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THE DEVELOPMENT OF AN ELEMENTARY SCHOOL MATHEMATICS CURRICULUM FOR INDIVIDUALIZED INSTRUCTION.

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DESCRIPTORS- *INDIVIDUAL INSTRUCTION, *MATHEMATICS CURRICULUM, DIAGNOSTIC TESTS, COMPUTER ASSISTED INSTRUCTION, SEQUENTIAL APPROACH, INDIVIDUALLY PRESCRIBED INSTRUCTION (IPI),

INDIVIDUALIZED PRESCRIBED INSTRUCTION (IPI), DESIGNED FOR GRADES 1-6, IS A SEQUENTIAL MATHEMATICS CURRICULUM IN WHICH EACH OBJECTIVE IS A DESCRIPTION OF SOMETHING A STUDENT SHOULD BE ABLE TO DO. EACH OBJECTIVE IS A PREREQUISITE TO THE LEARNING OF A LATER OBJECTIVE. STUDENTS ARE TESTED FOR MASTERY OF OBJECTIVES AND THEN PLACED SO THAT THEY ARE STUDYING SOMETHING NOT YET LEARNED BUT SOMETHING FOR WHICH THEY HAVE ALL THE PREREQUISITES. CONSIDERATIONS WHICH SHAPED THE SEQUENCE OF OBJECTIVES DISCUSSED ARE (1) THE COMMITMENT TO THE NEW MATHEMATICS, (2) THE NEED FOR AND THE STRENGTH OF OBJECTIVES, (3) THE EFFECT OF INDIVIDUALIZED INSTRUCTION UPON CURRICULUM PREPARATION, (4) SUBJECT MATTER ACCURACY AND LOGICAL PROGRESSION, (5) THE USE OF MEMORIZATION AND MASTERY IN THE MATH CURRICULUM, (6) LEARNING THEORY AND EDUCATIONAL EXPERIMENTS, (7) TESTING REQUIREMENTS, (8) INTERACTION BETWEEN LESSON WRITERS AND THE NEW CURRICULUM, (9) THE EFFECT OF A DEVICE (A LANGUAGE MASTER) FOR COMMUNICATING WITH NON READERS. SOME CHILDREN MAY NOT BE ABLE TO LEARN AS WELL BY INDIVIDUALIZED INSTRUCTION AS IN A CONVENTIONAL CLASSROOM. ONE-DAY-A-WEEK, CLASS ACTIVITIES IN A MATHEMATICS SEMINAR WAS THE APPROACH USED TO OFFSET THIS POTENTIAL PROBLEM. ACHIEVEMENT RESULTS FOR THE SCHOOL YEAR 1964-1965 SHOW WIDE RANGES OF ACHIEVEMENT FOR INDIVIDUAL PUPILS. THE FIRST GRADE CLASS SEEMS TO HAVE MADE DOUBLE THE NORMALLY EXPECTED GROWTH. A SUGGESTION THAT HAS MANY POTENTIALS IS THE USE OF COMPUTER ASSISTED INSTRUCTION WITH INDIVIDUALIZED INSTRUCTION. (HH)

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WORKING PAPER 7

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THE DEVELOPMENT OF AN ELEMENTARY SCHOOL MATHEMATICS CURRICULUM FOR INDIVIDUALIZED INSTRUCTION

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Abstract

A short description of a system of Individually Prescribed Instruction (IPI) for elementary school mathematics is followed by the considerations which shaped the writing of a sequence of behavioral objectives. The considerations discussed are: (1) the commitment to the New Mathematics, (2) the need for and the strength of behavioral objectives, (3) the effect of individualized instruction upon curriculum preparation, (4) subject matter accuracy and logical progression, (5) the use of high strength responses in the math curriculum, (6) learning theory and educational experiments, (7) testing requirements, (8) interaction between lesson writers and the new curriculum, (9) the effect of a device for communicating with non-readers. Finally, promising aspects of the program and potential problems are discussed.

The Learning Research and Development Center of the University of Pittsburgh (LRDC) is engaged in the development of programs of individualized instruction in elementary science, reading, and mathematics.

Our immediate goal has been to demonstrate that practical, effective programs of individualized instruction can be implemented in an elementary school--practical in that they are within the projected financial resources of the public schools, effective in the sense that each child proceeds at an acceptable rate through the curriculum in a manner dependent upon his own interaction with the subject and relatively independent of anyone else's ability.

Our present answer to the question of feasibility is the system we call Individually Prescribed Instruction (IPI) whose basic components are (1) a sequential curriculum, (2) placement and diagnostic tests, and (3) lessons (e.g. work page assignments or teacher directed activities). By a sequential curriculum we mean a list of behaviorally written objectives;

each objective is a description of something the student is supposed to be able to do, and is so ordered that it is prerequisite to the learning of a later objective. Students are tested for mastery of objectives often enough to place them so that they are always studying something that they have not yet learned, but something for which they have all the prerequisites. When a child is ready to learn an objective, a lesson sequence to teach that objective is individually prescribed for him by the teacher.

The developmenc of IPI began in May of 1964, and it was implemented last year (1964-65) at the Oakleaf Elementary School in the Baldwin-Whitehall School District near Pittsburgh, so that, at this time, we are in the second year of implementation. With the exception of small seminar and tutorial groups, a visitor at Oakleaf during math period sees 80 children seated in a large room in groups of from one to six. There are three roving teachers answering questions. When a child finishes a work sequence he has his work corrected by a clerk, or he finds the answer key and corrects the work himself. If the work was satisfactory, the student receives a prescription for a new sequence from one of the teachers. The prescription is based upon the work he has just completed, certain diagnostic tests, and the student's past history. If a student is at a point in the curriculum at which he is to be tested (on a unit of related objectives he has been studying) he goes to a teacher aide in an adjoining room to be tested. If the child's prescription simply calls for a certain workpage, workbook, or other lesson materials on an objective in the unit, he goes to get the materials in a second adjoining room.

The basic procedure in relating the student to the curriculum is as follows: A diagnostic placement test is given in each subject area (numeration, place value, addition, subtraction, multiplication, division, mixed operations, fractions, money, time, geometry, and special topics). After placement but before beginning work assignments in a given unit, the student is given a pretest in the lowest unit for which he failed to show mastery. This pattern persists; before beginning work in any unit, a pretest is given; if mastery is shown on the pretest, the student moves on to a pretest in the unit (perhaps in another area) most appropriate for him to master next. If he does not show mastery on the pretest, the student will receive work assignments and instruction until mastery is achieved. The net result is that the student never engages in formal instruction in areas whose objectives he has already learned.

It is interesting to quote a passage which, except in so far as all of us are in the same cultural stream, was written completely independently of this project. The article appeared in a recent issue of the American Mathematical Monthly, the Journal of the American Mathematical Association.

> For a long time psychologists and measurement specialists have told us that objectives must be stated in terms of performance-based behavioral patterns. Has this been done for elementary school mathematics? Almost always the answer is, 'N.' . . .
>
> Furthermore, the psy-hologists or learning theorists suggest a definite procedure for designing and creating blocks or units of learning materials for elementary school mathematics, or for any other subject matter. This may be summarized as follows: Specify the objectives of the materials in terms of behaviors which can be measured. When this has been done, materials should provide for individual differences so that students of varying abilities can achieve the objectives. Materials should be tested and revised, if necessary, until data can be provided to insure that the objectives are attained with the intended audience . . .
>
> It is expected, therefore, that if teaching materials tested for effectiveness of student learning is available, the classroom teacher can make intelligent choices as to what to use during the time a student or class is studying a particular lesson or topic. (Zant, 1965)

This is very close to our approach. If we were re-writing the above passage, we would lean more heavily on designing the system so that each child moves at his own best rate that the system can provide.

Role of curriculum objectives in the individualized program. The entire program is keyed to a set of behavioral objectives which define the curriculum. The hierarchical list of expected student behaviors is the basic document for the work of writing placement, diagnostic and achievement tests, gathering lesson materials, developing teaching procedures, and evaluating program effectiveness.

Curriculum Currently Being Evolved

The Oakleaf curriculum which is currently in use does not exhibit the full thrust of recent mathematics curriculum developments. There are two reasons: (1) it was felt that the teachers could not learn both the new procedures of instruction and also prepare themselves adequately for new curriculum content; (2) a curriculum utilizing all the concepts in the new curricula would require the preparation of hundreds of lessons designed for the individualized system.

Our future plans do encompass extensive development of objectives, lessons, and procedures to implement a new mathematics curriculum. The forces which shaped the objectives will be described below. Objectives were drawn principally from analysis of the following innovative programs: (a) Math Workshop by Wirtz, Botell, Sawyer, and Beberman; (b) Math Laboratory by Lore Rasmussen; (c) School Mathematics Study Group by a large number of teachers and professors; (d) Sets and Numbers by Patrick Suppes; (e) Elementary School Geometry by Hawley and Suppes; and (f) Minnemath Elementary School Program by Rosenbloom. The procedure was to examine the materials of the above programs and ask, "What is the student asked to do?" The answer, a statement of what the student must do in order to respond to the stimuli of the lessons and tests, became a behavioral objective.

From the set of all possible objectives which could be extracted from the above programs, those objectives were chosen which, in our judgment, developed mathematical concepts which seemed appropriate to the characteristics of elementary school students and which defined a well-developed sequence. A first draft was then turned over to the lesson writers, test writers, and teachers for comment and revision.

Need of different language for different groups. It might be thought that an objective, once written, would be adequate for all audiences. Different audiences, however, have different attitudes, goals, and backgrounds. Just as our students bring different entering behaviors to the learning task and thereby need different lessons, so the teacher, lesson writer, and test writer will respond to the words of the curriculum according to their separate backgrounds and job goals. The test writer wants criteria and examples which will define the test items to be written. The lesson writer wants a hierarchical structured sequence which will prompt the lesson writing sequence. The teacher responds most favorably to qualitative language which is related to the skills of the students who will be in the classroom. It should be emphasized that the three curriculum documents are merely different versions of the same basic statement of objectives.

Modification of objectives by lesson writers. The original first draft was written from an analysis of the programs listed above which are presently being used in schools. This guaranteed both a prototype lesson and that some students had been successfully taught the objective in question. This left the individualization of the lessons as our only goal.

The lesson writers found that for their purposes the behavioral steps were too large, that lesson writing was facilitated if each objective in the original list was expanded into several objectives which made explicit the behavioral sub-steps required. Tables 1* and 2 compare teaching objectives in the first draft with those resulting after sub-objectives have been included. Table 1 shows the main objectives. Table 2 shows the objectives with their accompanying sub-objectives.

Still another modification took place. The lesson writers were trained primarily in mathematics. As the lessons began to take form, there was a tendency to add objectives because it seemed possible to teach them in the sequence being developed. It seemed like a wasted opportunity not to teach a concept which apparently could be taught at each point.

Modification of objectives for communicating with non-readers. The problem of individualized instruction for non-readers is very complex. Information and directions must be transmitted to students who have few well-defined verbal concepts. Since they cannot read, information must be transmitted by aural means and by pictures. Certain responses can be cued by graphic means, e.g., always trace along dotted lines. But many directions and most instruction must build upon learned behaviors which cannot be cued by purely graphic means.

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*Tables 1 and 2 can be found at the end of the paper.

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Because of this "non-reading" obstacle, lessons for many objectives were difficult to prepare. At this point a commercial tape-reading device (Bell & Howell Language Master) became available to the lesson writers. Lessons which were formerly difficult to construct became a fairly straightforward matter because the device enabled communication which coupled graphic displays, e.g., cows to be counted, with aural instruction, "Count the cows." While the Language Master is not a complete solution to all instructional problems, it has provided enough flexibility to permit lesson writers to teach objectives which seemed impractical before the device was made available.

With the above introduction to the general steps in developing a mathematics curriculum, it is useful to consider the manner in which certain considerations shaped the curriculum.

Sources of Influence Upon the Curriculum

Briefly stated, the curriculum should attempt to meet the following conditions:

1. It should meet the needs of children whose education will terminate in high school as well as those who will go on to technical and mathematical education in college.

2. It should be comprehensive in scope. By this is meant that it will include not only the computational objectives of addition, subtraction, multiplication, and division but also objectives leading to mastery of concepts identified by a consensus of scholars working on elementary school curricula.

3. It should recognize that different children learn in different ways, and that in an individualized program a variety of ways of developing a concept must be provided if the students are to have a fair chance to generalize effectively.

Commitment to new mathematics. In order to avoid limiting a child's future opportunities, we attempt to give each child a curriculum which would enable him to become a mathematician or scientist. If we do this, we find the charge that a curriculum aimed at a career in mathematics is too much for

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the average student, that his time is taken up with the attempt to learn objectives he will not need, so that he winds up knowing neither the new math nor the old math. This attitude is typified by Sally of the comic strip "Peanuts" who cries out in a "New Math" class that all she wants to know is, "How much is two and two?"

We speak to this dilemma by arguing that in the world in which these children will live, both mathematical literacy and computational skill will be needed, that far from being antagonists these areas of the curriculum are complementary and can best be taught together, and that individualization makes practical the teaching of this expanded curriculum.

The need for and the strength of behavioral objectives. Most curricula are not written in terms of behavioral objectives. Usually curricula are defined in terms of conceptual areas of study rather than what the student must do to exhibit mastery. As learning psychologists interacted with groups preparing new programs of instruction they developed the procedure of first making explicit the behavioral objectives of the course of instruction (Mager, 1961). We have found the procedure indispensable. For our system to work, a great many people (including the students) must have a clear idea of what the student must do in order to move to the next unit of study or to stay and receive instruction.

Test writers, lesson writers, and teachers all get their marching orders from the sequential list of objectives. Through carefully written objectives, an independent group of test writers is able to write test questions for the program. By defining a clear goal which must be met by the student, the lesson writer has a limited area in which to work, from a known starting point to a given terminal state. The combination of behavioral objectives and feedback of student performance has, after lessons are in the school, given us a powerful instrument for lesson improvement.

The area of student evaluation and classification also speaks for the utility of behavioral objectives. The objectives define the entering behavior of the students so that any student can enter the program at any time and fit smoothly into the instructional program. In the reverse situation, a student leaving the school can leave with a complete record of what he is capable of doing.

Commercial sets of materials can be conveniently used in our program by assigning each page or lesson to one of the behavioral objectives. The commercial materials, once related to the program at a specific point, can then be assigned to a student for instructional purposes. All in all, we can agree with the following statement of Cronbach:

> The greatest novelty in the new curricula is not the content, not the instructional methods, not the grade placement of topics. The greatest novelty is the objectives from which all else stems . . . Objectives had better be identifiable and explainable if one is to avoid the absurd claim that his method achieves all ends at once, each in greater measure than any competing proposal. The new curricula, starting from the sound premise that different classroom activities reach different ends, deliberately sacrifice some ends for the sake of others. (Cronbach, 1965a)

A more succinct comment was made by one of the authors, "The objectives keep us honest."

<u>The effect of the need to measure entering behavior</u>. Many difficulties in instruction arise from assuming that a student has certain capabilities which in fact he does not have. We may ask a child to select the picture which is "the same as" one being shown. When the student fails to perform, we often assume that the student cannot make the discrimination required. The inability to perform may just as well result from not knowing the meaning of "the same as." The general solution is to determine the state of the learner at the beginning of instruction. Knowing the capabilities of the learner at the beginning of instruction, we can build upon these entering behaviors to lead the learner to a new advanced state. Since the definition of the entering state must be based upon the behaviors of the curriculum, we must make certain that any important entering behavior is listed as an objective. The question then becomes--how closely must we define the requisite hierarchy of behaviors in order to have a useful instrument for diagnosing the entering behavior of the students. In this way the problem of entering behavior is one of the determinants of the curriculum objectives. The problem is dealt with on the first approximation level simply by keeping the question in mind as objectives are being written. As the curriculum is

implemented and diagnostic scores are compared with eventual competence in learning, more quantitative measures can be used in refining the curriculum.

<u>Effect of individualization</u>. As a background for the arguments of this section, a few quotations are in order since the implications of individualization upon the curriculum were a matter of some discussion among the authors.

> A teacher should not concentrate on one particular way of explaining negative numbers. An explanation that satisfies one child does not satisfy another. (Sawyer, 1964)
>
> Clearly, people have characteristic cognitive styles that may strongly influence the way they learn and their learning efficiency. While "personalized" teaching may be impractical, it is at least theoretically possible that the most efficient teaching methods are those that are congruent with the learners' cognitive styles . . . (Mussen, 1965)

The above quotations are not proven research statements. However, the individualized program attempts to instill common concepts into students with varying abilities and backgrounds and properties as learners. Thus, the principal requirement which individualization imposes upon the curriculum is the requirement that there must be alternate paths to the same objective. In addition, we take the view that when a concept has been attained, the student will be able to display his mastery in a variety of settings. This variety of ways to reach a common objective can be explicitly stated by writing objectives which clearly describe the manner of reaching the objective stated. For example, consider the objective that the student add all two digit numbers. In this case, using a number line, using a place value code, or using the memorized addition tables with rules for carrying are different ways of exhibiting the objective. Rather than write three objectives, we can provide materials so that the teacher assigns each method as it is needed. A risk to this procedure is that a teacher will often avoid a method which either does not appeal to him or with which he is unfamiliar. Thus, the use of a place value code (e.g., abacus) may be avoided unless it is written as a separate objective, e.g., adds all two digit numbers using objects which represent a place value code.

Making alternate paths to an objective appear as clearly stated separate objectives has advantages: (1) test writing for transfer and (2) teaching procedure alternatives become more neatly defined operations. The manner in which a concept is being defined is more clearly visible to an outside observer and the system is more open to feedback and corrective change.

It might appear that the curriculum is open to an excessive number of objectives. However, not all alternate paths in existence are written into the curriculum. Only those methods which have given evidence of either teaching efficiency or usefulness to some later objective are used. Thus, the use of the number line not only serves as a method of teaching addition, but it is also an extremely important forerunner of analytic geometry and vectors. In addition to the above restriction of the numbers of objectives in the curriculum, the curriculum is rendered manageable because students can "pretest" out of any objective for which they show mastery. It is argued, then, that once a student acquires a concept through mastery of a variety of objectives, he will begin to test out of objectives which are merely variants of, or extensions of the concept he has achieved.

To summarize, an individualized program requires alternate paths to a major conceptual objective since not all students learn equally well by the same approach. For example, some students may need more practice, some students may need other instances of a concept (e.g., dividing actual arrays of objects to illustrate the concept of division). In turn, the curriculum can stand the additional objectives without becoming too long because students who have a major concept under control can test out of many related objectives.

Subject matter accuracy and logical progression. We have taken the position that one can usually avoid the necessity of making incorrect statements in catering to the undeveloped mathematics student. At the same time, however, an incorrect response by a student may be perfectly reasonable at an early stage (incorrect to the adult--not to the student). For example, the terms circle and disc are used correctly in lesson material, but a student who identifies a disc as a circle in a discussion is not asked to spend

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time in remedial purgatory. The concepts can be refined as the properties of plane figures and line figures and solid objects are developed. Usually the student can be protected from making errors in the early stages of learning by limiting the range of responses which he can make, e.g., the student may be asked to match geometric figures with names which are provided. Subsequently, the student must become more precise in his responses; he must learn to discriminate between the circle and the disc and be able to verbalize the distinction.

Logical progression refers to developing a sequence of behaviors so that later behaviors are a development of, an extension of, or an integration of earlier behaviors (Gagné, 1965). The extent to which the program departs from rote memorization of rules and tables depends, to a great extent, upon the degree to which the logical progressions is adhered to. For example, one can teach multiplication without having first taught the concept of addition; if by multiplication one means the memorization of the multiplication and addition tables and the memorization of the algorithm rules. Few will argue that this is to be preferred to a system which develops addition as implicit counting and multiplication as repeated addition. In the latter case the student can always ask himself, "Is this reasonable?" or he can check his answer in the early stages by repeated addition. The emphasis on logical progressions builds up multiple ways of arriving at the same conclusion, and this redundancy of method and approach, combined with mastery criteria for the earlier steps, results in confident self-evaluative learners who do not need to wait for their papers to be corrected before they have any idea as to whether they were correct.

<u>The use of high strength responses</u>. It is hypothesized that students who have a set of responses which they can make quickly (short latency) upon proper stimulus presentation will be able to use these responses readily in new learning situations. For example, if a student has memorized that 4 x 3 = 12 as a strictly rote skill, he should be able to call upon this response when he sees an array xxxx
xxxx
xxxx and if asked what multiplication problem this could represent, he should be able to respond, "Four things taken three

times is 3 x 4 = 12 altogether!" An insightful response may also occur with the thought, "Oh that's what 3 x 4 = 12 means." Another way of putting the sequence is that, having memorized the multiplication tables, the student is free to pay attention to other aspects of the concept of multiplication. Memorization leads to disaster only when it is mistaken for mastery of the concept and when it is forced according to a fixed timetable upon a student with a poor memory. In psychological language, we can say that a well developed response can easily be put under the control of a new stimulus (repeated addition or an array of objects).

The effect upon the curriculum is clear. Objectives are written which require mastery of addition, subtraction, multiplication and division combinations with short response latency. These are taught by flash cards, games which require quick answers, and timed drill. The objectives are verified by timed tests. However, if a student has difficulty with this phase of the work, he is allowed to progress into other objectives while the tables are assigned for ten minutes a day until the required mastery with short latency is achieved. It is argued that learning tables can interfere with other learning only if they are allowed to masquerade as mastery of the related concepts. A young child who memorizes a story and pretends that he is reading may have a valuable teaching aid unless he is allowed to continue the pretense until the act of learning to read seems painful and unreinforcing as compared to the pats on the head he got for reciting a memorized story.

Learning theory and educational experiments. Learning theory (operant conditioning) delineates the manner in which behavior may be shaped according to appropriate schedules of reinforcement. The superiority of positive reinforcement (rewards) over negative reinforcement (avoidance of punishment) is shown for animals and hypothesized for humans. Discussions of why students fail and learn to dislike school read like case histories of avoidance behavior. Just before there is danger of punishment (failure to pass an exam) the student works frantically (cramming). Immediately after an exam the student's response rate to the stimuli of the course falls to near zero. The trouble is that a good grade is supposed to be rewarding; and while this may be true for some, for others the grade as a reinforcing contingency has the defects of being too difficult of attainment for some, too removed in time from the learned behavior and too unrelated to the subject for others. In short, we must look for other reinforcers and other schedules of reinforcement. An alternative is to provide a sequence of tasks which are continually on the boundary between what the student knows and what he does not know. In such a system the odds for psychological success are high and the student has the chance of being reinforced by increasing mastery of the subject (Berlyne, 1965).

The effect of the above principles on the selection of objectives is somewhat oblique. Rather than determining special objectives, the concept of the objectives as defining mastery steps which will be reinforcing to the students without being punishing pervades the entire program. It may be pointed out that there is considerable variability in the time required to master objectives. Some objectives (by any given student) may be pretested out of in one day while other objectives may take as long as a month to master. This variability of the reinforcement schedule (if mastering objectives is reinforcing) has been shown to instill persistent responsive behavior in learning experiments.

In our experimental program some appealing results from other work in learning theory have been utilized in defining the curriculum. For example, (1) there is evidence that mastery of vocabulary as a system of coding concepts results in greater transfer, more rapid future learning (Cronbach, 1965a; Spencer, 1960). As a result, vocabulary to be learned in the context of the lessons is specified as an objective and is to be explicitly taught and mastery is to be verified. (2) Two of our outstanding educational psychologists (Cronbach & Carroll, 1965) in a recent conference on individual differences developed the theme that different students may learn a concept most effectively by the number line should also be able to display the concept in another, less congenial, setting. We agree with this idea for many reasons including the one above. Thus, a feature of our program is the enunciation of different approaches to a concept as separate objectives to be mastered. (3) Finally, there are the extremely useful results from Suppes (Suppes, 1965) and his co-workers which point out that transfer of concepts was effective if the learning situation required the learner to recognize the presence or absence of a concept in a number of possible responses. This leads to objectives which require the student to perform exercises in which his response is True or False, = or ≠, greater than, less than, or =, and find the set of numbers which will make an equation true and selecting the proper operation for a problem rather than merely selecting the correct answer for a series of multiple choices.

Testing requirements. The mastery tests (diagnostic achievement tests) and the diagnostic placement tests are prepared by a testing and evaluation group after the objectives have been written. This is done partly because of the expert skills of the testing group, partly because of the desire for an independent test prepared by someone who does not have a vested interest in the performance of the children, and partly to share the rather large work load of preparing the program. The result is that the objectives define the test questions which will define mastery and which will pace the student through the program. Thus, if a certain skill is critical for the future performance of the student, an objective must be written which unambiguously calls for this skill or the student may never be tested to show mastery of that skill. For example, if it is felt to be important that the student be able to display the ability to add using a number line, this must be stated, "Adds single digit numbers using a number line." In this way the mastery of test items could to a certain extent be controlled by the objective-writing procedure.

Interaction between lesson writers and the new curriculum. The curriculum presented to the lesson writers did not put a severe limit on their opportunity to interpret and be creative. Consider the sample item, ". . . Adds using the number line."

The statement that a child is to add using the number line is not very much more helpful to the lesson writer than saying the child should understand addition on the number line. The main burden of deciding how to go about teaching this to a child remains. The lesson writer for individualized instruction identifies with the teacher: All the things that a good teacher would say and do to teach a child to add on a number line must be made explicit, behavioral, and put into an order of tasks starting with what the child already knows how to do and terminating with what we

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want the child to finally be able to do. The lesson writers keep in mind that the lessons that they write will be deemed successful only if real children actually exhibit the desired behaviors; furthermore, since a given objective will form the foundation for a child's interaction with many future objectives, the lesson writers' decision of what adding using the number line will mean for a very young child must not be too narrow; i.e., if the stimulus used shows no variability, the student's response may not occur when the number line appears in different form in a later lesson.

Our unit on addition follows a unit on geometry and a unit on numeration. In the geometry unit, the children learn to identify straight line and many other things. In numeration, they learn to count from 1 to 5, recognize numerals, construct number line segments, and say what number comes after a given number. Based on these capabilities, the following objectives in Table 2 emerged as what we mean for a five or six year old to add using the number line: 3a, b, c, d, e.

<u>The use of an audio device for non-readers</u>. A five-year old must receive aural directions rather frequently in an instructional sequence. How does one teacher tell 30 five-year olds what to do every minute or so if each child must receive different directions? Our present solution, or partial solution as time will tell, is a piece of hardware called a "language master." The significance of this machine is that it permits the individualization of the instruction of a large group of very young children under the supervision of one teacher.

This machine is the size and shape of a portable typewriter (see Figure 1). When a card is placed in a slot at the top, it is moved from right to left through a tape recorder head and a pre-recorded message is broadcast. Only one inch at the bottom of the card, the part with the recording tape, is out of sight in the slot so that the picture on the card can be viewed as the aural message is played. We are using cards that vary in size from five by eight inches to eight by eleven inches depending on the picture and the length of the recorded message. Two messages can be recorded on a single tape, and an easily operated switch on the machine controls which track is played; there is also provision for a child to record his own vocal responses through proper use of a second switch on the machine.

The significant attributes of the machine are (1) simultaneity of aural and visual stimuli and (2) versatility of the types of directions that can be given. A sequence of short directions can be given by means of a sequence of cards; it is possible to ask a question on one track and give an answer on the second track of the tape on a single card. (3) The child is active and has control. If a child decides to put a card through the machine, he will probably pay attention; conversely, unless the child does put a card through, nothing happens--he is not a passive audience--he has to act; the child can put the same card through again, hear and see the same message over.

To illustrate the effect on the curriculum of our using this machine in our individualized lessons, we ask the reader to consider for a moment the classical tutorial situation: one teacher with one student. What is conspicuous to us is that most of the details of giving directions to the student are ad-libbed by the teacher; furthermore, the teacher must either present all of the instructions out of his head, or, if using prepared materials, he must fill in any gaps in the materials given to student. Indeed, filling in gaps and interpreting a non-behavioral list of objectives into behavioral terms is what we have traditionally expected teachers to do in the process of teaching. This is true whether teaching one student or a class of students. Even if our school systems were able to afford one teacher for each child, the system might break down because we could not hope to bring every teacher to the kind of comprehensive competence required. In our illustration, however, let us assume a very good teacher, wise and resourceful, who can give directions to a child and fill in gaps in the lessons and the curriculum as they turn up.

Contrast this tutorial system with our five-year old in front of his language master machine putting a card in that says, "The rabbit is inside the circle." (switch) "Put a big X on that rabbit." If the child doesn't know any of the words--rabbit, circle, inside, or put, he may not respond correctly or at all. A tutor would try to fill in these deficiencies on the spot; our machine obviously cannot. Our machine cannot ad-lib or fill in any gaps. The implication is that our curriculum must be so detailed and so ordered that the child will already have the behaviors prerequisite not only to learning the next academic objective, but to following all the directions in the lessons. So before the child puts the rabbit card through the machine, that is, before he receives instruction in the objective "inside and outside of circles," he must be brought to show mastery of making X's, recognizing animals, and responding appropriately on worksheets that accompany the language master cards. It is true that when a child in our program gets "stuck," he can raise his hand and receive help from the teacher. But if too many children get stuck too often, the system will break down: we have too many children and too few teachers to tolerate many rough spots in the lessons.

We will continue to refine the curriculum as we work with the program at our Oakleaf school; just how fine grained the curriculum will have to be made is not yet known.

Promising Aspects and Potential Problems

The range of achievement. The individualized plan of instruction has resulted in wide ranges of achievement during the school year of 1964-65. The most productive student completed 19 units of work; the slowest student completed but 3 units of work. The mode was 6 units of work (10 to 30 students completed six units of work). The average was seven units of work. Looking at the levels of achievement of the class as a whole, the first grade class at the end of the school year looks like the third grade class at the beginning of the year. This left open the question of loss of skill over the summer. It is encouraging that although some loss showed up in September testing, this was a short term effect. By October 15, 1965, the second grade class (last year's first grade) had an overall achievement level virtually the same as the third grade class had on October 15, 1964. That is, the second grade class after one year of individualized instruction seemed to be a year advanced. Similar comparisons can be made for other pairs of classes in the school. It seems most appropriate to make the comparison with the first grade class which in its school life experienced only the individualized program for its formal instruction in arithmetic.

The curriculum as a research instrument. An unexpected result of an instructional system based upon the condition of mastery of behavioral objectives is that revision to meet any given defect takes less than a month from the time that a problem is detected to the time new materials representing a new approach is at the school. This opens the way for many kinds of materials analysis if the concept of feedback between the lesson writer and the student performance is accepted. This time constant for revision may be reduced still further by computer techniques (see below). Possible dimensions of lesson analysis are, (1) different average time to mastery through different approaches, (2) persistence of study activity as measured by time spent actually working as opposed to time spent talking, dreaming, etc., (3) student enjoyment of work assignments as measured by choice to continue or escape when offered a choice of activities. While the approach suggested is empirical, the resulting information can be a rich source of clues for basic research into the learning process.

Curriculum as a base for attacking the non-cognitive domain. Many express concern for such affective aspects of learning as self-evaluation of learning, self-direction of learning activities, self-initiated activities in learning or in using learned skills. From a base of known mastery of a set of behaviors, it looks promising to ask, "What are the conditions which will promote the changes which will result in the desirable behaviors mentioned above?" Without a base of known behavioral capability, one never knows if lack of self-direction, initiation in mathematics is due to lack of skill or lack of interest or simply the lack of the concepts of choosing one's own objective, choosing one's own path, and choosing the means of travel. Perhaps some day a detailed curriculum will be written leading to dependable behavior in the affective domain. In our program we are beginning to build such a curriculum upon well-defined cognitive skills.

Integration of computer-assisted instruction with individualized instruction. As has been indicated, individualized learning such as described here adapts to individual differences in the following ways: (a) it starts each student from where he is on the learning continuum; (b) the instruction the student receives is differentiated according to his performance as he learns; and (c) quality control of student learning is maintained, and performance criteria are used as a basis for making decisions concerning the student's future course of instruction. These criteria can be the result of measures such as the mastery level attained, the rate of attaining this level, the difference between his initial score and his final score, performance on a retention test, etc. Adaptation to individual differences is carried out by detailed assessment of the learner history, on which basis decisions are made about the next learning step. These decisions are made in order to optimize a performance criterion or certain combinations of performance criteria. This is done for each student. In the individualized situation extensive data are available on each student, on which basis continuing instructional decisions need to be made. Such close monitoring of student instruction seems patently impossible with standard learning material, and it seems reasonable to assume that such instructional decision-making can be most efficiently made with the capabilities of computer monitoring and data processing. The concept of a computer-assisted classroom appears as a feasible consideration in carrying out individualized learning. Over the long run, it is estimated that such systems will not be prohibitively expensive for school systems to consider.

Some of the potentials of computer-assisted instruction can be briefly stated: (1) devices for the presentation of dynamic displays can provide a rich environment in which the learner can be presented with a highly manipulative and responsive instructional situation (see Glaser, Lipson, and Ramage). (2) Detailed records of the student's responses can be analyzed and up-dated. (3) Decision rules based on these responses can be implemented by appropriate computer programs. In some cases, a rule can prescribe the learner's subsequent instruction; in other cases, a suggested prescription can be printed for the teacher to use as information

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for further instructional decisions. (4) At any time any part of a student's learning history can be obtained for comparison, grading, and school evaluation.

An additional use of the computer is to assist the design of instructional materials. It seems possible that a future mathematics curriculum will be developed and validated on the basis of rapid feedback data obtained by the writer as a student learns. On the basis of almost instantaneous knowledge of student responses, revisions could be made on the spot. Alternatively, detailed analysis of the learning record over some period of time can provide data for subsequent curriculum revision.

Possibility of retardation of students by individualized instruction. Class averages and group performance does not answer the question, "Are there students who would do better in the conventional classroom setting?" By the very argument of individualization there may be some children who would do better if challenged and stimulated by conventional classroom activities. We attempt to provide some of these activities through one day of mathematics seminar in which the class engages in group activities. There may be other ways of identifying students who need various forms of group interaction in order to meet their educational requirements.

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Table 1

First Draft of Mathematics Curriculum

Level A. Introductory Level of Curriculum

Addition

1. Given up to five objects, identifies how many are in each of a number of subsets and "HOW MANY are there ALTOGETHER?"

   Response is
   a. oral
   b. by drawing a line to a numeral
   c. by making tally marks
   d. by writing the correct numeral

2. Responds
   a. orally
   b. by writing a numeral
   c. by selecting a numeral when asked questions of the type: "What is ONE MORE than ________" and "________ is ONE MORE than what number?" Numbers to 5.

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Table 2

Modifications of First Draft
Resulting from Lesson Preparation Analysis

A ADDITION

Prerequisites: A Numeration

Task Categories:

1. Joining sets and indicating how many.

2. Associating compound addition numerals with the joining of two sets (by selecting numerals, selecting and constructing sets).

3. Associating compound addition numerals with taking steps on a number line segment.

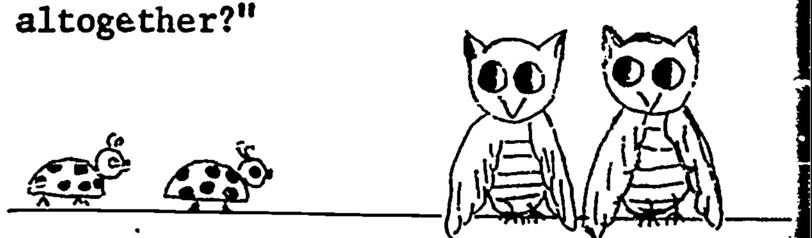
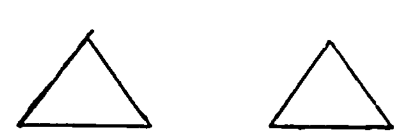
OBJECTIVES	SAMPLE TEST ITEMS
1a. Puts two sets together (pastes pictured sets, folds page, draws ring around all, shades in common background).	1a. "Put the two sets together by drawing a ring." "Put the two sets together by folding on the dotted lines."
1b. Indicates how many things are in each of two sets and then how many things there are altogether.	1b. "How many bugs are on the line?" "How many owls are on the line?" "How many animals are on the line altogether?" 
2a. Indicates that the component numeral of an addition numeral are counting numerals.	2a. "X the counting numerals: 1 2 + 2 Answer 1 2 + 2
2b. Presented with two sets with 0-5 things each, selects a compound addition numeral in response to being asked how many things there are altogether.	2b. "Circle the addition numeral for how many triangles you see." 1 + 1 3 + 2 5 + 0 

Table 2
(continued)

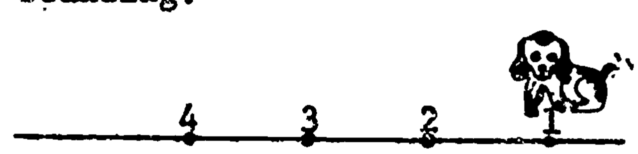
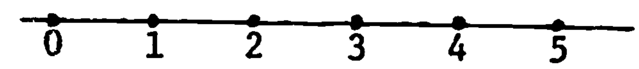
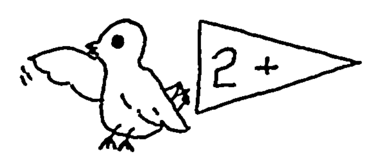
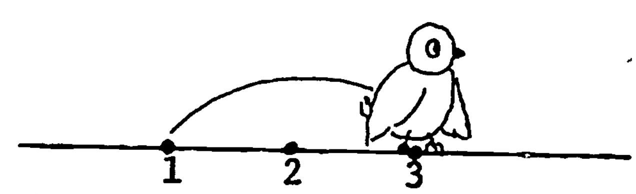
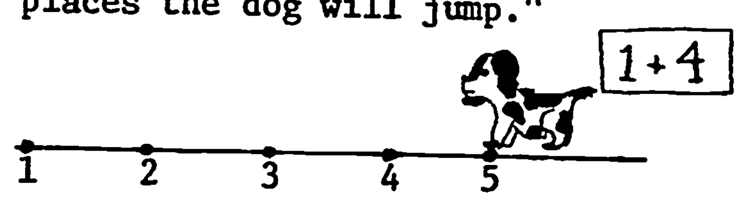
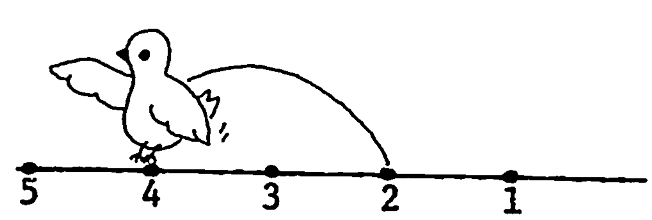
OBJECTIVES	SAMPLE TEST ITEMS
2c. Makes tally marks above each counting numeral in a compound addition numeral, numerals 0 + 0 through 5 + 5. Responds to the command, e.g., "Make four plus three tally marks (present numeral)."	2c. "Make this many tally marks." 3 + 5 Answer ''' ''''' 3 + 5
2d. Given a compound addition numeral whose sum is 5 or less, makes that many tally marks and then selects the counting numeral which represents how many tally marks there are altogether.	2d. "Make this many tally marks." 2 + 3 "Circle the numeral which says how many tally marks there are altogether." 0 5 4
2e. Selects the set with as many things as represented by a given compound numeral.	2e. "Put an X next to the set with this many stars." 2 + 1 
2f. Selects the compound addition numeral that is equal to a given counting numeral.	2f. Teacher points to each compound numeral in turn saying, "Make this many tally marks." "Now circle the addition numeral for this many tally marks." ' ' ''' ' 1 + 1 3 + 1 Answer 4
3a. Indicates all the numerals in view that come after a given numeral on a given number line segment.	3a. "Circle the numerals on this line segment that come after the two." 0 1 2 3 4 5 
3b. Selects the numeral on a number line segment at which an animal is standing.	3b. "X the numeral where the dog is standing." 

Table 2
(continued)

OBJECTIVES	SAMPLE TEST ITEMS
3c. Presented with the picture of an animal carrying one of the numerals 0+, 1+, 2+, 3+, 4+, 5+, and approaching a number line segment, indicates where the animal will stand and which numerals he will face.	3c.  "Where will the bird stand?" "What numeral will the bird face?" Answer (1) at the two (2) 3, 4, and 5 
3d. Given an addition numeral together with a pictured number line journey corresponding to that numeral, where the journeys started, how many places were jumped, where the journey finished.	3d. 1 + 2  "How many places did the chick jump?"
3e. Presented with a picture of an animal carrying an addition numeral, indicates the numeral telling where the animal will start, how many places the animal will jump.	3e. "Circle the numeral for how many places the dog will jump." 
3f. Selects the addition numeral corresponding to a pictured journey on the number line.	3f.  1 + 4 1 + 1 2 + 2 "Circle the addition numeral for what the bird did."

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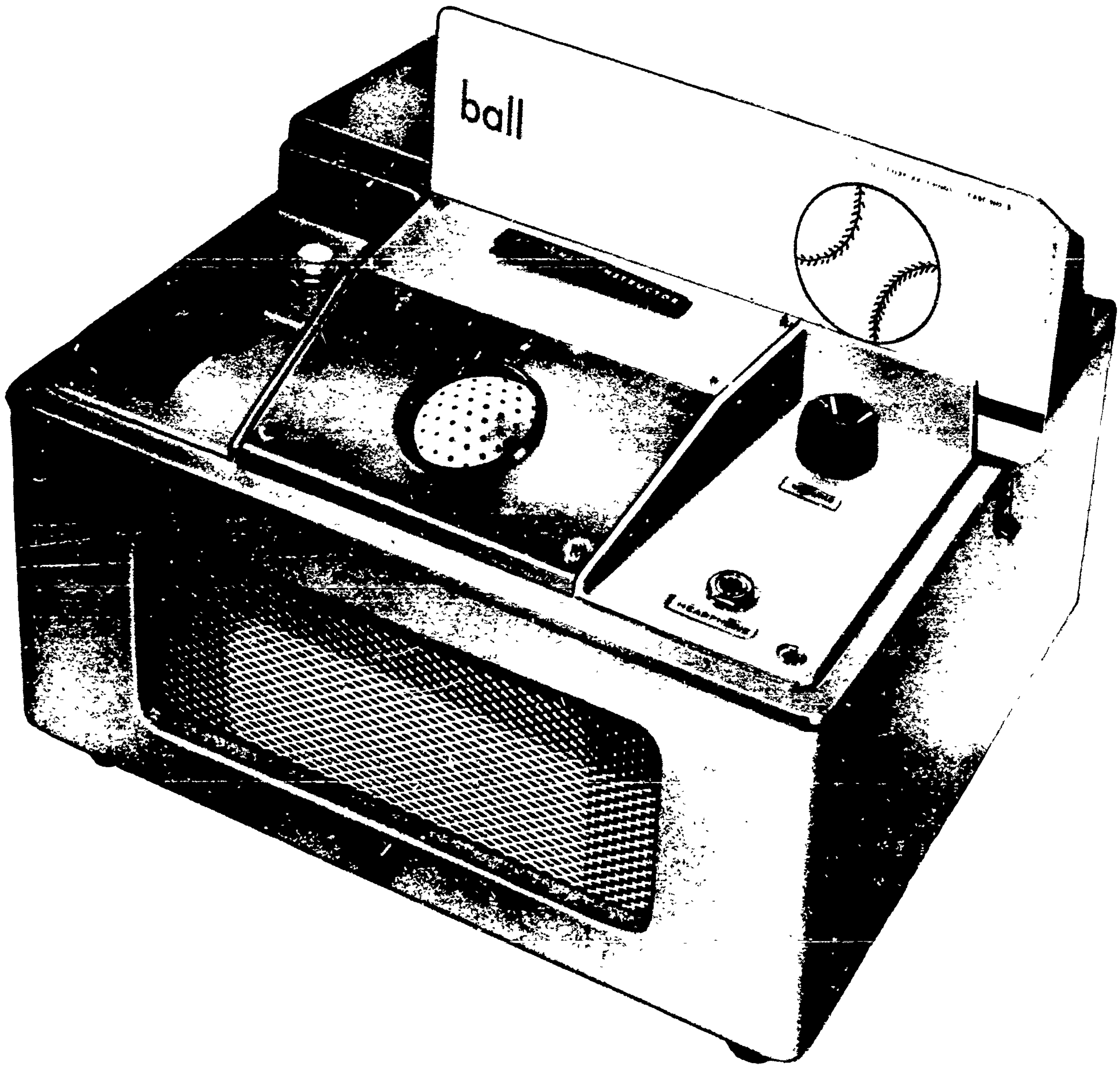

Figure 1: Bell & Howell Language Master