and means of transportation, and their socio-economic characteristics. The School Submodel (Box 3) is concerned with the school plant as described by its facilities, staff and programs. The Cost Submodel (Box 4), which is the purpose of this present note, is concerned with the estimation of the resource implications of the educational policy considered in its urban and school environment. The interaction (Box 5) of the above elements as measured by operational indices of benefits and costs are evaluated (Box 6) relative to educational goals and objectives. Since these goals and objectives are multivalued and no one policy is likely to be optimal relative to all objectives it is anticipated that several modifications (Box 7) on the initial policy will be made before a final policy is selected.

This present note is a further specification of the cost submodel of the total analysis procedure. It is assumed that the student population serviced (Urban Model) is known and the physical plant (School Model) is specified. The role of the cost model is then to measure the cost implications of the above elements.

The costing procedure is developed to the extent that new facilities and staffing cost are estimated independently of the existing system. In the actual implementation of the costing procedure the net resource expenditure will be of interest, that is, the comparison of existing capital worth with the cost of required additions or replacements. For example, the present age and condition of school plants will influence the decision for proposed capital investments and the present school or urban transportation investment will influence the mode and extent of transportation to be provided.



URBAN EDUCATION SYSTEMS ANALYSIS

Basic Model

Figure 1

Some analysis of the cost implications of educational decisions will be made in this note, such as "optimal" school size and "optimal" bus size but it must be remembered that this is only in terms of cost and that optimal cost decisions may bear no relationship to optimal educational decisions. That is, the educational benefits accruing from any cost incurrence must be examined, and these are planned to be examined in the further refinement and specification of Urban Education Systems Analysis, Technical Note No. 24.

Some of the cost elements that are discussed and estimated are:

1. the construction of new plants
2. the land acquisition of building sites
3. the personnel staffing
4. current operating expenses other than personnel
5. the acquisition of transportation
6. the acquisition of special equipment
7. the financing of capital.

These elements are to be investigated as initial costs and costs that develop as a function of time. Also the fixed and variable aspects of cost as a function of student size will be investigated.

Construction of New School Plants

A common measure used in the estimation of the cost of new buildings is dollars per square foot of floor space. This requires, assuming other than a proportional relationship between cost and space, the development of a function relating cost to total floor area. The cost to be related to floor area is defined as contract cost which is the actual cost of construction as shown in the contract between the local educational agency and general contractors. Included are costs of permanently fixed equipment and costs for plumbing, heating and electrical work. Not included are costs of movable school furniture and equipment, (See BOB Form No. 51-R507).

The remaining costs including such categories as legal and administrative costs, architect and engineering fees, furniture and equipment costs, and on-site improvement costs will be assumed to be proportional to the contract cost. This may tend to overestimate the total cost due to the fact that such factors as architectural fees decrease in percentage with increasing contract cost. It is believed, however, that the error will be small and that the further enumeration of these second order costs will not add additional insight into the determination of educational policy.

Therefore

$$\text{Total Construction Cost} = (1+b_1)\ a(k_1N)^b$$

where

b_i = proportion of contract cost contributed by legal, administrative, engineering and other non-contract costs

a = parameter of cost function

k_1 = average space provided per student

N = total number of students

b = parameter of cost function

The selection or estimation of the parameters a, b determines the underlying cost model. For example, the assumption of a constant construction cost of $20 per square foot which is independent of total floor space would be reflected in the model by the values a = 20, b = 1. In general the value b determines whether the unit cost per area stays constant, increases, or decreases with increasing floor space. Thus for positive b, less than 1, the average cost per area will decrease with increasing total floor area. For example, for $b = \frac{1}{2}$, the average contract cost per area changes as:

$$\text{cost per area} = \frac{a}{\sqrt{k_1N}}$$

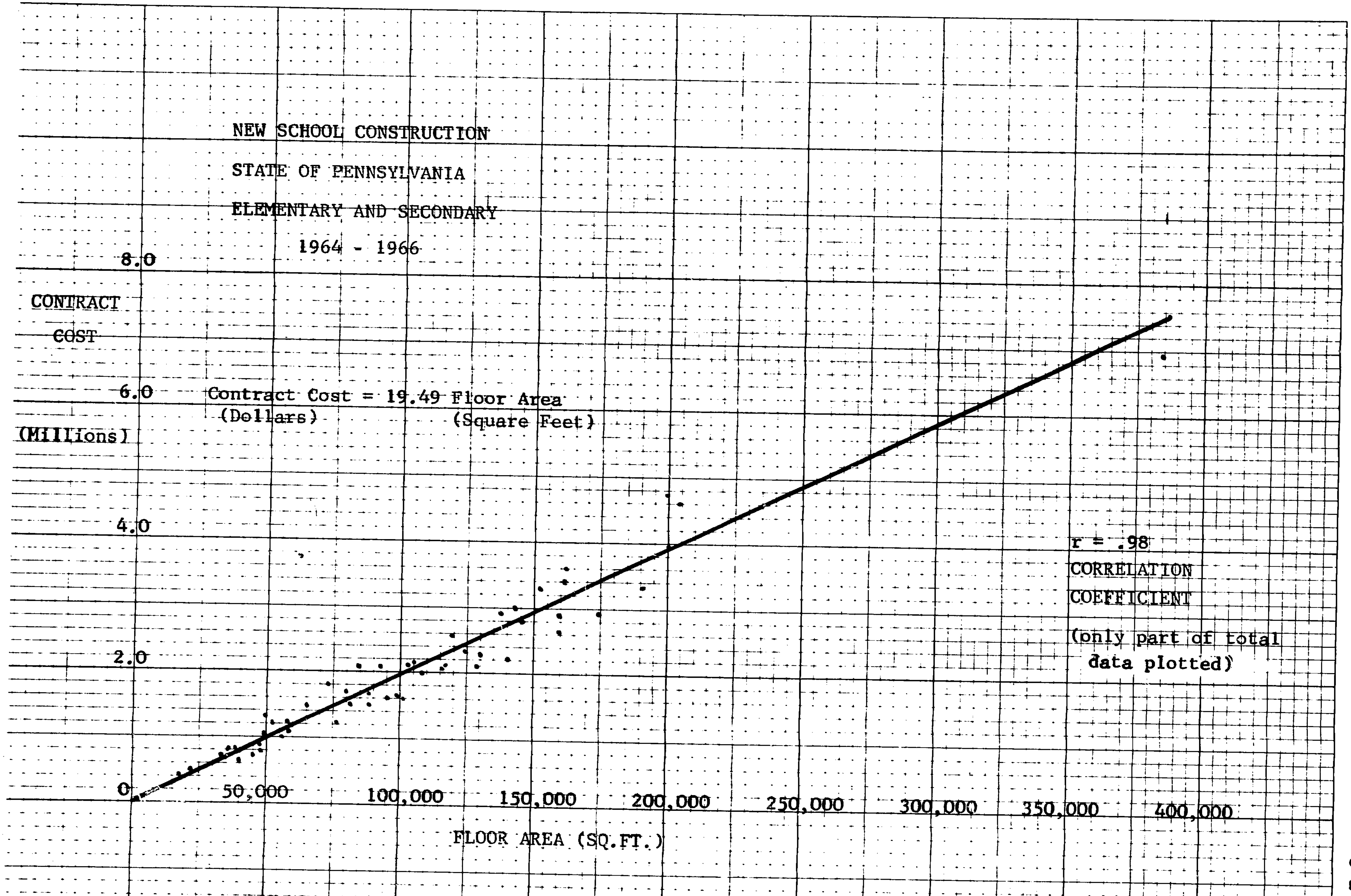
or in general for any b

$$\text{cost per area} = \frac{a}{(k_1N)^{1-b}}$$

Similarly a value of b greater than one indicates an increasing cost per area with increasing total floor area (or enrollment).

These functions can be developed by the analysis of recent school constructions through curve fitting techniques. An example is shown in Figure 2 for recent school constructions in the State of Pennsylvania (schools which participated in the School Assistance in Federally Affected Areas Program). It is seen that a linear model seems applicable. A curve fitting of that data (least squares regression technique assuming b = 1) yields the function,

$$\text{Contract Cost} = 19.49\ (k_1N)$$



NEW SCHOOL CONSTRUCTION
STATE OF PENNSYLVANIA
ELEMENTARY AND SECONDARY
1964 - 1966
CONTRACT COST (Millions)
8.0
6.0
4.0
2.0
0
Contract Cost = 19.49 Floor Area
(Dollars) (Square Feet)
r = .98
CORRELATION COEFFICIENT
(only part of total data plotted)
50,000
100,000
150,000
200,000
250,000
300,000
350,000
400,000
FLOOR AREA (SQ.FT.)

FIGURE 2

These data therefore (under the assumptions made) indicate no economies of scale in large school construction. Since these data, however, are fitted over elementary and secondary school constructions and no measure other than total floor area was considered, the quality of school as determined by the workmanship and type of facilities provided was not considered. If the larger schools are also the better schools, i.e., in terms of quality of facilities, then economies may be present which would not be revealed by these data. If this is considered possible a more detailed analysis will have to be made. The contract cost functions may be developed as a function of more than one independent variable, for example the precision of the cost estimate may be improved by adding independent variables as indicated that define the general shape of the structure, type of materials used and the type of space provided. These may be systematically examined where the appropriate data are available. The objective of this refinement would be to increase the precision and validity of the cost estimation and to provide additional data for the selection among educational policies.

One shortcoming of the statistical investigation of existing school structures is that the proposed sizes of educational plants may be considerably larger than those presently in existence and therefore the estimation procedure would involve considerable extrapolation of existing data. This is no doubt true of the data exhibited in Figure 2. It is recommended, if this be the case, that cost functions be validated and improved by comparison with industrial, commercial, or higher education building costs. Similarly radical changes in building techniques and sites, such as building over rail or highway space, will affect the cost precision and the estimates must be appropriately adjusted.

Land Cost

The acquisition of the building site is an important contributor to the overall cost of the program. It could also be a determining factor in the location of facilities both from the standpoint of land use patterns and cost. Sites currently considered unusable such as ravines or highway space may have to be utilized to lower cost and minimize the dislocation of present residences.

The cost of land is assumed to be

$$b_2 (c + dN)$$

where

b_2 = cost of land in dollars per acre

c = fixed land requirement associated with the level of school

d = variable land requirement per individual student

N = total number of students

The parameter c is associated also with policy relative to transportation. If for example busing is not provided and the public transportation is not adequate or convenient, then c must reflect the anticipated use of private automobiles.

Current Operating Expenses

The principal contributor of current operating expenses is represented by the salaries of personnel. The cost estimate used will be simply the sum of average salaries times the number of staff by the different personnel categories. A breakout of personnel categories is the following:

1. Administrators
2. Teachers
3. Other instructional
4. Administrative Secretaries and Clerks
5. Instructional Secretaries and Clerks
6. Health Personnel
7. Operation Personnel
8. Maintenance Personnel

The staffing relative to these categories will be based on the number of students (e.g. student teacher ratio), the level of students (e.g. elementary, secondary) and the type of students (e.g. special counseling needs based on the socio-economic character of the school population.) This staffing may be based on a priori decisions resulting from one's experience and judgment or based on statistical standards of existing school systems.

If based on statistical standards the staffing will be related to the number of students in a general form to allow for the increasing or decreasing rate of staffing with changing student enrollment.

The contribution of other sources (other than staff) to current operating expenses is assumed to be proportional to the cost of staff. It is not believed much insight will be gained by the delineation of these cost categories nor is the precision of the current operating estimate appreciably affected.

Therefore the estimate of current operating expenses is

$$(1 + b_3) \sum_u \sum_v e_{v,u} N_u^{f_{v,u}} Z_{v,u}$$

where

b_3 = proportion of current operating expenditures contributed by non-staff sources

$e_{v,u}$ = parameter of staffing function for the v^{th} staff category and the u^{th} school level

N_u = number of students at the u^{th} level of school

$f_{v,u}$ = parameter of staffing function for the v^{th} staff category at the u^{th} level of school

$Z_{v,u}$ = average staff salary for the v^{th} staff category and the u^{th} level of school.

The function of eN^f (omitting subscripts) allows for the selection or estimation of constant, increasing and decreasing values of teacher student ratios. The value of f, as was similarly shown for the construction cost function, determines the rate of change of expenses with respect to enrollment.

Transportation Cost

The transportation costs will be estimated by determining the fixed and variable costs required to transport the students by bus to the school location. The actual mode and requirements of transportation will of course be highly dependent on the existing urban transportation system of the region under study. It is nevertheless of interest to estimate the relative contribution of this factor to the total cost equation and it is believed that the estimation of busing requirements will accomplish that objective.

The cost of transportation by busing is given by:

$$(b_4 + b_5)\frac{k_2 \cdot N}{k_3 \cdot b_6} + \frac{k_2 \cdot N}{k_3 \cdot b_6} \cdot b_7 \cdot S \cdot P$$

where

b_4 = unit cost of a bus of given capacity

b_5 = annual fixed cost of bus maintenance and storage

k_2 = proportion of students that require transportation

N = total number of students

k_3 = factor representing effective capacity of bus (means for estimating shown in Appendix A)

b_6 = capacity of bus

b_7 = dollars per mile to operate bus

S = speed of bus

P = time allowed for collection of students

The first term in the above cost equation reflects the initial investment cost and annual maintenance cost of busing requirements. The factor k_3 is used to represent the impact the size of school attendance area and the distribution of students within that area will have on busing requirements. A method for estimating this factor is given along with a general busing requirements model in Appendix A. The second term in the above equation simply represents the collection cost of the students.[1/] It is seen that this is allowed to vary with a restriction on collection time and the allowable speed of the bus.

Special Equipment Cost

In this final cost category the fixed and variable costs associated with the procurement of major equipment items such as computers, language and science laboratories, library facilities, visual aid, sound equipment, and recreational equipment will be estimated.

The cost function for these categories is given by:

$$\sum_{w} (g_w + h_w N_w)$$

where

g_w = fixed cost of the w^{th} equipment

h_w = variable cost of the w^{th} equipment

N_w = number of students utilizing the w^{th} equipment

The fixed costs and some (if not all) of the variable costs are probably readily available from the manufacturers on these items. Some statistical information should be available on the variable cost of the use of particular pieces of equipment through school and manufacturer's sources.

1/ Note this term is daily one-way collection cost and must be multiplied by twice the number of school days in the year to obtain the annual collection cost. This is demonstrated in a later example.

Summary of Cost Functions

1. Construction

$$C_1 = (1 + b_1)\ a(k_1N)^b$$

2. Land

$$C_2 = b_2\ (c + dN)$$

3. Current Operating

$$C_3 = (1 + b_3) \sum_u \sum_v e_{v,u}\ N_u^{\ f_{v,u}}\ Z_{v,u}$$

4. Transportation

$$C_4 = \frac{k_2 \ .\ N}{k_3\ .\ b_6} (b_4 + b_5 + b_7\ S.P)$$

5. Special equipment

$$C_5 = \sum_w (g_w + h_w\ N_w)$$

Analysis of Cost Function

The above procedure is an attempt to measure the resource implications of given educational decisions. A generalized model is presented that may embody both statistical and deductive or judgmental inputs. Functional forms are hypothesized that allow for the representation of relationships between variables that either have been experienced or seem reasonable.

To demonstrate some of the conclusions and analysis that may be drawn using the cost equations presented, an example will be given using what is considered realistic data. In the discussion of this example some of the time dependent implications of the cost expenditures will be made through an examination of initial time and recurrent costs and the cost of securing capital.

First, however, the cost equations will be examined in terms of the relationship of student size to total cost. This is done not in the hopes of determining an "optimal" school size by the criterion of minimum cost but to examine where possible economies may lie, and to identify what specifically must be changed to achieve economy. It is not believed that the endeavor to optimize school size on a minimum cost basis will yield by itself any meaningful result. The usual assumption made by those who attempt this is that "other things being equal" and this is of course not true, even approximately so. Major economies are achieved by changing the basic inputs of education that is teachers, programs and facilities. To assume that one may measure various levels and combinations of these inputs with no significant change in the educational process taking place seems destined to failure.

One interest then in the consideration of the cost function, is the relationship of the cost per pupil to the total enrollment size, that is, the much discussed "optimal" school size. This "optimal" size is defined as the enrollment size at which the cost per pupil is the smallest.

If we divide the total cost function by the total number of students, N, we obtain, after rearranging the terms by those which are dependent on N and those which are independent of N, the following:

<u>Cost per Pupil</u>

$$= (1 + b_1)\, a(k_1)^b\, (N)^{b-1} + \frac{b_2 c}{N}$$

$$+ \frac{(1 + b_3)}{N} \sum_u \sum_v e_{v,u}\, N_u^{\;f_{v,u}}\, Z_{v,u} + \sum_w \frac{g_w}{N}$$

$$+ \sum_w \frac{h_w N_w}{N} + \frac{k_2}{k_3 \cdot b_6} (b_4 + b_5 + b_7\, S.P)$$

$$+ d$$

The term $(1 + b_1)\, a(k_1)^b\, (N)^{b-1}$ measures the contribution of the cost of school construction to the total cost per pupil. Whether this function decreases, increases or stays constant with respect to enrollment size depends on the value of b. One would intuitively believe that this function would decrease with increasing N (that is b less than one). The data exhibited in Figure 2 indicates a linear function (b=1), that is no economies are achieved in construction cost through control of the enrollment size. This data, however, is quite limited in sample points that are of the size in which these economies may become evident and in the size of enrollment that is being considered. When more data are available on the larger school enrollment sizes it is likely that the cost per pupil will decrease with increasing enrollment size or at worst be independent of school enrollment size. The economies implicit in this term are due to construction efficiencies in large scale building operations and the economy due to the lower per pupil space requirements achieved through the potential from the larger enrollment in the more efficient scheduling and use of facilities. The space economy of the cost function is reflected by the value k_1 which measures the space requirements per pupil. (The development of k_1 will be examined in the school submodel.)

The term, $\frac{b_2 c}{N}$, assumes that there is a fixed land requirement independent of school size and the contribution of this term to the total cost will decrease with increasing school enrollment size. It is not anticipated that this will be a significant part of the total per pupil cost.

The term $\frac{(1 + b_3)}{N} \sum_u \sum_v e_{v,u} N_u^{f_{v,u}} Z_{v,u}$

is the contribution of salaries and remaining current operating expenses to total per pupil cost. The economies implicit in this term are those due to the relationship of staffing to school enrollment size. Basically this term reflects the educational decision of student teacher ratio and student support staff ratio. The limited statistical data and the educational standards published indicate that the instructional staffing is proportional to enrollment size (that is f equal to one) and the non-instructional staff is approximately so. If this is truly the case, there are no economies apparent in the larger school attendance areas from staffing cost and other current operating expenses. It should be pointed out again, however, that the statistical data available are not of the school sizes that are currently being contemplated and therefore the extrapolation of existing data to these large school enrollment sizes is of doubtful precision. Another approach, is the deductive construction of staffing based on educational, administrative and maintenance defined requirements and functions. In this organizational approach some economies (or diseconomies) may become evident in the larger enrollment sizes.

The term $\sum_w \frac{g_w}{N}$ is the cost contribution of fixed charges of major equipment procurement. The economies implicit in this term are due to the fixed charges that would be charged independent of size of the facility. An example might be the installation charge for a computer. It is not anticipated that this will add significantly to the total cost per pupil.

The term $\sum_w \frac{h_w N_w}{N}$ will tend to remain constant if we assume the value $\frac{N_w}{N}$ tends to remain constant with increasing N.

The term, $\frac{k_2}{k_3 . b_6}$ $(b_4 + b_6 + b_7 \text{ S.P})$, which measures the contribution of transportation procurement and maintenance to the total per pupil cost appears to be independent of the enrollment size. Its appearance in the cost formula, however, assumes the need for transportation, that is a school enrollment of sufficient size. Given then the need for transportation the model assumes that cost is proportional to the enrollment size. There are, however, other factors such as the distribution of students within the region that will affect the factor k_3 of this cost element. The sensitivity of this factor may have to be examined for the specific region of interest.

The remaining term is seen to be independent of N and therefore does not affect "optimal" school size.

In review then some economies seem possible through construction efficiencies, fixed land and equipment charges, and possibly through staffing requirements. These economies, however, do not seem fully demonstrable on statistical grounds and the appeal to these economies are partially on intuitive grounds. The diseconomy is the transportation charge and though it appears to be proportional to enrollment size (that is, not affecting the cost per pupil) it actually is essentially zero until a student population of a certain size distributed in a certain way over a given region is reached. Some of the other factors present in the estimation of transportation cost will be discussed in Appendix A.

Example of Cost Estimating Procedure

Continuing the illustration and examination of the cost function, consider its use in the estimation of the cost of a large educational facility, such as an educational park, which is to service the following population:

N = total school population

= 10,400

N_1 = elementary school population (grades 1-4)

= 2,800

N_2 = intermediate school population (grades 5-8)

= 3,600

N_3 = secondary school population (grades 9-12)

= 4,000

1. Construction cost

$$c_1 = (1 + b_1)\ a(k_1N)^b$$

$$= (1 + .15)\ 20\ \ (85)\ (10{,}400)$$

$$= 1955N = \$20{,}332{,}000$$

where

b_1 = .15

a = \$20/sq. ft.

k_1 = 85 sq. ft./pupil (averaged over grades)

N = 10,400 students

b = 1

2. Land Cost

$$c_2 = b_2\ (c + dN)$$

$$= 50{,}000\ \left[60 + .01\ (10{,}400)\right]$$

$$= \$8{,}200{,}000$$

where

b_2 = \$50,000/acre

c = 60 acres

d = .01 acres per student

N = 10,400 students

3. Current Operating Cost

$$c_3 = (1 + b_3) \sum_u \sum_v e_{v,u}\ N_u^{f_{v,u}}\ Z_{v,u}$$

Assume as slight modification of this formula that the dependence of N on the student grade level is removed by weighting the number of students at each level as

$$N_o = N_1 + N_2 + 1.1\ N_3$$

so that the resulting equation is

$$C_3 = (1 + b_3) \sum_v e_v N_o^{\ f_v} Z_v$$

This modification was necessitated by the source of the data and normally is not recommended.

Then

$$\begin{aligned} C_3 = (1 + .18) \Big[& (.0003)\ (13,000) + (.0400)\ (6900) \\ & + (.0038)\ (9300) + (.0010)\ (4700) \\ & + (.0028)\ (3600) + (.00045)\ (6100) \\ & + (.0056)\ (4500) + (.0013)\ (5300) \Big]\ 10,800 \\ = \$ & 4,649,712 \end{aligned}$$

where

$b_3 = .18$

$f_v = 1$

$e_1 = .0003$, administrators per student

$Z_1 = \$13,000$, average administrator salary

$e_2 = .0400$, classroom teachers per student

$Z_2 = \$6900$, average teacher salary

$e_3 = .0038$, other instructional staff per student

$Z_3 = \$9,300$, average other instructional salary

$e_4 = .0010$, administrative secretaries and clerks per student

Z_4 = \$4,700, average administrative salary for secretaries and clerks

e_5 = .0028, instructional secretaries and clerks per student

Z_5 = \$3600, average instructional secretaries and clerks salary

e_6 = .00045, health personnel per student

Z_6 = \$6100, average health personnel salary

e_7 = .0056, operation personnel per student

Z_7 = \$4500, average operation personnel salary

e_8 = .0013, maintenance personnel per student

Z_8 = \$5300, average maintenance personnel salary

N_o = 2,800 + 3,600 + 1.1 (4000) = 10,800, weighted number of students.

4. Transportation Cost

$$C_4 = \frac{k_2 \cdot N}{k_3 \cdot b_6} (b_4 + b_5 + b_7 \text{ S.P})$$

$$= \frac{(.96)\ (10,400)}{(1.75)\ (60)} \left[20,000 + 5000 + .15\ (10)\ (1)\right]$$

$$= 95 \left[20,000 + 5000 + 1.5\right]$$

$$= \$1,900,000 + \$475,000 + \$142.5$$

where

k_2 = .96, proportion of students requiring transportation (based on distance)

k_3 = 1.75, factor reflecting effective capacity of the bus (based on size of area serviced and distribution of students within the area)

N = 10,400, total number of students

b_6= 60, bus capacity

b_4 = \$20,000, unit cost of bus

b_5 = \$5,000, annual cost of bus maintenance and storage (other than collection)

b_7 = \$.15/mile, cost per mile to operate the bus

S = 10 miles/hour, average speed of bus

P = one hour, time by which it is required to collect all students

5. Special Equipment Cost

$$C_5 = \sum_w (g_w + h_w N_w)$$

$$= \sum_w g_w + \sum h_w N_w$$

$$= 100{,}000 + 50{,}000 = \$150{,}000$$

where

$\sum_w g_w$ = \$100,000, total fixed cost of special equipment

$\sum_w h_w N_w$ = \$50,000, total variable cost of special equipment

Grouping the above costs into initial and current operating costs per year we obtain the following:

Initial Investment:

Construction	\$20,332,000
Buses	1,900,000
Special Equipment	100,000
Land	8,200,000
	\$30,532,000

Current Operating (Per Year)

Salaries	$3,965,900
Other Operating	713,900
Busing	
Operation	475,000
Collection1/	51,400
Special Equipment	50,000
Total	$5,256,200

If we assume that the $30,532,000 capital investment cost is financed at 6% per year for 20 years, then we may consider the yearly impact of these cost elements. Repeating the above table now on a per pupil basis with the cost of financing included we obtain the following:

Category	Cost Per Pupil Per Year ($)
Initial	
Construction	$171
Buses	16
Special Equipment	1
Land	69
Subtotal	$257
Current	
Salaries	$381
Other Operating	69
Busing Operation	46
Busing Collection	6
Special Equipment	5
Subtotal	$507
TOTAL	$764

Realizing that this is hypothetical data (which, however, has been attempted to be made realistic) and that the nature of the existing system and the educational and social benefits accruing from these expenditures are of paramount importance, one may still point out some elements of interest. One is the sum of $30,532,000 that must be acquired as an initial investment in this facility. An expense of this magnitude should be justified by real, demonstrable, educational or social benefits. Some clear statement of one's goals and the degree to which they are being met must be made. The cost of the new building and land (assuming the financing as given) costs $240/pupil/year over a twenty year period. One should consider whether the new facility yields benefits that justify this expense or whether the

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1/ Based on school year of 180 days and daily cost of 2($142.5) developed above.

application of this amount to other educational programs will produce more fruitful returns. Similarly, consider the expense of $68/pupil/year to provide transportation. This is an expense which yields no direct educational return and is due to the size of the facility. The comparable benefits (or other economies) acquired through this size must be demonstrated or stated.

The data may also be used to simply compare the cost of the proposed system with the current expenditures in the school system. For example, the estimated $507/pupil current expenditure may be compared with the expenditures experienced with the existing system.

In summary the cost model presented enables one to make an estimate of some of the resource implications of educational decisions. The analysis or conclusions to be drawn must also be examined in terms of the existing system, in terms of the educational benefits to be acquired and in terms of one's social and political objectives.

APPENDIX A

Bus Requirements Model

The objective of this model is to estimate the number of buses required to service a given population of students, N. Let

$$N^1 = N\left(1 - \frac{r_o^2}{r_{max}^2}\right)$$

where

N = total number of students

r_o = radius within which busing is not required

r_{max} = radius which defines the area to be serviced

N^1 = total number of students requiring transportation

This equation assumes that the determination of students requiring transportation is only a function of the distance from the school. If other knowledge of the alternatives of transportation are known the value of N may be further reduced. The area serviced is assumed circular with the school located at the center.

Assume that the population N^1 is distributed around rings of radius r_i from the school. We may assume different densities on the rings or assume as we shall for the demonstration of this model that there is a uniform density on each ring, then

$$\bar{d} = \frac{N^1}{\sum_{i=1}^{max} 2\pi r_i}$$

where

$\bar{d}$ = average number of students per mile

N^1 = number of students requiring transportation

r_i = distance from the i^{th} ring to the school in miles

π = 3.1416

Assume a bus of capacity c students with rate of speed S miles per hour, then assuming the bus is in a position to load passengers, the bus will fill up to capacity in m, miles and time t hours, where

$$m = c/\bar{d} \qquad \text{and}$$

$$t = \frac{c/\bar{d}}{S} = \frac{c}{\bar{d}S}$$

Assume that it is required that all passengers be loaded within P hours, then if the bus starts on the r_i th ring the total time to make the initial trip and the subsequent round trips, X, is

$$\frac{r_i}{S} + \frac{c}{\bar{d}S} + X\left(\frac{2r_i}{S} + \frac{c}{\bar{d}S}\right)$$

It is required that this time be equal to P. Setting this time equal to P and solving for the number of round trips, X, we obtain

$$X = \left(P - \frac{r_i}{S} - \frac{c}{\bar{d}S}\right) / \left(\frac{2r_i}{S} + \frac{c}{\bar{d}S}\right)$$

= number of round trips bus will make in time P on ring r_i.

The number of people carried on ring r_i in time P is then

$c\ (X + 1)$ (accounting for initial trip)

The number of people on ring r_i is $2\pi r_i \bar{d}$, thus the number of buses, B_i, required on ring r_i to pick up all passengers is

$$B_i = \frac{2\pi\, r_i \bar{d}}{c(X + 1)}$$

and substituting in the value of X we obtain:

$$B_i = \frac{2\pi r_i\ (2\ \bar{d}\ r_i + c)}{c\ (SP + r_i)}$$

and the total number of buses required over all rings is

$$\sum_{i=1}^{max} B_i = \sum_{i=1}^{max} 2\pi r_i \frac{(2\,\bar{d}\,r_i + c)}{c\ (SP + r_i)}$$

Consider the following demonstration of the above model. Assume we have reduced the original number of students, N, to $N^1 = 10{,}000$ either by the formula presented for N^1 or by other information concerning the needs for transportation. Assume further the following parameters:

1. r_i = i miles where i = 1, 2, 3, 4, 5
2. c = 60 students
3. S = 10 miles per hour
4. P = 1 hour

Then

$$\bar{d} = \frac{N^1}{\sum_{i=1}^{5} 2\pi r_i} = \frac{10{,}000}{2\pi(15)} = 106.1 \text{ pupils per mile}$$

and

$$B_i = \frac{2\pi r_i\ (2\,\bar{d}\,r_i + c)}{c\ (SP + r_i)}$$

$$= \frac{2\ (3.1416)\ r_i\ (2\ (106.1)\ r_i + 60)}{60\ (10\ (1) + r_i)}$$

The calculations are summarized in the following table:

Distance from School r_i	Number of Students on Ring r_i	Number of Trips Per Bus to Service Ring r_i	Total number of Students Carried Per Bus on Ring r_i	Number of buses, B_i, Required on Ring r_i
1	667	4.3	258	2.59
2	1333	2.6	156	8.54
3	2000	2.0	120	16.67
4	2666	1.6	96	27.77
5	3334	1.4	84	39.69
Total	10,000			$\sum_{i=1}^{5} B_i = 95.26$[1/]

Thus under the assumptions made, the busing requirements are 95 buses to service the population of 10,000 students under the restriction of one hour service time.

Given that the assumptions made in this general model are realistic, one may examine the effect of busing requirements caused by changes in the parameters. For example one may examine the effect of bus capacity. As the bus capacity increases, the number of buses required will obviously decrease; however, when the unit bus cost and the operating cost are considered it may be possible to determine an "optimal" bus capacity that is one that minimizes total bus procurement and operating cost. Similarly the effect of bus speed, size of area serviced and density of students may be examined for the specific region of interest.

1/ Fractional values are carried to demonstrate the calculations. In practice integral values should be used in determining final requirements.