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A FIRST STEP TOWARDS THE IMPLEMENTATION OF THE CAMBRIDGE MATHEMATICS CURRICULUM IN A K-12 UNGRADED SCHOOL.

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*MATHEMATICS CURRICULUM, *CURRICULUM RESEARCH, *UNGRADED SCHOOLS, *CONFERENCES, *SYSTEMS ANALYSIS, EXPERIMENTAL PROGRAMS, ABILITY GROUPING, INDIVIDUALIZED CURRICULUM, INSERVICE TEACHER EDUCATION, TEACHING METHODS, PROGRAM DEVELOPMENT, NOVA SCHOOL, BROWARD COUNTY, FLORIDA, TALLAHASSEE

A SERIES OF THREE CONFERENCES WAS HELD TO EXPLORE THE FEASIBILITY OF IMPLEMENTING A LONG-RANGE CURRICULUM DEVELOPMENT PROJECT FOR AN UNGRADED, K-12 SCHOOL, BASED ON RECOMMENDATIONS OF THE CAMBRIDGE CONFERENCE ON SCHOOL MATHEMATICS. OVER 50 MATHEMATICIANS, MATHEMATICS EDUCATORS, AND PERSONS INVOLVED IN THEORETICAL AND APPLIED PSYCHOLOGICAL RESEARCH, EDUCATIONAL SYSTEMS ANALYSIS, AND ACADEMIC GAMES DEVELOPMENT PARTICIPATED. PRINCIPAL AREAS OF CONCERN IN THE PROJECT WERE--(1) CURRICULUM DEVELOPMENT, (2) TEACHER TRAINING, (3) IMPLICATIONS OF LEARNING AND COGNITION THEORY, AND (4) RESEARCH AND EVALUATION. RESULTS OF THE CONFERENCES SUGGESTED THAT THE SUBJECT MATHEMATICS CURRICULUM COULD BEST BE DEVELOPED THROUGH AN EDUCATIONAL SYSTEMS APPROACH, USING BASIC FUNCTIONAL SYSTEM COMPONENTS OF INSTRUCTIONAL RESOURCES, SYSTEMS MODIFIERS, SYSTEMS ANALYSIS, DATA STORAGE AND PROCESSING, AN ACTION MONITOR, AND A VALUE ESTIMATOR. ADDITIONAL DISCUSSION CENTERED ON THE AREAS OF CURRICULUM CONTENT AND MATRICES, STUDENT GUIDES, AND AN INSERVICE TEACHER-TRAINING PROGRAM. IT WAS RECOMMENDED THAT A PILOT PROJECT BE CONDUCTED TO INVOLVE A SMALL PORTION OF THE SCHOOL POPULATION PRIOR TO OVERALL PROGRAM IMPLEMENTATION. (JH)

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A FIRST STEP TOWARDS THE IMPLEMENTATION OF THE CAMBRIDGE MATHEMATICS CURRICULUM IN A K - 12 UNGRADED SCHOOL

Cooperative Research Project No. S-405

Garrett R. Foster
Burt A. Kaufman
William M. Fitzgerald

Florida State University
Tallahassee, Florida

1966

The research reported herein was supported by the Cooperative Research Program of the Office of Education, U. S. Department of Health, Education, and Welfare.

FOREWORD

During the fall of 1965 a series of conferences ~~were~~ was held at Nova High School, Fort Lauderdale, Florida, to explore the feasibility of implementing a long-range curriculum development project for a non-graded, K - 12 school based on the recommendations of the Cambridge Conference on School Mathematics. A cross section of mathematicians, mathematics educators, and researchers attended these conferences. This is a report on those proceedings and the conclusions and recommendations reached.

This report goes well beyond the Cambridge Conference Report in suggesting the ultimate requirements of an educational system which will be needed to implement an optimal mathematics curriculum for all students. The components of the system are essentially teacher training, materials production, information processing, research and evaluation. The proposed system provides for the integration of these components into the daily activities of a school, thus granting these important functions full membership in the educational process.

The optimal mathematics curriculum for all students is here defined as one which allows unique tracking through a sequence of mathematical experiences for each student. The concept of a curriculum matrix as a framework on which one can build such an individualized program seems to offer a new direction in curriculum development in general and mathematics curriculum development in particular.

It is hoped that this report will be of interest to a wide variety of people. It should be especially valuable to:

(1) those who contemplate curriculum revision in mathematics in the spirit of the Cambridge Conference on School Mathematics.

(2) those who seriously desire to "individualize" instruction.

(3) those who believe that vertical acceleration of the faster learning students is a necessary part of an optimum program for all students.

It should be evident from the list of conference participants that the views and ideas expressed in this report will demand serious consideration by the entire mathematics education community in this country. Although complete agreement on all phases of the project was not reached, it is significant that there was wide-spread agreement that a long-range curriculum development project such as outlined in this report should be seriously attempted as soon as possible.

Dexter A. Magers,
State Supervisor of Mathematics,
Florida State Department of Education.

ACKNOWLEDGEMENTS

The authors are deeply grateful to Dr. Lauren Woodby and Dr. Hazen Curtis for their encouragement and support both in getting these conferences started, and in guiding them to a successful conclusion. We are indebted to the U. S. Office of Education for its financial support of these conferences and its confidence in the directors. A special word of thanks is due to Mr. Dexter Magers and the Florida State Department of Education for the supplementary financial aid contributed as well as the many hours of help given us in conducting the conferences and the writing of this report.

It goes without saying that the success of a series of conferences such as these depends mostly on the contributions of the participants. No one could have asked for better cooperation or dedication to the tasks at hand. We were most impressed by their willingness to give so much of their valuable time and energy to see that these meetings were successful in laying the groundwork for a long range development project. We were most fortunate in having the continual guidance of the personnel of the Madison Project and in particular its director, Dr. Robert Davis. His inspiration to all of us was one of our chief assets. We are also grateful to Dr. Warren Winstead and the Nova University of Advanced Technology for their willingness to act as host for our conference banquets. To Mr. Fred Lica, our conference coordinator and Mrs. Lois Armetta, the conference secretary, Paul Armetta and Robert Buckley, conference recorders, go our heartfelt thanks for making the entire proceedings a very smooth operation.

There is one group of people for whom it is most difficult to find any words which would adequately express our appreciation. The Nova students of the three demonstration classes presented to the conference participants an existence proof of what students can and will accomplish once liberated from the confines of traditional curriculum patterns. After watching these youngsters perform at a rather high level of mathematical maturity, the conferees seemed convinced of the feasibility and desirability of our long range plans. These students were the best public relations agents imaginable. Their contribution must be rated as most important.

Burt A. Kaufman

Garrett R. Foster

William M. Fitzgerald

NOVA MATHEMATICS CONFERENCE PARTICIPANTS

<u>August Conference</u>

Layman Allen, Yale University

Ned Bryan, United States Office of Education

Robert Davis, Madison Project, Webster College, Syracuse University

William M. Fitzgerald, Florida Atlantic University

Garrett R. Foster, Florida State University, Nova Schools

Abraham M. Glicksman, Bronx High School of Science

Royce Hargrove, Science Research Associates

Julius H. Hlavaty, DeWitt Clinton High School

Burt A. Kaufman, Nova Schools

Dexter Magers, Florida State Department of Education

Herman Meyers, University of Miami

Edwin E. Moise, Harvard University

Walter Prenowitz, Brooklyn College

Charles R. Rickart, Yale University

Gerald Rising, University of Minnesota

Harry Ruderman, Hunter College

Charles A. Whitmer, National Science Foundation

<u>October Conference</u>

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Charles Brumfiel, University of Michigan

Robert Burrows, Florida State Department of Education

Robert Exner, Syracuse University

William M. Fitzgerald, Florida Atlantic University

Jack Freeman, Florida Atlantic University

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Vincent Haag, Franklin and Marshall College

Paul Halmos, University of Michigan

Phillip Jones, University of Michigan

Burt A. Kaufman, Nova Schools

Kathy Kharas, Madison Project, Webster College

NOVA MATHEMATICS CONFERENCE PARTICIPANTS

October Conference (Continued)

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Eugene Nichols, Florida State University

David Robinson, Greece Central School System

Paul Rosenbloom, Columbia University

George Springer, Indiana University

Marshall Stone, University of Chicago

Carolyn Teixeira, Educational Services Incorporated

Frank Van Atta, Madison Project, Webster College

A. D. Wallace, University of Florida

Lauren Woodby, United States Office of Education

November Conference

Robert Allen, Nova High School

Ned Bryan, United States Office of Education

J. Robert Cleary, Educational Testing Service

Donald Cohen, Madison Project, Webster College

Hazen Curtis, Florida State University

J. A. Easley, U.I.C.S.M., University of Illinois

William M. Fitzgerald, Florida Atlantic University

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F. J. King, Florida State University

Fred Lica, Sunbeam Electronics

Dexter Magers, Florida State Department of Education

Barbara Mills, Nova Schools

Edward Palmer, Florida State University

Joseph M. Scandura, Florida State University

David Sulzinski, Sunbeam Electronics

Joseph Ward, Lackland Air Force Base

PROJECT PERSONNEL

Co-Director	-	Garrett R. Foster, Assistant Professor, Department of Educational Research and Testing, Florida State University.
Co-Director	-	Burt A. Kaufman, Mathematics Coordinator, Nova Schools.
Chief Consultant	-	William M. Fitzgerald, Associate Professor of Mathematics, Florida Atlantic University.
Administrative Assistant	-	Fred Lica, Methods and Procedures Administrator, Sunbeam Electronics.
Project Secretary	-	Lois Armetta, Nova High School.
Project Recorders	-	Paul Armetta and Robert Buckley, Nova High School.

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APPENDICES

BACKGROUND OF THE NOVA MATHEMATICS CONFERENCES

Nova is a non-graded K-12 public school in Broward County, Florida. The high school began operation in September, 1963 and the elementary school opened in September, 1965.

Mathematics is taken seriously at Nova. The entire faculty feels that children can and will learn much more mathematics than they now learn if the appropriate provisions are made within the school. The philosophy of the mathematics department at Nova school is described in the following statement excerpted from the Scope, Sequence and Philosophy of the Nova School Mathematics Department.

> Nova's purpose is to prepare the minds of students for "work that does not yet exist and whose nature cannot even be imagined". Frontiers of mathematics are changing daily. The curriculum of 50, 20, or even 10 years ago no longer meets all the needs of our society. Space missiles are only symbols of the great explosion of scientific knowledge of the Twentieth Century. One of the most important factors contributing to this explosion is the revolutionary advance in both the development and use of mathematics. New requirements are being placed on mathematics in the behavioral and the life sciences as well as in the fields of physics, chemistry and engineering.
>
> Modern applications of mathematics require less manipulation of formulas and equations but greater understanding of the structure of mathematics and mathematical systems. There is less emphasis on human computation that can be done by machines and more emphasis on the construction of mathematical models and symbolic representation of ideas and relationships. The role of mathematics is not only to grind out answers to engineering problems but also to produce mathematical models that forecast the outcomes of social trends and even the behavioral changes of the individuals and social groups. Such important new uses and interpretations of mathematics require that students have a program with greater depth than the classical program designed for Nineteenth Century education.
>
> Because of these new uses, mathematics in its broadest sense should be woven firmly into the fabric of the national culture. Mathematics can and must be taught in a way which focuses on its fundamental structure; emphasis should be put on discovery and encouraging mature mathematical discussions. The unifying ideas of mathematics should be introduced to students early in their school life and re-examined and re-inforced during succeeding years. Although the child's mind is far from ready to master these ideas in all their generality, seeds of each can be planted early. They will grow year by year in the right climate to mature understanding. The mathematics necessary for personal every day use requires a basic minimum of learning for all students. This is further developed and deepened as the student progresses in his general education, and into this basic body of knowledge certain topics are introduced to fit the needs of the individual.

For the student who intends to pursue higher levels of mathematics learning, provision is made to build upon these basic concepts and acquire much additional knowledge. The curriculum should meet the needs of all students, regardless of their academic aims and emphasizes those principles of mathematics which are essential for a firm foundation in the subject.

OBJECTIVES

1. Provide a program which will challenge the ability and develop the interests and talents of every student.
2. Develop understanding of the concepts of mathematics and proficiency in mathematical skills and techniques.
3. Create the desire for further study of mathematics.
4. Introduce patterns of thinking as illustrated in the structure of mathematics. The abilities to recognize various kinds of reasoning and to reason logically must be developed. In particular, the student should be acquainted with deductive and inductive reasoning as applied to the development of mathematical theory.
5. Create intellectual enjoyment of mastering challenging problems, discovering mathematical ideas and developing analytical insights.
6. Develop the habit of estimating and checking all mathematical computations.
7. Develop the ability to create mathematics which is original in terms of the student's past experience.
8. Develop the ability to talk and write about mathematics in a mature fashion.
9. Help the student become familiar with and be able to use universally accepted symbolism. Experience is provided for students to create some symbolism of their own as one way of appreciating how mathematics is created.
10. Develop the ability to read mathematical treatises on the student level and criticize mathematical writing. In particular, students must learn not to rely totally on textbooks and should be encouraged to be critical of such books. The practice of believing that the book is never wrong should be abandoned.
11. Encourage students to use many books and other research materials in their learning process.
12. Teach appreciation for the significance of mathematics as applied in other fields.
13. Impart cultural enrichment through appreciation of the historical growth of mathematics and the works of great mathematicians.
14. Develop related abilities such as effective study habits, ability to organize one's work, skill in self-expression and good habits of neatness and accuracy.

From the very inception of the Nova program in September, 1963, the vehicle through which the above stated philosophy and objectives were to be achieved has been that of vertical acceleration for all students. Almost immediately it became apparent that the type of vertical acceleration desired here was not that normally found in most schools where all students study the same material but at different rates of speed. Initial experiences seemed to indicate that an optimum program of mathematics education for each student would eventually require extensive individualization of the mathematics curriculum.

As an ungraded school, Nova has been an ideal institution in which to begin a serious attempt to individualize mathematics instruction. Many of the boundary conditions which have impeded similar projects in the past were not present because of the dedication of the Nova administration and staff to breaking educational lock-steps. Among other things students were grouped purely by ability in mathematics without regard to grade level or age. The normal calendar pressures with respect to completion of courses, final exams, beginning new courses, etc. has been largely eliminated. Students begin new courses immediately upon completion of the old one regardless of when this occurs during the year. The first attempts at individualization was the creation of a 3 track trichotomy outlined on the following page.

It should be noted again that the tracks outlined present mathematics in 3 different levels of sophistication. For the mathematically talented it was felt that even the common concepts of the 3 tracks had to be presented in a totally different fashion.

Even with this limited effort at individualizing instruction, many of the track 1 students have attained levels of mathematical maturity exceeding that of many advanced college mathematics students. For some, this maturity level has been reached as early as 14 years of age. It was hypothesized from this that more concerted efforts in individualization could achieve breath-taking results. Furthermore, such a concerted effort might also produce amazing results for the less talented. It certainly should go a long way in raising the mathematical literacy level of the total student population.

Nova Secondary School Mathematics Curriculum Flow-Chart °

Entering 7th Year Students

◇

Basic Contemporary Math I

↓

Basic Contemporary Math II

Elementary Contemporary Mathematics I

↓

Elementary Contemporary Mathematics II

Tracks 2 and 3

↓

Elementary Contemporary Mathematics III

Track 3

↓ Entebbe Geometry I

↓ Entebbe Geometry II

↓ Intermediate Contemporary Mathematics I

↓ Intermediate Contemporary Mathematics II

Track 2

↓ SMSG Geometry I

↓ SMSG Geometry II

↓ Introduction to Modern Algebra

↓ Pre-Calculus I

↓ Pre-Calculus II

Track 1

↓ Foundations of Modern Algebra and Geometry

↓ Number Systems of Algebra *

↓ Introduction to Math Analysis I *

↓ Elementary Geometry from an Advanced Standpoint I *

↓ Elementary Geometry from an Advanced Standpoint II *

↓ Elementary Geometry from an Advanced Standpoint III *

↓ Introduction to Math Analysis II *

↓

Δ

Calculus I *

↓ Calculus II*

Abstract Algebra I *

↓ Abstract Algebra II *

↓ Abstract Algebra III *

° See Appendix A for course descriptions.

◇ Entering seventh year students will take Elementary Contemporary Mathematics I if they have the proper background. If their preparation in elementary school is weak, they will take Basic Contemporary Mathematics I and/or Basic Contemporary Mathematics II first and then they will enter the regular sequence starting with Elementary Contemporary Mathematics I.

* College Level Courses.

Δ The Calculus and Abstract Algebra sequences can be taken simultaneously or the track 1 student can elect either one or the other. It is anticipated that by 1967 or 1968 many students will have finished Introduction to Mathematical Analysis II during their 10th year in school and will be able to complete both of these sequences without having to do them simultaneously. In this case the Abstract Algebra sequence will be done first.

The Nova Mathematics staff expected that it would take many years to attract the support needed for such an effort. However, it was apparent in the reading of the Cambridge Report on Goals for School Mathematics that the curriculum that was being envisioned for the schools 50 years from now was being approximated to some degree by the existing Nova Program. Certainly the pedagogical principles and philosophy of the Nova staff were in complete conformity with that of the Cambridge Report. Therefore, each member of the Cambridge Conference was invited, in September, 1964, to propose ways in which Nova could become a development and demonstration center for the Cambridge curriculum.

The response was quite enthusiastic and, fortified by this support for seriously undertaking such a project, a proposal for a planning grant was written. In the summer of 1965, a grant was awarded by the U. S. Office of Education for a series of conferences to develop long range plans for the creation, implementation and evaluation of a K-12 mathematics program based on the suggested content and consistent with the stated goals of the Cambridge Conference Report. Out of these conferences came the following design for a comprehensive school mathematics program and specific recommendations for initial efforts in developing such a program.

CONFERENCE PROCEDURES

The list of persons invited to participate in the conferences was constructed from nominations made by members of the Nova Mathematics Advisory Board. An attempt was made to select participants with a wide diversity of experience and interests in the many fields which should contribute to a comprehensive mathematics curriculum. While the great majority of conference participants were mathematicians and mathematics educators, a special effort was made to include persons involved in theoretical and applied psychological research, educational systems analysis and academic games development.

Prior to the first conference, a tentative outline for the entire Nova Comprehensive Mathematics Program (NCMP) was developed in order to focus discussion on the principal areas of concern in the project:

(1) Curriculum development,
(2) Teacher Training,
(3) Implications of learning and cognition theory, and
(4) Research and evaluation.

The outline and an accompanying list of specific questions for discussion were sent to each participant prior to the conference. These materials were intended to stimulate and guide discussion within highly flexible boundaries.

The first two conferences (August 23 - 26 and October 25 - 28) were attended by mathematicians and mathematics educators, and focused primarily on curriculum development and teacher training. The third conference (November 15-November 19) was attended by researchers and focused primarily on overall systems analysis and program development, with emphasis on:

(1) the nature of the individual components (activity packages) of the curriculum matrix and,
(2) the role of research and evaluation within the overall system.

The first day of each conference was devoted to familiarizing the participants with the Nova Schools. After a brief description of the school program and a tour of the facility, an opportunity to evaluate the potential of NCMP was provided through the observation of a demonstration class. In the demonstration classes, Burt Kaufman led Nova students in a discussion of topics in modern algebra at a level of sophistication usually found in the more advanced undergraduate college-level courses. The students in the demonstration classes ranged in grade level (in the traditional sense) from grade nine to grade twelve.

Upon completing the visit to the Nova Schools, the participants returned to the conference headquarters at the Lago Mar Hotel, in a lovely beachfront setting, to devote the remaining three days to discussions of the project. In order to obtain maximum contributions from individual participants as well as consensus on major points, small group discussions were alternated with total group discussions. Specific topics were determined by consensus within the groups. All meetings were recorded on tape and the small group discussions were summarized for the total group by one of the participants.

In each conference time was set aside for individual written contributions by the participants. Some of these contributions appear in the appendices, and others have been used to help shape the main body of this report.

PROVISIONS FOR THE DEVELOPMENT OF THE NOVA COMPREHENSIVE MATHEMATICS PROGRAM

Hopefully, this report constitutes at least an approximation of a workable, if not ideal, integration of the recommendations contributed by the 52 participants of the three planning conferences. The program outlined below represents the direction NCMP now expects to follow as a result of the advice and recommendations of these conferences.

The reader should bear in mind that the developmental program described in this section constitutes a long range goal and that, in the authors' opinion, many of the incorporated recommendations should not be acted upon until the successful completion of an exploratory effort on a smaller scale.

The Curriculum Matrix

As was pointed out in the FOREWORD, the fundamental objective of NCMP is the provision for each child to learn the mathematics most appropriate for him at all times. This calls for the collection and production of a large quantity of diverse teaching materials which can be used in a wide variety of learning situations. The curriculum matrix is provided here as a conceptual framework for such a collection of teaching materials in the mathematics program.

The elements of the proposed Curriculum Matrix are the basic units of instructional activity, hereafter referred to as activity packages. The dimensions of the matrix consist of the several ways the activity packages would vary to meet the diverse needs of the students, teachers, and the system as a whole.

Two of the three dimensions of the Curriculum Matrix are normally considered in curriculum development efforts. These are the content dimension and the spiral or sophistication dimension. Since NCMP intends to emphasize individualization of mathematical instruction, it is apparent that at least one dimension concerned with the characteristics of the learner needs to be included in the Curriculum Matrix.

The central dimension along which the activity packages vary would be the nature of the content included. Some packages will deal with algebraic systems such as the Number Systems of Algebra text now being used in the Nova School, while other activity packages would deal with other areas of mathematics such as, for example, the measurement of volume. The content areas listed on this dimension most likely will be those listed in the Cambridge report with modifications based on the recommendations of future planning conferences.

Packages must vary in levels of sophistication of the mathematical ideas. One package might deal with group theory by manipulating wooden triangles while another version might present group theory from a more sophisticated axiomatic point of view. From the experience already gained at Nova it is clear that sufficient materials do not exist at this time for high school students which deal with mathematics at the college level of sophistication. Some materials have been produced and are in use at Nova, but much more effort needs to be directed to these ends. Also, it seems widely agreed that many advanced concepts can be successfully introduced in a more elementary form at the lower grade levels.

By means of this sophistication dimension NCMP hopes to achieve the spiral type of curriculum recommended by the Cambridge report. This axis will also enable the project personnel to see at a glance whether provisions have been made for all levels of sophistication judged appropriate for the various mathematical concepts on the K-12 content dimension.

The third dimension, characteristics of the learner, requires the packages to vary in social context, vocabulary, and other similar ways. A dull high school student may need a much different kind of material than does a bright elementary school child even when they are studying the same mathematical concept at the same sophistication level. Moreover, different needs and inhibitions can require markedly different situations for optimal learning by students of the same intellectual capacity. Thus, the characteristics of the learner dimension will require many alternative packages which treat the same content at the same level of sophistication. For example, academic games centered packages might be designed specifically for informal peer group interaction, while packages dealing with similar content at the same sophistication level might be designed for individual use with or without a tutor. A package will also vary in the degree of application of the mathematical concept, and the disciplines to which the mathematical concepts are applied. There may be a package dealing with graph theory in the abstract, and others dealing with applications of graph theory to molecular structure or production trends in industry.

Students also differ in the extent to which they require physical manipulation of materials for optimal learning. Some packages therefore, will include simple manipulative materials such as structural rods, or more complex materials such as minivac computers along with work sheets, prog- ..ed instruction manuals or open-ended challenges.

Ultimately the characteristics of the learner dimension should include all those attributes found to significantly differentiate students in their individual approaches to the learning of mathematics.

With regard to the total Curriculum Matrix, it should be noted that the existence of a three dimensional matrix with as many as 10,000 to 100,000 possible intersections does not imply that 10,000 to 100,000 activity packages are necessary to provide a highly individualized mathematics program. While each intersection in the matrix indicates a potential activity package, activity packages will be developed only as the need for them becomes apparent to project personnel.

Programming Students through the Curriculum Matrix

Programming or guiding students through the curriculum matrix is primarily a matter of periodically matching the characteristics of an activity package or a series of activity packages with the characteristics of a given student. The long range objective of this procedure is to provide for maximum student success in terms of some terminal set of overall achievement criteria. If such long range evaluation of pay-off is to be a continuous, integral part of the system, it is necessary to assign an estimated pay-off value to each package or sequence of packages considered in the matching process. Initially these estimates of worth or value may be quite arbitrary, especially at the early grade levels. As data are collected on a longitudinal basis, however, it should be possible to identify certain types of networks of activity packages which culminate at distinctly different levels of "mathematical literacy" - the overall outcome. The judged value of this outcome and the values of more advanced packages may then be reflected back on the more basic packages in order to optimize decision making in the early stages.

Discrete diagnostic and achievement assessment instruments for each activity package would be required. Eventually, matching or scheduling procedures would become more effective as histories of specific activity packages and students are developed and made available on a routine basis for scheduling. The probability of the success of a particular child with known characteristics and history under known circumstances with a particular activity package could be predicted with more precision as more data are acquired. As these predictions become more precise and reliable, it will become less and less excusable to continue to provide students with inappropriate instruction.

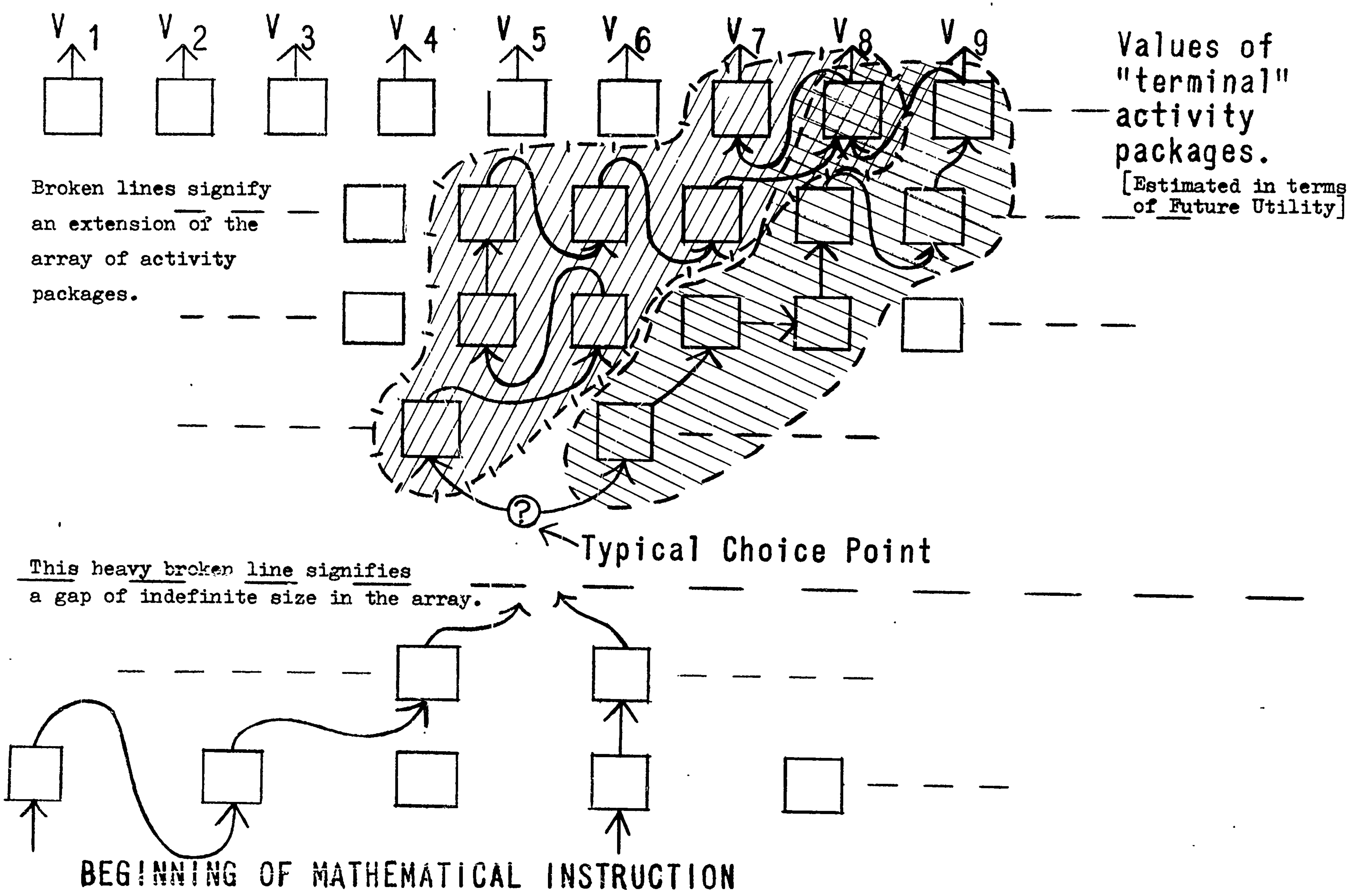

Fig. 1. Hypothetical flow of students through one region of the total set of activity packages.

Moreover, the accumulation of evidence that the "best available" instruction the system can provide is in all probability of little or no benefit to certain students should provide the impetus for continual development and refinement of the curriculum matrix. Thus, the development and improvement of activities packages must be viewed as an unending process based on the growth of knowledge external to the system and the accumulation and feedback of information within the system.

A graphic description of paths students might follow through the array of activity packages is shown on page 11 and further discussion of this diagram is given in Appendix C.

The Systems Approach to NCMP

The mathematics faculty at Nova has accumulated two and one half years of experience in developing and implementing a non-graded mathematics curriculum. A product of this experience has been a growing awareness of the complex requirements of a system which would provide an optimal program for all children.

It is a basic pedagogical concept that as the instructional environment is enriched, the differential rates of human learning increases the variance among the students. Thus, the task of providing optimal learning opportunities for all children becomes increasingly more difficult in an "efficient" school program. This fact is verified in the mathematics class scheduling at Nova. The number of different levels and different classes has continually increased since the beginning of the school operation in response to the needs of the students as perceived by the faculty. Each new deficiency detected in the curriculum must be met with a regrouping of the students, a re-scheduling of the classes, a re-assignment of the staff and, frequently, a search for new and more appropriate teaching materials.

Moreover, in contemplating the extension of the individualization via the Curriculum Matrix, it became apparent to the participants at the third conference that the creation of a totally new educational system would be the most appropriate approach to NCMP.

Taking a systems approach means essentially to describe all of the necessary functions and interactions of functions required to accomplish the goals of the project in such a way that the new system might evolve in an orderly manner. Consideration must be given to such things as selection and development of teaching materials, staff competencies, space allocation, the collection, storage and retrieval of information, research, evaluation and modification of the system.

A rather detailed description of such a system is provided by Ward and Easley (see Appendix C). A summary of the system, with minor modifications is presented below and a diagram of the functions which compose the system is provided on page 14.

The system contains five components; instructional resources, systems modifiers, systems analysis, data storage and processing, and action monitor and value estimator. The Instructional Resources block includes all available resources for achieving the educational goals. These resources can be thought of as four basic types; the indirect contact staff, the direct contact staff, the physical facilities, and the students. In the indirect contact staff group are included such personnel as authors and teacher-trainers. The direct contact staff includes teachers, counselors, librarians, and any others who have direct contact with students on a routine basis. The physical resources refer to the rooms, the data gathering instruments, the laboratory equipment (and all other plant facilities), and the activity packages or curriculum modules. The activity packages are the system's basic units of instructional activity and may ultimately number in the hundreds or thousands. These units continually function in the decisions to regroup students, teachers and activities. The last, but most important physical resource is the student body.

The second block is called the Systems Modifier. This is a function that will bring about any changes in the overall system as determined from the next block which is Systems Analysis. Systems Analysis contains the research function and questions of importance about the overall system will be analyzed and investigated within this function.

Data Storage and Processing which constitutes the fourth block, receives information from all other functions in the system for the purpose of routine decision making. This is the central function of the system for it accomplishes the matching of characteristics of the learner and the activity packages (see page 10) in the process of scheduling all resources. Given adequate resources, this function achieves individualization of instruction by optimizing the scheduling of resources, i.e., by selecting the most appropriate activity package, teacher, location, etc., for a given student at a given time. Optimal scheduling would be approached as the routine evaluation of all previous schedules (i.e., associations of resources) provides a more adequate data bank on which to base predictions of the value of new associations of resources. Much of the data storage and processing may be handled by clerks

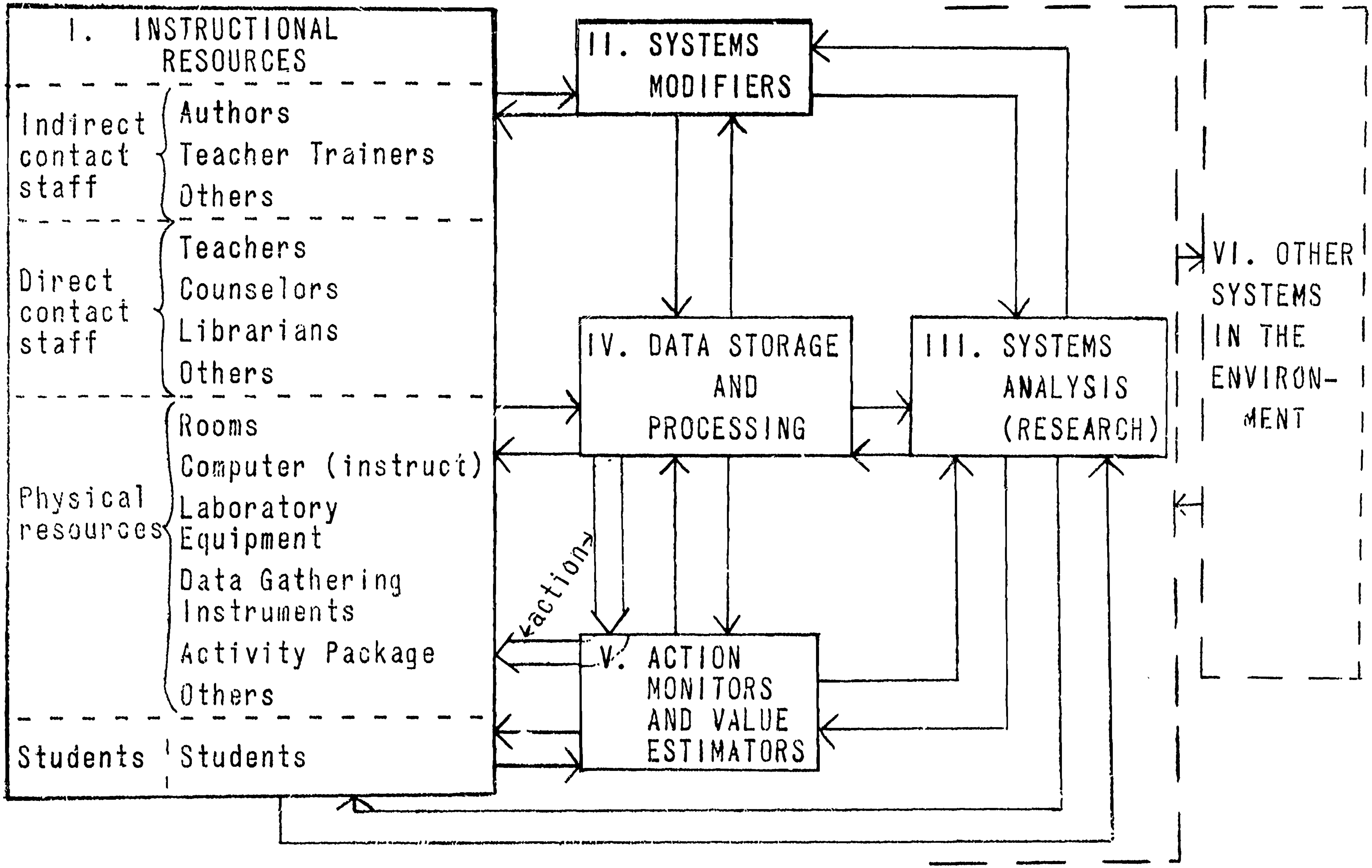

Fig. 2. Diagram of the major functions and interrelationships of functions in the Systems Approach to NCMP

initially, but this function would ultimately require computer facilities.

The fifth box is called the Action Monitor and Value Estimator. The action monitor aspect of this function is a continuing surveillance of the results of the data processing function to be sure that the actions that have been suggested as a result of the analysis are appropriate as viewed by the monitor. Another aspect of this function is value estimation. This represents the judgments and estimates of worth of whoever, or whatever group of people, are performing this function. It is important to realize that this may be one person monitoring a rather specific decision, or it could be quite a collection of people such as teachers, administrators, school board members and other members of our society who are properly involved in the determination of the overall adequace of the system. Again it should be emphasized that the components of the system are functions and a given individual may successively play almost any of these functional roles. Typically the project director would find himself serving alternately as teacher-trainer and author, systems modifier, a value estimator, etc. As the system develops, however, more specialization may be needed in terms of individual staff responsibilities.

The dotted box to the right in the diagram represents all other systems within the environment. These systems might include the social studies, science, athletics, band, and any other environmental activities whose presence affects the operation of the mathematics system on the left. The arrows between the other system and the mathematics system represent the intercommunication and information flow among all parts of the system.

The curriculum matrix systems approach to individualization as stated above is admittedly an over-simplification of a highly complex and undoubtedly problematical program. Indeed it may constitute an overstatement of the case for individualization, for there might well be a relatively small number of optimum sequences of activity packages to be matched with relatively few "types" of children with particular attributes. Such a conclusion, however, is not indicated by the work in progress at Nova, and could only be drawn after extensive experimentation carried out in a project such as proposed here.

Teacher Training

As noted in their report, the Cambridge Conference deliberately gave little or no consideration to the current problems of teacher training in the design of their far reaching school mathematics program. The Cambridge Report noted, however, that an intensive teacher training program will have to be developed simultaneously with the curriculum in order to insure continuing program reform of a significant nature.

The reasons in support of this contention seem, for the most part, obvious.

1. As early as the 7th grade, the content suggested in the report is more sophisticated than material now found in advanced senior high school courses. In the later grades much of the content involves concepts now found only in advanced undergraduate and beginning graduate school courses. The majority of present secondary school teachers have never formally studied many of these topics and will need to do so if they are to make major contributions in long range curriculum development projects of this nature.

2. The same type of problem as mentioned above exists and is even more serious with elementary school teachers. As stated in the report, "It is common knowledge that the average elementary teacher knows, at most, formal arithmetic narrowly construed and some teachers now entering the profession have a proficiency in arithmetic which is below 8th grade norm." [1]

3. As presently envisioned, NCMP will make far heavier demands on teachers than ever before. Drill, per se, would be largely omitted from NCMP materials. Emphasis would be given to mathematical ideas and these ideas would further serve as the natural instructional context for the necessary mechanical skills of arithmetic. Teachers must be trained to teach ideas. They must develop a method of teaching mathematics as the "pursuit of truth by a process of inquiry". [2]

Past experiences at Nova clearly indicate that many students are capable of learning far more sophisticated ideas than educators have given them credit for. The only restrictions encountered so far have been the staff's own limitations and the availability of materials, not the r [illegible]ive ability of the students.

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1. Goals for School Mathematics, The Report of the Cambridge Conference on School Mathematics. Houghton Mifflin Company, Boston. p. 26.

2. Ibid.

4. The task of training teachers to elicit inquiry among their students is far more difficult than training them to teach by drill. Among other things, teachers need to recognize, as quickly as possible, the validity of unexpected responses. Even if incorrect responses are given, the teacher needs to know whether or not a valid idea was implied which should be pursued in the discussion. Such skills are much harder to develop in a teacher than the ability to lecture. For this curriculum, teaching will have to become an art. The only way to eliminate the normal teacher behavior that accompanies student responses not accounted for in "the book", is to "alleviate the purely intellectual incomprehension which forces them into it".[3]

These concerns for improved teacher training become immediate concerns for NCMP in view of the fact that teachers are in the best position to develop the desired pedagogical techniques for the project. Research mathematicians must carry the major responsibility for writing instructional materials and guiding the entire development, but it would be folly if classroom teachers weren't considered as equal partners in this endeavor. The teachers' contributions, however, will be of little value if they don't know more than just small fragments of the content being developed.

With respect to the teacher training aspect of the project, the participants at the first and second conferences were in agreement that a year's head start on the main body of the curriculum development project was needed. They also felt that through this project a precedent should be set for an intensive in-service training effort on a released-time basis. It was agreed that school teachers seriously involved in professional development need the same type of working day now enjoyed by college professors. In fact, in terms of their intellectual backgrounds, many public school teachers today may need school time for study far more than their college counterparts.

It was suggested by the participants that a separate proposal for an intensive teacher training program for the fall of 1966 be developed immediately. This proposal would request funds to hire an additional 15 full time teachers to the Nova Elementary School staff for one school year. These 15 teachers, together with the 30 regular staff members would be released approximately 2 hours per day to study mathematics for the entire year. Approximately 4 hours per week would be devoted to formal classroom lectures with the remaining time to study sessions and problem seminars. The teachers would be grouped into 3

3. _Goals for School Mathematics_, _loc. cit._

classes according to ability and the course work organized in an ungraded manner similar to the present organization for students. It was felt that the emphasis at the beginning of this training should be aimed at improving these teachers for their present tasks and getting them into the spirit of the project. As the year progressed, it was hoped that at least one of the classes could achieve a level approximating the level 1 recommendation of CUPM[4] for elementary teachers. From this group would come future project staff.

There was concern, however, about the feasibility of developing such highly competent mathematics teachers as long as they had to teach four or five other disciplines. Thus, the conferees expressed a desire to see extensive departmentalization in the elementary school so that this mathematics training could be reserved for those teaching just mathematics, leaving time for the others to engage in similar training activities in the other disciplines. However, in the absence of a highly departmentalized program they felt that the training program as outlined above was essential for all teachers.

Research and Evaluation

The term research is being used here in a general sense to mean the systematic collection and analysis of reliable evidence on which to base decisions concerning the development of the program, and to draw conclusions about certain theoretical underpinnings. NCMP does not aspire to be an experiment in the strictest sense of the word, but, within the systems approach, NCMP can provide a laboratory for field research ranging from exploration and observation to controlled experimentation.

The first research objective of NCMP, in the opinion of the participants at the third conference, should be a pilot or feasibility study. This effort would focus on the further development, at a concrete operational level, of the conceptual models for the entire systems approach, the curriculum matrix, and the activity packages. While a detailed discussion of the role of research and evaluation in a long term project would be premature prior to such a pilot study, there are several general recommendations worthy of comment at this point.

With regard to overall program evaluation of a formal nature, there seemed to be agreement among researchers as well as mathematicians that the study of the accomplishment of essential objectives of NCMP against well-defined

4. Committee for the Undergraduate Program in Mathematics. A committee of the American Mathematical Association which recommends mathematical curricula appropriate to the pre-professional training of undergraduate students.

standards of excellence would be preferable to the typical control group design. There appeared to be a fair degree of consensus among mathematicians and researchers, however, that emphasis should be given to internal research and evaluation which would facilitate the development and continuous operation of NCMP. Such a cybernetics model for research was therefore built into the system discussed in Appendix C.

Evaluation of student progress in a K-12 program would perhaps most appropriately include both the routine, diagnostic assessment built into the acticity packages and the long term developmental assessment of more global objectives such as "mathematical literacy", Piaget's developmental stages, or Bloom's taxonomy of educational objectives in the cognitive and affective domains. In this connection, it has been suggested (see Appendix L) that Guttman's scalogram technique and the concept of "content standardization", where the test score conveys highly specific information about content (or processes) attained, would be appropriate models within the systems approach to NCMP. The content standardization model appears most appropriate to the task of recording, in terms of specific measures, a highly detailed academic history of the individual student. The Guttman technique would provide, at any given time, a reading of the developmental stage (or level of mathematical literacy) attained by the student. Unfortunately, the Guttman scalogram model would require a high degree of homogeneity of student response patterns and may therefore prove to be somewhat less feasible in a program devoted to individualization of instruction.

The systems approach provides additional research possibilities to the extent that important attributes of the major components of the system are described in concise terms and data related to these attributes are collected and stored in a systematic manner. Researchable hypotheses would consist of statements made in terms of these important attributes. Thus, most researchable hypotheses could be tested by treatments of the data which automatically exist within the system. Hypotheses might be tested concerning student behavior and learning, material effectiveness, teacher characteristics and behavior, effects of physical spaces, to name a few.

Important longitudinal research could be performed by observing individual students over their entire K-12 school career. Virtually no substantial research of this kind is available now aside from the National Longitudinal Study of Mathematical Ability project of the School Mathematics Study Group which will cover a five year span. Other beginnings in this field can be found in the earlier work of the Gessell Institute and the University of Michigan Laboratory School. In none of these cases has the observation been integrated into so ambitious a program as proposed here.

Another important area of research which could become an integral part of this project would be the area of concept development of a Piagetian type. This increasingly influential work focuses on and may lead to the basic knowledge essential to a useful theory of materials development and curriculum administration. NCMP research on materials development should also be approached from the concept of "learning sets" as illustrated in the recent work of Gagné and the University of Maryland Mathematics Program. The production and testing of materials for "inquiry training" such as the work by Suchman, American Association for the Advancement of Science Elementary Science Program, Minnemath, and the Webster College Science and Mathematics Center could also be a natural part of the research program.

A brief review of related research is presented in Appendix K, and selected comments on research and evaluation made at the first conference may be found in Appendix B.

THE NEXT STEPS IN THE DEVELOPMENT OF THE NOVA COMPREHENSIVE MATHEMATICS PROGRAM:

The question of getting started on the NCMP with something manageable in the near future was thoroughly discussed at each of the three conferences. Out of the first two conferences came the outline of a proposal for teacher training and curriculum planning (see page 16). The anticipated impact of such an intensive teacher retraining program on a relatively new faculty was such that it seemed advisable to temporarily postpone this effort.

As the participants at the third conference came to grips with the central question of program implementation it became obvious that the major aspects of the proposed program could not be further integrated as a system without the benefit of preliminary try-outs in the field. Therefore, of the many alternatives for initiating the project discussed at the conferences, the most feasible suggestion seemed to be that of beginning with a pilot study which would serve to further define and develop the major concepts of NCMP.

The aspects of the overall program which should receive considerable attention in such a pilot study include (1) the analysis of a suitable educational system for implementing NCMP, (2) the long range planning of the curriculum content and teacher training program, and (3) the development of an operational, small scale curriculum matrix. These three activities are presented below as initially distinct but inter-dependent efforts which would gradually merge into a unified comprehensive K-12 mathematics program.

I. Analysis of the Educational System Requirements Necessary for the Implementation of NCMP

NCMP would most probably be implemented initially at some small segment of the school such as either the K-2 or 5-6 level of the elementary school. Conversion to the NCMP curriculum in the classroom would occur when a set of curriculum components (i.e., a section of the K-12 Curriculum Matrix) designed to provide at least two years of material for all students reaches a stage of completeness such that it could be substituted for the existing mathematics program. At this stage of development the degree of success with NCMP will depend almost entirely on (1) the extent to which the mathematics curriculum matrix can function effectively as a system and (2) the extent to which the curriculum matrix system can be integrated into the existing educational system of the entire school. The implication here, of course, is that a precondition to the successful integration of the mathematics system and the total school system is a thorough analysis of both systems and a clear understanding of the requirements and limitations imposed on both systems by certain elements of each.

A second role for systems analysis which should be initiated in the near future is that of advanced systems planning for the entire K-12 program. This might best be accomplished via computer simulation of a total school facility designed according to a set of educational specifications for a general curriculum matrix of which the mathematics program would be but one segment.

II. Long Range Planning of Curriculum Content and Teacher Training.

These two aspects of the overall project appeared to be very closely associated in the thinking of both the mathematicians and mathematics educators. Indeed, the outline of a proposal which came out of the first and second conferences combined curriculum content planning and an extensive teacher training program.

Ultimately the teacher training program should be quite intensive in the sense of requiring mastery of a major segment of the Cambridge Conference curriculum, and rather extensive in the sense of providing in-service training for at least one entire K-12 faculty and a teacher-training institute for representatives of other faculties desirous of developing a program along similar lines. The total impact of such a teacher training program in the experimental school must be given careful consideration and far more planning must necessarily precede a teacher training program of this magnitude.

In the meantime, valuable experience in teacher training can be gained through initial efforts in the pilot study. This approach would involve approximately eight teachers in the project on a release-time basis. The training program would consist of:

1. formal content training via regularly scheduled courses for credit.
2. seminar meetings and problem sessions.
3. demonstration classes taught by project personnel.
4. individual conferences with project personnel concerning their daily mathematics lessons and how these can be developed for optimum learning experiences for students. Project people will serve in supervisory roles for the ongoing elementary program.
5. experience with curriculum development via participation in project planning sessions and development and testing of experimental activity packages.

III. Activity Packages

During the next two years, major effort should be devoted to the initial development of a set of activity packages which could function as a small scale curriculum matrix. The first task would be to identify that segment of the present Nova mathematics curriculum which could most readily be developed into a relatively self-contained small scale curriculum matrix. This segment of the curriculum should then be analyzed to determine:

1. discrete segments of materials such as a text, a chapter of a text, an original Nova mathematics unit, etc., which could be considered as part of a potential activity package.
2. accessable objectives or goals of each potential activity package in terms of the major curriculum matrix dimensions.
3. the student population for which each potential activity package is appropriate, i.e., those student attributes, including academic attainments which constitute student "readiness" for the materials at hand.

At this point, the chosen segment of curriculum should be hypothetically structured as a curriculum matrix and carefully scrutinized for major gaps along any of the major dimensions for a given student population. When such gaps are identified, a survey of materials available elsewhere would be undertaken or, in many instances, new materials would be written to fill the gap. In either case, the project would rely heavily on the mathematicians involved in the long range curriculum planning at this point.

When there is raw material sufficient to construct a curriculum matrix reasonably complete in its major dimensions, the next step would be the conversion of such material into activity packages. This task would include:

1. describing the appropriate method of instruction necessary to attain the stated objectives of each activity package.
2. developing instruments to diagnose student readiness for beginning specific activity packages.
3. developing instruments for assessing achievement of goals of each activity package. In high sequential material the readiness assessment instrument for one package may consist of short forms of the achievement assessment instruments for previous or lead-in packages.

The third task would be to begin trials of activity packages on a small scale. The system should be developed as the need for new activity packages for maximizing individual achievement becomes apparent to the project staff.

Until computer facilities are available, a great deal of information processing would be done by research assistants in order to continuously determine the extent to which the curriculum matrix is meeting the needs of the individual students. New activity packages would be required as:

1. the teachers find existing materials inappropriate for a group of students.
2. the teachers run out of appropriate materials in a given student's sequence of activity packages.
3. specific deficiencies are recognized in students' learning.
4. it becomes evident that mathematical ideas of significance have been ignored in existing materials.

The curriculum matrix would grow as more activity packages are introduced into the system. It is hoped that, with constant feedback and assessment occurring, most of the changing needs of the children, the teachers, and other aspects of the system can be met by the addition, deletion or alteration of the activity packages. Nonetheless, one of the most important functions of the pilot study will be to indicate specifically where the boundaries of the existing Nova school program place severe limitations on the extent to which the objectives of NCMP can be realized.

APPENDICES

The intention of the Nova Mathematics Conferences was to bring together people with diverse backgrounds so that many important perspectives required to effectively implement the goals of the program could be brought into focus. Obviously, all of the different ideas could not be captured in the main body of the report. Indeed any attempt to do so would only distort the ideas as they were presented.

Many of the ideas were submitted to the authors in writing by the participants. Excerpts of these written reports are presented along with additional explanatory materials in these appendices to provide for the reader a more detailed presentation of some of the views set forth at the conferences.

APPENDIX A

DESCRIPTION OF PRESENT COURSES IN THE NOVA MATHEMATICS CURRICULUM

Basic Contemporary Mathematics I

Text: Eicholz, O'Daffer, Brumfiel and Shanks: Basic Modern Mathematics, First Course.

Tracks 2 and 3.

This is a basic mathematics course, with a review of the fundemental principles of arithmetic, designed for students who are not ready for the ECM series. The full course will take about 3 trimesters; however, when the student shows sufficient maturity and capacity, he may be transferred to ECM I.

The Units are as follows:

Unit I:	Place Value
Unit II:	Addition and Subtraction
Unit III:	Adding and Subtracting
Unit IV:	Multiplication
Unit V:	Division
Unit VI:	Measurement
Unit VII:	Basic Principles
Unit VIII:	Multiplying
Unit IX:	Dividing
Unit X:	Number Theory
Unit XI:	Fractions
Unit XII:	Rational Numbers

Basic Contemporary Mathematics II

Text: Eicholz, O'Daffer, Brumfiel and Shanks: Basic Modern Mathematics, Second Course.

Tracks 2 and 3.

This provides a continuation of the study of basic principles begun in BCM I. The full course will take about 3 trimesters. It is anticipated that students completing this course will be prepared to enter ECM I. However, if there develops a need, a third course in this series will be organized.

The Units are as follows:

Unit I:	Place Value and Number Bases
Unit II:	Addition and Subtraction
Unit III:	Multiplication and Division
Unit IV:	Measurement
Unit V:	Special Products and Quotients
Unit VI:	Estimation
Unit VII:	Multiplying
Unit VIII:	Dividing
Unit IX:	Number Theory
Unit X:	Fractions
Unit XI:	Rational Numbers
Unit XII:	Addition and Subtraction of Rational Numbers
Unit XIII:	Multiplication and Division of Rational Numbers
Unit XIV:	Decimals and Percents

Elementary Contemporary Mathematics I

Text: Keedy, Jameson, and Johnson: Exploring Modern Mathematics, Book I

Tracks 1, 2, and 3.

This is a pre-algebra, pre-geometry course which introduces some elementary algebraic concepts together with intuitive geometry. It is expected that the accelerated student will finish the course in two trimesters or less, while the less capable will take considerably longer. The units included are:

Unit I:	Numeration Systems
Unit II:	Properties of Whole Numbers
Unit III:	Properties of Points, Lines and Planes
Unit IV:	Factoring and Prime Numbers
Unit V:	Mathematical Systems
Unit VI:	The Number System of Arithmetic
Unit VII:	Plane Geometric Figures
Unit VIII:	Percents, Decimals, Measures and Applications

Elementary Contemporary Mathematics II

Text: Keedy, Jameson and Johnson: Exploring Modern Mathematics, Book II

Tracks 1, 2, and 3.

This course is a continuation of the preceding course and contains more formal algebra and two more units of intuitive geometry. The accelerated student will probably complete Elementary Contemporary Mathematics I and II in four trimesters or less, while the track two or three student could conceivably take as long as 6 to 9 trimesters.

The content of this course includes the following units:

Unit I:	The System of Integers
Unit II:	Congruence, Constructions and Circles
Unit III:	Exponents and Scientific Notation
Unit IV:	The System of Rational Numbers
Unit V:	Areas, Regular Polygons, and Circle Graphs
Unit VI:	Applications of Integers and Rational Numbers
Unit VII:	Solving Equations
Unit VIII:	Polynomials
Unit IX	Applied Problems and Conjunction of Equations

Elementary Contemporary Mathematics III

Text: Keedy, Jameson and Johnson, Exploring Modern Mathematics, Book III.
(Elementary Algebra)

Tracks 2 and 3.

This course completes the elementary algebra sequence and also has two more units of intuitive geometry. The Units contained in this course are:

Unit I:	Similar Figures
Unit II:	Polynomials in Several Variables
Unit III:	Number Sentences and Proofs
Unit IV:	The System of Real Numbers
Unit V:	Fractional Phrases and Equations
Unit VI:	Geometry in Three Dimensions
Unit VII:	Radical Notation and Number Sentences
Unit VIII:	Graphs, Relations and Fractions
Unit IX:	Probability

Entebbe Geometry I and II

Texts: Brumfiel, et al.: Entebbe Geometry

Moise and Downs: Geometry

Track 3

This course is a less rigorous, Track 3, version of the SMSG Geometry Course. Much of Entebbe Geometry I uses special material written by Dr. Brumfiel and others in the African Mathematics Project, and provides an introduction to the spirit of geometry and logical reasoning. This course is aimed at the average student. The rest of the course (after Unit VI) will follow the chapter sequence of the Moise and Downs text, but will dwell only on the major concepts with the more precise (but, of course, not less important) mathematical points receiving less attention.

Entebbe Geometry I

Unit I:	An Introduction to Deductive Reasoning and Postulates in Geometry
Unit II:	The Betweenness Postulates
Unit III:	Congruence, Inequalities, and Measurements for Segments
Unit IV:	Congruence, Inequalities, and Measurement for Angles
Unit V:	Congruence of Triangles
Unit VI:	Logic
Unit VII:	Perpendicular Lines and Planes in Space
Unit VIII:	Parallel Lines and Planes, Parallelograms

Entebbe Geometry II

Unit I:	Areas of Polygonal Regions
Unit II:	Similarity
Unit III:	Plane Coordinate Geometry
Unit IV:	Circles and Spheres
Unit V:	Characterizations and Constructions
Unit VI:	Areas of Circles and Sectors
Unit VII:	Volumes of Solids

Intermediate Contemporary Mathematics I and II

Text: Dolciani, Berman and Wooton: Modern Algebra and Trigonometry, Structure and Methods, Book 2.

Track 3

This course and its sequel, Intermediate Contemporary Mathematics II, will follow Entebbe Geometry II and is an extension of the Elementary Contemporary Mathematics sequence also including several units in trigonometry. It will, in all probability, be the last formal mathematics courses taken by the track 3 student.

Intermediate Contemporary Mathematics I

Unit I:	Sets of Numbers; Axioms
Unit II:	Open sentences in One Variable
Unit III:	Systems of Linear Open Sentences
Unit IV:	Polynomials and Factoring
Unit V:	Rational Numbers and Expressions
Unit VI:	Relations and Functions
Unit VII:	Irrational Numbers and Quadratic Equations
Unit VIII:	Quadratic Relations and Systems

Intermediate Contemporary Mathematics II

Unit I:	Exponential Functions and Logarithms
Unit II:	Trigonometric Functions and Complex Numbers
Unit III:	Trigonometric Identities and Formulas
Unit IV:	The Circular Functions and their Inverses
Unit V:	Progressions and Binomial Expansions
Unit VI:	Polynomial Functions
Unit VII:	Matrices and Determinants
Unit VIII:	Permutations, Combinations and Probability

SMSG Geometry I and II

Text: Moise and Downs: Geometry
Karmos: Introduction to Logic

Track 2

This pair of courses is for the above average student who has completed Elementary Contemporary Mathematics III. The units included are:

SMSG Geometry I

Unit I:	Introduction to Logic
Unit II:	A Review of Sets and the Real Number System, Lines, Planes and Separation
Unit III:	Angles and Triangles
Unit IV:	Congruences
Unit V:	Proofs, Perpendiculars in a Plane, and Geometric Inequalities
Unit VI:	Perpendicular Lines and Planes in Space
Unit VII:	Parallel Lines in a Plane, Parallelograms, Parallel Lines and Planes in Space

SMSG Geometry II

Unit I:	Areas of Polygonal Regions
Unit II:	Similarity
Unit III:	Plane Coordinate Geometry
Unit IV:	Circles and Spheres
Unit V:	Characterizations and Constructions
Unit VI:	Areas of Circles and Sectors
Unit VII:	Volumes of Solids

Introduction to Modern Algebra

Text: Hinshaw: Introduction to Modern Algebra

Track 2

This course is for Track 2 students who have completed SMSG Geometry II. It will include a study of the algebraic structure of the real number system and the complex number system, as well as the elementary structure of abstract groups, rings and fields, elementary number theory, and a brief introduction to field extensions. Much of the material in this course duplicates, in a sense, material studied in the ECM series, but it is covered here from a more sophisticated college-level point of view. The units of this course are:

Unit I:	Real Numbers and Fields
Unit II:	Linear Equations and Inequalities
Unit III:	Elementary Number Theory
Unit IV:	Roots, Radicals and Exponential Notation

Unit V: Rings and Polynomials
Unit VI: Quadratic Polynomials
Unit VII: Complex Numbers
Unit VIII: Algebraic Structures

Pre-Calculus Mathematics I and II

Text: Shanks, Brumfiel, Fleenor, and Eicholz: Pre-Calculus Mathematics

Track 2

These courses will probably be the last courses in mathematics to be taken by the track 2 student at Nova. **They are the equivalent of a college** freshman mathematics course in most universities and it is expected that the student, after completing these courses, will score well enough on placement tests to begin with calculus in college.

Pre-Calculus Mathematics I

Unit I: The Plane
Unit II: Vectors in the Plane
Unit III: Space
Unit IV: Vectors in Space
Unit V: Circles, Cylinders, and Spheres
Unit VI: Elementary Functions and their Graphs
Unit VII: Finite Mathematical Induction, Sequences, and the Binomial Theorem
Unit VIII: Systems of Linear Equations and Determinants

Pre-Calculus Mathematics II

Unit I: The Circular Functions
Unit II: Applications of the Circular Functions
Unit III: Analytic Trigonometry
Unit IV: Inverse Trigonometric Functions and Trigonometric Equations
Unit V: Angles, Lines and Planes
Unit VI: Conics
Unit VII: Other Coordinate Systems
Unit VIII: Parametric Representation of Curves and Surfaces
Unit IX: The Problem of Tangents and the Problem of Areas

Foundations of Modern Algebra and Geometry

Text: Karmos: Foundations of Modern Algebra and Geometry

Track 1

This course is for the track 1 students and is much more sophisticated than Elementary Contemporary Mathematics III. Most of the topics are developed quite formally. However, similarity of triangles, area and volume, continuous functions, trigonometry, and probability are done informally. The accelerated student should complete ECM I, ECM II, and Foundations of Modern Algebra and Geometry in approximately five trimesters.

Unit I: Logic and Sets
Unit II: The Real Number System
Unit III: Similarity of Triangles and Right Triangle Trigonometry
Unit IV: Introduction to Exponents
Unit V: Factoring, Quadratics, and Solutions to Equations
Unit VI: Three Dimensional Geometry
Unit VII: Open Sentences in Two Variables
Unit VIII: Relations and Functions
Unit IX: Intuitive Probability

The Number Systems of Algebra

Text: Kaufman: Number Systems of Algebra

Track 1

This course will be given to track 1 students and will provide an excellent introduction for these students to the spirit of more advanced mathematics courses. As the title indicates, this course will develop the number systems of algebra with full mathematical rigor. The units of this course are:

Unit I:	The Rational Number System
Unit II:	The Algebraic Extension, $Q(\sqrt{2})$
Unit III:	The Real Number System
Unit IV:	Roots, Radicals and Exponential Notation
Unit V:	Quadratic Functions
Unit VI:	Finite Fields and an Introduction to Elementary Number Theory
Unit VII:	The Complex Number System
Unit VIII:	Introduction to Algebraic Structures
Unit IX:	Introduction to Matrix Algebra

Introduction to Mathematical Analysis I

Text: Kaufman, et al: Introduction to Mathematical Analysis

Track 1

This course and Introduction to Mathematical Analysis II are similar in content to the track 2 courses, Pre-Calculus Mathematics I and II, but are done from a much more rigorous standpoint. Since the track 1 student has studied relations and functions in Foundations of Modern Algebra and Geometry, this course is, for the most part, a study of special types of functions.

Unit I:	Finite Mathematical Induction, Peano Postulates, Sequences, and the Binomial Theorem
Unit II:	Poly Functions and Polynomial Functions
Unit III:	The Logarithmic Functions
Unit IV:	Counting Procedures, the Multinomial Theorem, and an Introduction to the Theory of Probability

Elementary Geometry from an Advanced Standpoint I, II, and III

Texts: Moise and Downs: Geometry
Moise: Elementary Geometry from an Advanced Standpoint

Track 1

These courses will be similar in basic content to SMSG Geometry I and II, but will discuss the concepts from a far more sophisticated level. All three courses will be treated as college level courses. The units in these courses are as follows.

Elementary Geometry from an Advanced Standpoint I

Unit I:	The Algebra of the Real Numbers Incidence Geometry in Planes and Space Distance and Congruence of Segments
Unit II:	Separation in Planes and Space Angular Measure
Unit IV:	Geometric Inequalities
Unit V:	The Euclidean Program: Congruence without Distance
Unit VI:	Three Geometries Absolute Plane Geometry

Elementary Geometry from an Advanced Standpoint II

Unit I: The Parallel Postulate and Parallel Projection
Unit II: The Similarity Relation on Triangles
Unit III: The Construction of an Area Function
Unit IV: Perpendicular Lines and Planes in Space
Unit V: Circles and Spheres
Unit VI: Rigid Motions and Cartesian Coordinate Systems

Elementary Geometry from an Advanced Standpoint III

Unit I: Constructions with Ruler and Compass
Unit II: From Eudoxus to Dedekind
Unit III: Arc Length and the Area of a Circular Sector
Unit IV: Jordan Measure in the Plane
Unit V: Solid Measuration: The Elementary Theory
Unit VI: Hyperbolic Geometry
Unit VII: The Consistency of the Hyperbolic Postulates
Unit VIII: The Consistency of the Euclidean Postulates and The Postulational Method.

Introduction to Mathematical Analysis II

Texts: Haaser, LaSalle, and Sullivan: Introduction to Analysis
Kelley: Algebra, A Modern Introduction

Track 1

This course coordinates the two textbooks named above in order to present analytic geometry from a vector point of view. Trigonometry and an introduction to linear algebra are also discussed.

Unit I: Introduction to Vector Geometry I
Unit II: Introduction to Vector Geometry II
Unit III: Rigid Transformations
Unit IV: Introduction to Linear Algebra
Unit V: Graphs of Equations, Conic Sections, and Reduction of a Quadratic Form to Diagonal Form
Unit VI: Plane Analytic Trigonometry

The Calculus and Abstract Algebra sequences following can be taken simultaneously or the track 1 student can elect either one or the other.

Calculus I

Text: Haaser, LaSalle, and Sullivan: Introduction to Analysis

Track 1

Relying upon the concepts found in the Introduction to Analysis courses, the foundations of calculus are rigorously developed in this course.

Unit I: Limits and Continuous Functions
Unit II: The Derivative of a Function
Unit III: A Closer Look at the Least Upper Bound Axiom
Unit IV: Applications of the Derivative of a Function

Calculus II

Text: Haaser, LaSalle, and Sullivan: Introduction to Analysis

Track 1

Unit I: The Definite Integral
Unit II: Applications of the Definite Integral
Unit III: Elementary Functions
Unit IV: Methods of Integration

Abstract Algebra I, II, and III

Text: Mostow, Meyers and Sampson: Fundamental Structures of Algebra.

Track 1

This sequence can be taken concurrently with the Calculus I and II or, if not simultaneously, will precede the Calculus sequence. Abstract Algebra will be taught as a college level course, and it is quite possible that in the near future students enrolled in this course will receive college credit from a Florida university.

Abstract Algebra I

Unit I: Binary Operations, Groups and Rings
Unit II: Integral Domains, The Integers
Unit III: Fields, The Rational Numbers
Unit IV: The Real Number System
Unit V: The Field of Complex Numbers
Unit VI: **Polynomial Rings**
Unit VII: Rational Functions

Abstract Algebra II

Unit I: Vector Spaces
Unit II: Affine and Euclidean Spaces
Unit III: Linear Transformations and Matrices
Unit IV: Groups and Permutations
Unit V: Determinants

Abstract Algebra III

Unit I: Rings of Operations and Differential Equations
Unit II: The Jordan Normal Form
Unit III: Quadratic and Hermitian Forms
Unit IV: Quotient Structures
Unit V: Tensors

APPENDIX B

Comments on Research and Evaluation

Following Dr. Robert Davis' comments emphasizing the importance of the early decisions about what not to do in the development of the Nova Schools, Dr. Edwin Moise opened an informal discussion of what should not be done in the research and evaluation of NCMP. Although no attempt has been made to identify the speaker or to ascertain the degree of consensus on specific points, the following excerpts definitely reveal some of the major concerns of the participants in regard to research and evaluation.

> "Very often it's hard to put down just what your objectives really are because they are highly intuitive and highly subjective. Subjective doesn't mean bad, subjective means good here."

> "... if you claim that all education is definable in behavioral terms, then I will say, yes, you are probably right. But if you say all education which I, today am unable to describe in behavioral terms is unreal and mystical - if you say people should stop pursuing any educational objectives which they haven't described in behavioral terms yet then it seems to me that education is in very serious danger because most of its most important aspects have not been so described."

> "I think there's another good reason for not wanting to develop our objectives in too high a detail, it sort of leaves us free to change our minds."

> "... the modern psychologists have a term I must confess I don't fully understand, but I believe is relevant here - the term is criterian behavior ... the point I am trying to make here is that there are many aspects of our program which are going to be a little hard for us to define in this criterian behavior, and I think it will only be after we have developed the curriculum itself that you can even begin to look into this and try to determine what the so-called criterian behavior is, and then you decide whether you are going to construct your tests."

> "... I want to reiterate something I said a few minutes ago - to attempt to give behavioral descriptions in the technical sense, in helping one to succeed, this is one thing I agree with. To claim that until you have succeeded in this, everything is all a-wash and a-drift, this is entirely another matter, and I think it is the difference between these two viewpoints that makes it so important for curriculum development to depend in its exploratory phases on the direction of the people who do it."

> "... I can't imagine a more invalid experiment than allowing an author to develop his own tests - because he could rescue the text three times in every class lesson without even realizing that he is doing it. I think testing is surely needed when you are trying to tool up for mass production, when you are trying to find out what to do to curricula, what to do to text material, what to do to apparatus - this, that or the other. I am not arguing that nobody

should have measuring, but what I was arguing against is introducing a bias in favor of measurable things while the curriculum is being designed and I believe that you introduce this bias if you use testing during the creative process."

"Now there is one more aspect of this that I think is even more important and I think it is appropriate to make this comment while some of the people are here from the U. S. Office and National Science Foundation, and that is that very often we are forced into the position of selling a curriculum development program on the basis of research whereas what you are really interested in is the development aspect of this program. I think this is extremely unfortunate ... I would hope that we would be able to - in a case like this, for example - make a realistic proposal here of your development program, which is really what you are talking about, and sell it on the basis of that."

On the final day of the conference the floor was again opened to the discussion of research and evaluation, and at this time the conference participants were asked to indicate the kinds of problems that might be dealt with through research and evaluation.

"The most desperate need for assessment that I see arises as we attempt to develop more individualized programs. I would say that in any teaching situation in any school in the country there is an awful lot of waste that comes in teaching things to people who are not ready yet to learn them. Now the only way we can eliminate this waste is to devise ways whereby the classroom teacher can make rather efficient assessments of the facts that a child knows in regard to some specific topic, know what they don't know, and then have the materials available, readily available to provide for the children to learn what they want to learn at their level. But this requires a tremendous amount of instantaneous assessment more like a diagnosis and treatment."

"... much of the research in this area indicates that sequencing of materials might be much more important than the way in which you present them, and I think we need to find a way to determine the best sequences for various types of students."

"I think that today alot of the troubles we find ourselves in stem from our overconcern with sequence."

"I would say that perhaps each learner has his own sequence."

"We didn't feel that a kid could learn ruler and compass constructions until he had had simpler constructions, I mean we wanted to build up to this. He might very well have some work in arithmetic interweaving the geometry and you might very well start a sequence depending on the interest that the kid seems to take for the geometric things. You might start there, whereas if he seems to take more to sets than to arithmetic, you could start there. I think there could be a good deal of interweaving of separate spirals."

"I think the kind of expression I made earlier could be used as a justification for not soul-searching - as a justification for saying "I'm here, I'm the master of my class and nothing else matters." and I think that's a great danger. I think we should engage in soul searching about our objectives. I think that psychologists who do this kind of work could help us and other mathematicians who have different intuitive views of what they're trying to achieve - by discussion with them we could go through soul searching and help to make our own ideas more precise."

"I object to the proposition that objectives must be stated before you can do anything else. You tend to try and find nice fancy words for the objectives to try and satisfy the people who demand that you write them. I object to that terribly. I think the objectives are often there and you just cannot state them in English, you just feel them. In the long run you might be able to verbalize, but you shouldn't feel that that should prohibit you from starting."

"On the other side of this is the fact that whenever the objectives for the teaching of mathematics have been written down they included at least two things - one is the teaching of critical thinking and the other is the appreciation of mathematics and its role in culture. And yet, so far as I know, there has been very little attempt to measure these, consequently there has been very little attempt to see if this kind of thing has been accomplished. Assessment makes us all a little more honest."

"In one sense, I'm not suggesting this, but I suppose it would be possible that if we devised tests that test creative behavior, we might see more creative behavior in curriculum."

"This is a nice point, make people more conscious of what creative behavior is. They might find that they have in mind doing this kind of thing but by giving them examples and illustrations and descriptions they appreciate a little more sharply what they were vaguely groping for."

"I would say that what I'm concerned about in teaching mathematics primarily is attitude; is points of view which I would like the students to develop and appreciate. This probably has to be tested in terms of time series."

"... you won't get two or three math teachers around the table or anywhere, before one of them says, "Now I once had a boy who -" or "I had a girl who -" and so on. Illustrations of something that happened that we hoped would happen but rarely have evidence of happening. Or, in some instances, it illustrates something which we never expected to happen, but it's a good thing that it happened. Now, for these broader and deeper objectives maybe something could and should be done, and maybe the only way of doing this at the present time is anecdotal record."

"Let me ask another question where research might actually help out. The other day Burt mentioned that some of his problems involve discipline cases. Why do discipline cases arise? This is a place where I think you could serve a very useful function. What makes a child dislike mathematics, what makes a child love mathematics, let's find out these things. Why is it that some children are affected by certain things and not by other things?

These are things that I feel belong to the field of learning, intrinsic learning. What is there about the learning of mathematics that sometimes creates blocks towards learning the mathematics?"

"Would anybody accept the fact that there is anything, any characteristic in mathematics itself, that creates negative attitudes towards mathematics, or are negative attitudes created by people?"

"It may be this female (negative) reaction to mathematics is somewhat tied in to some of the evidence we have that females do not achieve for intellectual satisfaction as much as for social approval."

"There's the tremendously important factor that no teacher is as poorly prepared and is as anxious and full of feelings of insufficiency as the mathematics teacher."

"Maybe beyond a sex difference, there's a characteristic of mathematics, the relatedness concept, so that if you somehow get out of the main stream it may be harder to plug back in - I'm thinking of some of the kids who just don't participate, they aren't tuned in to the discussion in class and the achievement will be very low just because what is going on is over their head. This goes back to your question - is there something about mathematics? - Other fields are perhaps not as cumulative and interrelated."

APPENDIX C

A Systems Approach to NCMP

by Joe H. Ward, Jr. and J. A. Easley, Jr.

The first topic that we wish to discuss is what we call the evolutionary approach of the Nova mathematics project. It is suggested that a basic structure of the system for carrying out this project be defined in such a way that it will not change as the development proceeds in an orderly fashion. In the beginning, the definitions of the elements in this structure may be quite fuzzy and not clearly defined; however, the details will become clearer as each specific aspect in the system is studied. We will describe the general system for the project in terms of Figure 2 (page 14) and then we will go into details of the components of the system as we see them at this time. In the description of the system and the detailed components we will assume an advanced state of development, describing the functions in midstream so to speak; however, it should be clearly understood that the system doesn't come into full-blown operation all at once.

We will first present a very general conception of the problem of educational systems in general and then, in particular, the Nova conception of a system for mathematics education.

Components of a general educational system:

We might consider this system as composed of the following major components. We can think of such things as teachers, teacher trainers, counselors, and other people associated together with rooms, physical plant, activity packages, and data gathering facilities as the resources of the system. All of these resources are in association with the students. We might then say that we desire to bring together all of these resources, or objects, in such a way that we tend to maximize some objectives, some pay-off, or some utility.

In thinking about combining the people, physical things, and students in various associations so as to maximize some pay-off, we find that it is necessary to talk about various characteristics or attributes associated with all these people, physical objects, and students. The reason it is necessary to consider characteristics or attributes is, of course, that when we are thinking about the evaluation of the utility or the pay-off associated with particular activities, it is necessary to associate with each combination of attributes considered in a decision some estimates as to what each of the particular activities is worth. It is important, for example, to know what characteristics of the activities are important and what characteristics of the students are important so that, when considering students engaged in learning activities, we can compare one activity-student combination with another activity-student combination in terms of its worth or utility. If we have no way of describing students and activities, we have no logical or reasonable way to discuss their judged worth or utility.

One must have some long-range conception of the kind of students that a new educational program will turn out, even though this conception is, of necessity, extremely vague, since no students of this type have ever been produced before. A notion like mathematical literacy may serve to assist in the estimate of pay-off of each package-teacher-student combination which is required for the on-going decisions in the system.

The Nova Mathematics Project System

Referring to the systems block diagram, Figure 2, (page 14) the system contains five major components; instructional resources, systems modifiers, systems analysis, data storage and processing, and action monitor and value estimator.

The instructional resources block includes all of those resources that are available to us for achieving our educational goals. These can be thought of as of four basic types; the indirect-contact staff, the direct-contact staff, the physical facilities, and the students. In the indirect-contact staff group we include such people as authors and teacher-trainers. In the direct-contact staff we include teachers, counselors, librarians and any other persons who may be involved in the educational process. In the physical resources we think of the rooms, the data gathering instruments, the laboratory equipment and many other physical resources, but most important of all are the activity packages. These are the basic units of instructional activity to which we refer in our discussion. The activity packages in the system may ultimately number in the hundreds or thousands. These are the products of the authors, and it is these units which provide the basic framework for the continual re-grouping of students, teachers, and activities. The last, but certainly most important, resource is the students themselves. They are not merely a resource in the sense of raw materials, but also it is from their responses to instruction that activity packages and teacher training operations are improved.

The next block we refer to is called the Systems Modifiers. This represents the functions that will bring about changes in the over-all system as determined in the next block - Systems Analysis. Within systems analysis we have the research function; any questions of importance about the over-all system as well as detailed evaluation of each activity package, will be investigated within this function.

The next block is called Data Storage and Processing. This function includes all the systematic analyses of the outcomes of the association of all resources, as well as prediction activities that are required in the estimation of the value or utility that may result from the association of the resources. This data storage and processing function may or may not be carried out by a computer facility.

The fifth box is called the Action Monitor and Value Estimator. The action monitor aspect of this function is a continuing surveillance of the results of the data processing function to be sure that the actions that have been suggested as a result of the analysis are appropriate. The other aspect of this function, the value estimation aspect, represents the judgments and estimates of worth of whoever, or whatever group of people, are assigned to perform this function. (It is important to realize that this may or may not be one person, but it could be quite a few people such as a mixture of teachers, administrators, school board members and any other members of our society that wish to be involved in the determination of the adequacy or quality of our system.)

Again it should be emphasized that the components of the system are functions and that single persons may successively play almost any of these functional roles; typically the project director would find himself serving as teacher-trainer and author, systems modifier, a value estimator, etc. As the system develops, more specialization may be needed in terms of individual staff responsibilities, however we will continue to describe the functions without making any assumptions about the actual distribution of personnel.

The dotted box to the right in Figure 2 (page 14) represents all other systems within the environment. These systems might include the social studies, science, athletics, band, and any other environmental activities whose presence affects the operation of the mathematics system. The arrows between the other system and the mathematics system represent the intercommunication and information flow among all parts of the entire school system. We proceed now to discuss each function in detail.

The Instructional Resources Function

The first sub-division of this block is the indirect-contact staff. Within this group are the authors. Authors will have various characteristics and attributes that are relevant in our thinking about their relation to specific system demands. Examples of these attributes might be the level of mathematical competence, the area of mathematical specialization, the extent of interest in applications of a certain type, and any other information that becomes important. The thing to remember in the terms of attributes is that we will include those attributes required in the understanding and clarifying of the system and all the interrelationships of its components. Some attributes will be added or subtracted as the system grows through time and the addition of attributes associated with certain of these components will stimulate the need for, or modification of attributes associated with other components.

The next component within the indirect-contact staff are the teacher-trainers. Again the knowledge of the characteristics of the teacher-trainers is required in the effective working of the system. Teacher-trainer characteristics might be described as, (1) having experience in certain broad areas of mathematics, (2) as specializing in applications, (3) the number of years of teaching experience a person has had, (4) whether or not they have trained teachers of a certain type previously, or many other relevant characteristics. An important idea here is that functions, such as systems analysis, anticipating the role of certain attributes in the evaluation and utility-estimation process, will decide that it is appropriate to gather certain information about attributes of some of the system components; in particular, information about authors and teacher-trainers might be gathered that would perhaps not be considered in the action decisions.

The next group includes the direct-contact staff. The first block is appropriately concerned with instructional function -- the teachers. Attributes to be associated with teachers might include indications of previous experiences, qualifications with regard to certain mathematical topics, and, as the activity packages are developed, attributes associated with teaching experiences utilizing particular activity packages, and perhaps even descriptions of the types of students with which a teacher has been associated. For example, if a teacher has worked with fast groups more than slow groups, this might be very relevant in the analysis process.

The next direct-contact staff member would be a counselor. It seems, again, that the kind of information appropriate would include the counselor's formal background for, or interest in, the mathematically related activity packages, since he would have to deal with complaints from students that they have been misassigned. It should be remembered that all of these components have interactions, and they are not shown explicitly in the diagram. The counselor, certainly, has interactions with all of the components within the instructional resources block. As another example of counselor activity, a situation would arise in which the data processing facility sent a message to the counselor to collect certain kinds of information from students or from teachers. In this way the counselor functions as a data-input source and possibly functions as an evaluator also.

Other direct-contact staff such as librarians, tutors, and any others functioning in this way can be added as required in the development of the system.

The next group of components is the physical resources. One example would be the rooms or spaces available in the school facility. Characteristics associated with these rooms would include the number of students that can be accommodated, whether or not there is a projector available in the room, and any other relevant characteristics. Another component of the physical environment would be the specific laboratory equipment that may be important in the educational process. Still another type of physical component is the data-gathering material such as test instruments and inventory devices. It is essential that data-gathering instruments be able to present detailed and diagnostic information needed in deciding which activity package should be assigned next to which student. The data-gathering instruments may be as short as can provide reliable information for these decisions and should be treated as discreet instruments for particular outcomes rather than summative instruments of general achievement for purposes of grading. This is not to exclude tests of general intelligence and other characteristics that could be useful, but to indicate the minimum essential in a working system. This requirement means that authors must continually be developing new instruments as new activity packages are developed.

Probably the most important component is the activity package. The activity packages are the basic units about which center the activities of our students and teachers. The important requirement is to know characteristics of the activities that will permit evaluations and value judgments regarding their contributions to the outcomes of the program. Examples of attributes associated with activity packages might be the following: First, each activity package might be described in terms of its contents, that is, the areas of mathematical activity that are involved; another attribute would be the application areas that might be involved; another attribute might be the sophistication of the material involved; another attribute might be the characteristics of the learner that are required for him to engage in this activity; another attribute might be teacher requirements, and it should be noted that these attributes may be modified as the whole system evolves through time. It is important to realize that many different content topics could be found together in one activity package as well as several application topics, so these attributes should not be considered as mutually exclusive categories; we may have mixtures. One other attribute needed before making decisions about the grouping of students and activities and teachers is the expected time requirement, or time required as a function of student characteristics.

There are two attributes of activity packages which are of special interest to the systems-analysis function. These are (1) the conditions of learning under which the activity is conducted, for example, group discussion, game playing, or individual study, and (2) the expected immediate outcomes of these conditions in terms of behavior, content, applications, and the like. The descriptions of these two attributes represent hypotheses concerning the function of the activity package which are testable by data generated in the system, in contrast to the hypotheses concerning the long-range influence of a package which must be represented only by the values assigned to it by the action monitor.

It will probably be very appropriate that some activity package might have the characteristic of being simply an idling package, for there are certain times when it might be valuable in terms of the system as a whole to have a student engaged in what might be called an idling activity. One hopes to be able to create components of this system as necessary in dealing with the problem at hand.

Another physical component of the resources block could be a computer-aided instruction facility. This computer facility should be distinguished in function from the data-processing and storage block since it is part of the instructional resources. The use of the computer as an instructional resource can be thought of, perhaps, in at least two ways. One way is to assist in the teaching of materials involved in the activity packages. This might be considered as the traditional computer-assisted instruction aspect. Another possibility is the use of the computer as a device to model a mathematical structure and, through suitable interaction with students, provide a different insight into some mathematical process. The way that such a computer will be employed will require extensive study, and some of the work coming out of laboratories for computer-assisted instruction research will certainly be relevant to this particular aspect of the system. It is appropriate also to mention the possible ways in which a teacher, a computer, and a student may come together in this learning activity. It is certainly not necessary to consider whether a teacher is engaged in teaching or a computer is engaged in teaching, but it seems most reasonable to consider both the teacher and computer together in the instructional process. This means that the teacher-computer interaction with the student may result in something quite different from either one of these components working individually, and it is quite likely that the study of ways in which the teacher can interact with the computer in the instructional process will be a very important contribution of the Nova mathematics project.

The next component is the students. Students have many characteristics that are relevant to the determination of actions in which they should be involved. Their age, interest preference, and other characteristics are quite obvious, but characteristics that will begin to emerge from the operation of the system include such things as experiences with certain activity packages and certain teachers and achievement in certain contents and abilities. As the student moves through time his historical information will increase, and at many decision points in the student's career it will probably be relevant to refer to an accurate description of what activities have been completed and the degree of success achieved in those activities.

The language used to describe the activity packages, namely, the contents, the application, the level of sophistication, and the particular behaviors associated with these can be the same language used to describe the student's accomplishment, and these must be the same for the matching of activity packages with student readiness and need.

The Systems Modifier Function

Systems modification could involve bringing into the system different types of authors, different numbers of authors, different types of teacher-trainers, different types of teachers, counselors, and so forth. It may also involve the modification of rooms, bringing into the system other physical resources, bringing about possible changes in time modules in the daily schedule, bringing about changes in transportation arrangements that might be relevant to educational activities, and bringing about changes in the overall components of the system that seem necessary to make the system operate better in some sense.

The Systems Analysis Function

Systems analysis includes what is commonly thought of as instructional research and curriculum evaluation. The activities of systems analysis involves the determination of what is going on in the system; such questions as why the system is not working well and why particular aspects might not be contributing to the overall objectives. The systems analysis activity will also suggest modifications to be made in the system. It is apparent that in this analysis activity we have potentially all of the usual research activities that could be considered appropriate. One may wish to determine whether different activity packages will work with other types of students than have been tried in the past. This may lead the systems analysis activity to request the generation of other kinds of information relevant to the evaluation of the effectiveness of the proposed experimental activities. The systems analysis activities can be organized on several different levels. We have earlier mentioned the evaluation of the conditions of instruction as a means of attaining particular goals for particular activity packages. At this level, researchers may wish to design variations on a given package in order to attempt to optimize its functions. At the gross level systems analysis personnel may be studying the distribution of students in terms of levels of sophistication, may suggest other types of assessment or data-gathering procedures in order to provide better feedback to authors who are engaged in revision and in the development of new activity packages. The systems analysis function may certainly be expected to grow along with the system, and it would be inappropriate to set any limits on it at the outset.

The Data Storage and Processing Function

The data storage and processing activity will involve the selection of all relevant information as required in our system as well as the processing and analysis that is required. The type of data that will be required for storage can be stipulated either by the data-processing function itself or by other components in the system. Besides just gathering data, it is most important to decide what is to be done with the data that is selected. The kinds of processing will become apparent as the system evolves. A most important type of processing in this function is the prediction of the outcome of the associations among the instructional resources as well as choosing the method for taking action in order to maximize the accomplishments of the system. The analysis requires first predicting outcomes and then recommending actions that would lead to a more optimum condition, if the predictions are correct.

One aspect of this data processing and analysis function is the systematic capturing or simulation of the value judgments or worth judgments made by the action monitor and value estimator. One way in which this can be accomplished is the following. We can require the action monitor to make value judgments continuously of the particular associations that are now going on in the system. The action monitor can be given information or characteristics of these particular associations of instructional resources, and in terms of this information he can make general decisions regarding the relative adequacy of each particular association of resources. Without his having to describe for the data-processing function exactly how he is putting this information together, a mathematical model can probably be found which will adequately predict how he would judge new associations of the instructional sources, based on a small sample of his judgments. The computer programmed with this mathematical model can predict the judgment in other cases in which students, activity packages and teachers are associated, make an action decision on the basis of this prediction, and present it to the monitor for his review.

There are several important advantages in this approach. It is certainly obvious that, if we can adequately capture the judgment policy of the value estimator and action monitor function, we will be in a position to apply this policy to a very large number of activities which might be impossible otherwise. This is particular importance when the combinations of instructional resources result in a very large number of possibilities. Since the policy will have been described in terms of appropriately defined variables, it is possible to predict what the judge would have said if he had seen every possible combination of instructional resources. We must realize, of course, that in the operational situation it may not be necessary to make predictions about all of the very many combinations of the resources, but from a conceptual point of view this unlimited possibility is desirable.

In addition to the value of allowing the computer to assist in the routine, time-consuming task of predicting the values of these many associations of our instructional resources, the same mathematical model also makes it possible to find out what information and attributes are contributing to the judgment process. As a specific example, if a value estimator looks at all the characteristics associated with a combination of instructional resources and makes value judgments, it may turn out th t only one of these pieces of information may be required in predicting outcome. It may be that the evaluator is merely using a score that results from some particular activity as his value judgment of the activity.

This is frequently the way that judgments are made, but the more general approach gives us the possibility of getting at more intuitive and fuzzily defined kinds of value judgment. The difficulty in the past has been that people can frequently tell you what information is important in their decisions, but cannot tell you how to put this information together to replicate their judgment.

In situations where it turns out that this approach cannot adequately describe the judgment process we can, however, proceed systematically to get a grasp on those reasons that keep us from successfully predicting judgments. One reason might be that there is missing information about the decision or evaluation situation. If the evaluator has information not known to our information system, then he may be using that information in his value judgment. Another reason may be that we have not adequately defined a model that can capture this process. In many processes, experience has indicated that the functional form is relatively easy to specify because people's analyses, and the ways they put information together are much less complex than it first appears. A third possible reason is that the evaluator has no policy and maybe tending toward random, inconsistent decision processes. The important point is that we can pursue each of these difficulties systematically in an effort to improve the system.

The decisions that are made, or simulated by data processing systems, concern the assignment of a particular student to a particular activity and a particular teacher. In the beginning these assignments can be made, perhaps primarily, on the basis of estimated worth of the particular activity packages together with other pertinent mechanical-type data such as room assignments, availability of teachers, etc. However, as the number and complexity of the activity packages grows, it becomes possible to consider the network of activity packages in which one package leads to, or prepares the way for another package, so that it is possible to reflect back into the earlier packages the values attached to later packages and to utilize this information in making the decisions. Figure 1 (page 11) attempts to suggest the kinds of networks of activity packages which may need to be compared for value decisions. At the lower levels of the diagram, which represent earlier times, many possible paths through higher levels must be considered in order to incorporate in the decision the potentiality of each choice in the long run progress of a student toward mathematical literacy - the overall expected outcome.

With regard to the problem of choosing paths through the activity "space" in order that the students achieve some goal, it might be appropriate to consider the potential usefulness of such techniques as those of network flow, dynamic programming, "program evaluation and review techniques" (PERT), and others that have been developed for this type of problem.

Another important data-processing function is the ability of the data-processing facility to answer questions raised by such other functions as systems analysis, systems modifiers, and the action and value estimators. Of course it may be that counselors, teachers and students will also inquire from this system when appropriate. This would make it highly desirable to build into the data processing function a program that will enable appropriate questions to be received, interpreted, and properly processed to satisfy these needs.

The Action Monitor and Value Estimator

The action monitor activity centers on the large arrows going from the data processing function to the action monitor and then over to the instructional resources. The action monitor will look at the recommended actions and check as required whether or not these actions seem appropriate according to his goals for the system. It is not necessary to have a complete monitoring, and the amount of monitoring will vary as the situation changes. The value estimator role has been described in more detail in the previous discussion of data processing. This function is very significant because this is where the extent to which the system is achieving its goal is determined. We must point out that many kinds of people may operate functionally in this type of activity. These may be principals, teachers, school board members, citizens and anyone else who is concerned with the appropriateness of the educational activities.

Another important feature of this function is the interrelations between the action monitor, the data processing facility, and the instructional resources. It may be that actions should not be taken until certain pieces of information are collected from the students, teachers, or other components of the instructional resources. If the action monitor indicates to the data processing facility that it should gather more information prior to a proposed action, then the data processing function will communicate directly to some resources component in order to obtain the information required to justify a recommended action.

It should be emphasized that all of the functions represented by blocks in the system diagram actually could be carried out in the beginning by one person, and it is not necessary in the early stages to have a piece of hardware for the data processing and storage facility. This implies, of course, that this way of conceptualizing a problem could be put into operation immediately and that it provides a framework that would be useful in guiding the development of the educational system as rapidly as possible toward the goal of efficient, large scale operation. However, it is desirable to consider the possibility that hardware will be involved in the data processing function, and it would be appropriate to think about the information flow in this context. It would be much easier to insert computers into the system if the data conversion problem were minimized by planning in advance.

It should be stressed that this system is evolutionary and modifiable, it is a framework into which components can be put as they are developed in order to facilitate understanding their functional relationships. Its most important feature is that it provides a systematic way of recording information and communicating among all of the important components of the system. It is perhaps not appropriate, at this time, to discuss the detailed requirements of personnel in the various functions. It seems that the most desirable way to proceed would be to start with a minimal system and obtain the financial capabilities to expand the system as seems appropriate from experience in the operation of the system.

It's probably necessary to have some advanced projections of budgetary requirements and personnel re[illegible]ents; however, this should not be planned too far in advance since one [illegible] not be able to fulfill the desired requirements when the actual operation begins.

One of the outcomes of this project might well include the development of more systematic approaches to instructional processes. As this system develops, various modifications will be required in the conceptualization of the problem, and these developments should be of considerable value to future projects, not only in the mathematics area, but in any general area of educational activities. Some of the other outcomes will be primarily in the activity packages area. These activities will then be available for use by other people in other systems. Also it will be possible to determine how teacher-training activities can be best undertaken to achieve these goals of the project. Another important outcome will be the experiences that will be gained in the data processing and storage area. New developments in educational data systems will certainly come out as the policy-capturing, action direction, and information inquiry activities take place.

Other outcomes of the project will include results of research studies conducted within the system. These may take the form of general contingencies between types of activities and educational outcomes or they may take the form of systems characteristics which optimize certain outcomes. Another long-range outcome of this project will certainly be called for, and that is a study of the students after they have left the system and gone on into colleges, universities, or other activities. Their characteristics and achievements can be studied as a long-range evaluation of this type of program - that is to say, the type of program which evolves as a result of the value estimations enter into it. The process of defining variables and recording information will enable everyone concerned to get a better grasp on the judgment-utility-value system.

Overall Administration

The system, or any system of this capacity must have a director, and perhaps a general advisory board. These administrative functions are only partly represented within the different function blocks. In order to eliminate possible misunderstanding of this proposal as describing a system that functions autonomously, it needs to be emphasized that there always remains some overall administrative concern for the system as a whole.

APPENDIX D

Comments by Marshall H. Stone, University of Chicago

What is most striking at the South Florida Education Center ("Nova") is that it combines - perhaps uniquely in the world - the opportunity, the desire, the will, and the facilities to develop a unified modern program of instruction for grades K-12. What Nova needs in order to produce such a program is to add to this combination the money, the staff and the time without which a practical realization of theoretical educational goals would remain impossible. So far as the mathematical components of such a program is concerned, its nature is clearly foreshadowed and its feasibility guaranteed by what has been done theoretically and experimentally during the past ten or fifteen years. With the start which has already been made at the Nova High School under Burt Kaufman's inspiring leadership as mathematics coordinator, and with adequate resources, an outstanding modern program could be developed, I believe, in five to ten years. Such a program would meet the challenge of the Cambridge Report on "Goals for School Mathematics", not in 1994 or in 1984, but in something like ten years earlier than the latter date. A Nova mathematics program in being would itself offer a greater challenge than any report and would serve as an inspiring model for the schools of America.

In planning the development of the Nova program it will be necessary to agree at the start upon handling in some orderly fashion the different components out of which a complete program would be built. It is of the greatest importance that work in the elementary school be begun as soon as possible. Among those who have studied the problem of realizing a modern program in which each student can achieve the fullest development of his mathematical knowledge and capabilities there is agreement that a radical change in the elementary part of the mathematics program is essential. It is not possible to crowd into grades 7-12 alone all that should go into a modern mathematics school program such as that envisaged in the Cambridge Report and at Nova High School. The foundations of the program have to be laid down in grades K-6. Furthermore it is not merely a question of the mathematical content of the program. The fundamental motivation and attitudes of students are of equal, perhaps even greater concern. Consequently the development of the elementary part of the Nova program will involve deep pedagogical considerations as well as bold curricular studies and experimentation. In order to draw together at the elementary level the curricular and pedagogical innovations indispensable for the creation of a modern program of school mathematics, it will be necessary to prepare the participating teachers psychologically and mathematically for their tasks and to modify, where necessary, the organization of the school in which they are to work. Practically considered, the central problem is not so much that of developing new curricula or new methods - there is a wealth of valid proposals along these lines - as it is that of working out in detail in an actual school situation a skillful articulation of the ingredients of which a fully developed program is to consist.

Ideally it would be best if the staff which is to work together on the elementary part of the Nova program could spend a summer together in study, discussion and planning before starting any class-room work with students. Once work with students has begun, the group will need to aid current tentative evaluation to its study, discussion, and planning. To what extent it will prove to be necessary to prepare materials for the use of students and teachers is not easy to estimate. There is a wealth of materials available, but some new materials would almost certainly have to be developed on the spot. In any case a good deal of what might be called "working materials" would be indispensable - books, planning documents, outlines, summaries, reports, teacher training papers, student work sheets and student texts, various laboratory materials and instruments (i.e., desk computers) would also have to be produced or acquired. Adequate provision for these material needs must be made in the budget along with those for salaries, travel, secretarial assistance, fringe benefits, and so on.

Simultaneously the work already in progress at the Nova High School must continue. The curriculum must be expanded to include new subjects, and new approaches to subjects already included must be tried out. In my opinion one of the most difficult problems in designing a good modern mathematics program is encountered in the field of geometry. A satisfactory solution to the problem of teaching school geometry demands the introduction of much more physical or intuitive geometry at the elementary level and the elaboration at the secondary level of an axiomatic treatment radically different from that now being followed at Nova High School. Probably this treatment should be based in part on the concept of transformation. Nova should not postpone too long an attack upon this problem.

A very big problem is posed by the need to coordinate the school mathematics program with the programs in physics and other sciences. Since the program at which Nova is aiming in mathematics is to be a flexible one adapted to the needs of the individual student, it should be a relatively easy matter to commence the work of coordinating the program with those of other departments at any time they are ready to cooperate. However, it would be well to anticipate such cooperation and to prepare for it by studying the opportunities for introducing into the mathematics curriculum topics and materials relating to scientific and technical applications of mathematics.

Now, whatever else is to be expected from the work to be done in the South Florida Education Center, a mathematics program for universal adoption across the country is not a valid or a practical goal. In fact, in the teaching of mathematics the aim must be to achieve a rich diversity in an essential unity. Perhaps modern mathematics programs, as they will be developed in different places over the years, will resemble each other in broad outline, but there are too many possible ways of organizing, stressing, and teaching the important topics in mathematics to fix on one program as preferable to all others. Furthermore, the advances in mathematics and the extensions of its applications will certainly necessitate the continuing revision of mathematics programs, however satisfactory they may appear to be at any particular time. Thus what Nova is doing now or aims to achieve after ten years time will only serve as the basis for new work to be done when the present plans have been brought to fruition.

APPENDIX E

Comments by George Springer, University of Indiana

I think that all of my comments should be based on the assumption that money will be forthcoming from your school system and from the U. S. Office of Education to enable you to set up an ideal program. If this were not the case, there would be nothing further to discuss, for then Nova will be just another good school system which is giving its students a modern curriculum. With sufficient funding, Nova can become one of the most important curriculum development centers in the world, for you have the facilities to conduct a long range study on one student body from K to 12.

This should be your next step in curriculum planning. You already have a program which gives each track very good training. Your own Number Systems of Algebra followed by such books as Moise's Geometry, Haaser, LaSalle and Sullivan's Calculus and Mostow's Algebra give your track 1 students a good head start in college mathematics. Now is the time to begin with the first grade and develop materials that will give the young children insight into mathematics and prepare them for abstractions and generalizations. I do not want to view this as a crash program that must be completed in a short time to meet a current demand. Since you have a curriculum that is working fairly well, you can afford to plan a long range program.

You will need first of all the help of the most competent people to write the new materials. Finding these people and giving them the working conditions they need is one of your hardest problems. In this connection, the new Nova University will be a help and you will have to tie your long range plans in with them. Your writing "team" will probably hold some kind of joint appointment with Nova University and have the experimental facilities of the elementary and high school available.

If this group of writers, headed by Burt Kaufman, produces materials from K on up based upon the spirit of the Cambridge Conference, it has the classes available to try out these materials at once. But clearly you will need competent teachers in all grades. Here the assumption of adequate financing is important, for you will have to release your teachers about one-third time for study, meaning a sizeable increase in staff. Once again you will have to rely upon an affiliation with a University to provide the seminars for these teachers. We provide such an in-service program for both elementary school and the junior high schools in Bloomington, but our goals are not nearly as ambitious as yours. We meet with the teachers only once a week, whereas your program will have to be more intensive and give a broader background in mathematics, even beyond level 1 of CUPM* for the Cambridge Conference materials will be more demanding with respect to mathematical maturity. Having the teachers criticize the new materials as they are produced and try them out will also add to their training, but this is only in connection with more intensive seminar work.

I think that one problem you will have to face is whether to make this a universal program in the lower grades as a multi-track program even in early elementary grades. I would hope that your new program will be aimed at the highest level of achievement and that those students who cannot keep up should be dropped into your present curriculum where they fit it.

* See Footnote 4, page 18.

APPENDIX F

Comments by Bernard Friedman, University of California at Berkeley

Any proposal for support should be for a program which is designed for every child with ample provision for individual differences. I believe that the majority of children can be taught much more mathematics than has been done before. There is no need to restrict oneself to the top 15 or 25 percent.

I would like to make some general comments about the philosophy that should be considered in teaching. Because of the short length of time that is available for education, it is not enough to keep the children amused and interested. We must try to do things as efficiently as possible and to present material which will lead to later generalizations. An illustration of this philosophic attitude is my belief that high school geometry should not be based on Euclid's or Hilbert's postulates because this approach does not lead to generalization in future mathematical activities. A better approach to high school geometry would be by means of transformation and group theory.

In our teacher training we must show the teacher what are the fundamental ideas and methods in mathematics instead of insisting on teaching a certain amount of subject matter. In that way whenever a teacher does something in class he will understand the future objectives of the devices he uses.

There is a great need for more geometry and more geometric ideas in elementary school teaching. By this I don't mean the kind of geometric mensuration which leads to arithmetic. I do mean the study of such topics as symmetries, shadow projections of geometric figures, Euler's Theorem and the study of invariance properties under certain kinds of transformations.

APPENDIX C

Comments by Layman E. Allen, Yale University

I think that your efforts in developing the mathematics program at Nova has potentialities for influencing other important aspects of education, in addition to making a unique (and extraordinarily useful) contribution to pre-college mathematics education. For example, the planned released-time program for in-service training of elementary teachers in mathematics could turn out to be the seeds of a more general program for all elementary level subject matter - and perhaps eventually, for a continuing program of in-service training throughout a teacher's career. If the Nova project begins to arouse public expectations that pre-college teachers also should have time for reflection and study, just as those at the college and university level do, you may be generating change of the most significant kind. It is nothing less than a fundamental value perspective of American society that may be vulnerable to beneficial change, resulting in an enhancement of education and ideas in our total scheme of values.

I think you know my feelings correspond with those of James Colemar of John Hopkins that academic games may also have a useful role in producing similar effects on value perspectives by affording an opportunity for public display and recognition of intellectual skill achievement in a way that parallels current practice with respect to athletic skill achievement. It is of particular interest to me that you are planning such activity as an integral part of the mathematics curriculum and that there is already under way at Nova an academic games program for other subject matter, as well as mathematics. It seems to me that the odds are stacked nicely in favor of Nova's emerging as a pattern-setter for such efforts throughout this country. If there is any way that I can help out with this, I'll certainly do whatever I can to be available.

I guess what I'm saying is that although you are posing merely as revolutionaries in mathematics education, the prospects are pretty good that your program will cut even deeper and more fundamentally into the fabric of American society. And further, I am all for the direction in which you are moving.

APPENDIX H

Comments by David B. Robinson, Greece Central School System, New York

The report which is enclosed represents my observations as a school administrator, not as a mathematician nor as a mathematics educator. I express these opinions in the firm conviction that the uniqueness of your program is based on sound objectives, guided by admirable ideals, and lead by an energetic, highly competent and truly dedicated individual.

1. The conference was well planned and organized. The variety of conference consultants represented an excellent national cross-section of individuals in the forefront of math education. It was particularly valuable to have individuals present representing E.S.I., Madison Project and Minnemath.
2. The Nova program has an energetic, vigorous and well-trained math staff upon which to base its ambitious objectives.
3. Track 1 students are obviously highly motivated, exceedingly well grounded in theoretical algebra, devoted to their teachers, and surprisingly well-adjusted to the non-graded approach being developed.
4. Physical facilities at Nova High School are poorly planned and vastly over-crowded for the nature of the math program desired.
5. Teacher load has been extended much too far beyond normal course requirements, not even considering the ambitious program foreseen.
6. Greatly expanded staffing must be provided for pupil instruction, curriculum development, material writing and preparation, and in-service training for teachers.
7. Increased teacher remuneration must be provided to recruit and retain the well trained and highly competent staff desired and needed for this program.
8. Continuance of the Nova mathematics instructional program concepts will demand up-grading of other academic departments to fulfill Nova High School's goals, to avoid pupil frustration in these other subjects, and to avoid the seemingly inevitable intra-school strife that will result from inter-departmental comparisons and competition.
9. Much greater administrative and public support must be forthcoming if these program goals are to be achieved. These programs should not be considered "experimental", they are from my brief observation, practical and realistic. System-wide support is not only desirable, but essential, otherwise this "unique" situation will be subject to constant public controversy, and open to each economy drive of public officials.

10. Freedom to further develop your objectives should not be jeopardized by too close an alignment with any one national program or university. Hopefully, you should have all the resources available at your command, and the possibility to incorporate those that appear most promising.

Specific Conclusions Relative to Proposal Development

1. All planning, curriculum development, teacher training, instruction and program evaluation should be on a K-12 basis.
2. All planning and development should intimately involve the teachers.
3. All program development should have definite objectives which can be readily seen and understood by the teachers.
4. All curriculum content should have significance for the students, and meaning for each subsequent phase or stage of curriculum development.
5. Teachers should not be trained in "pure" mathematics without relating it to its significance and meaning for pupils.
6. The preparation of new materials and content should involve not only competent mathematicians and psychologists; but teachers as well. As each new draft is prepared it should be evaluated in the classroom and then redrafted as necessary.
7. Released time for teachers during the school day should be incorporated into the proposal to insure time for their instruction and study in basic mathematics concepts, work with mathematics consultants, writers, etc.
8. Some means of involving administrators and other subject area staff (especially science) should be sought to give these people a vested interest, to insure their future support, and to spread the concept of what is being done.
9. Procedures should be developed for evaluating the program in terms of what happens to pupils in terms of their attitude and behavior. Video tape facilities at Nova should be used to record pupil reactions and progress for future reference.
10. Careful consideration should be given to program objectives in terms of determining
    a. to whom this program is directed. What kind of students? what level of ability is expected? etc.
    b. how all pupils may eventually benefit - not just the top 15 to 20 percent.

APPENDIX I

Comments by Phillip S. Jones, University of Michigan

I personally would be very much pleased to see two different types of experimentation at the elementary school level. One of these would be a well-defined and documented attempt to teach mathematics in an "ungraded" system. A report on the successes and failures, problems and ways of meeting them in such a situation would be very useful to persons concerned with elementary mathematics education all over the country. Even a thorough report without any of the fancy research paraphenalia of control groups, etc. would be useful.

Secondly, I would be interested in seeing how far children of considerable capability can be led to progress if they have competent teachers and none of the handicaps of the non-homogeneously grouped elementary school class. It seems to me that there is a possibility that the Nova Schools are in a unique position to do these two different but complimentary tasks. You are not weighted down by tradition, you are interested in experimentation, you have a new school with an interest in an ungraded program, you have an example of the potential for acceleration in the things that have already gone on at the secondary level. I would support the desirability of experimentation of this sort for the welfare of education in general and mathematics education in particular.

APPENDIX J

<u>Comments by Charles Brumfiel, University of Michigan</u>

For the next several years the math program in 7 through 12 should be given priority. It is well established that even with conventional math backgrounds in K-6 Mr. Kaufman can get phenomenal results from his students during the high school years. It is not at all certain that much significant acceleration can be obtained by playing with the program of the elementary grades.

The crucial need is for math teachers with training and teaching skills comparable to Mr. Kaufman's and for available "resident mathematicians".

The development of new materials for use in the K-6 program seems to me of vastly less importance than the effort to develop techniques of working with the abler students so as to move them quickly through the mathematics program of the elementary school and develop substantial algebraic skills.

APPENDIX K

Comments by F. J. King, Florida State University

The Nova Mathematics Curriculum Project is unique in that it recognizes the need for individualization of instructional content, methods, and materials. The curriculum matrix with its three dimensions of content, applications, and learner characteristics insures that provision for individual differences among students will be central to the development of the curriculum.

A number of investigators have foreseen the importance of student ability - curriculum interactions for achievement of mathematics. The following quotation from Gagné (1960, pp. 49-56) implies that instructional materials in mathematics might be modified to capitalize on different student abilities.

> ... it should be possible to verify a number of hypotheses, not only about the nature of abilities involved in mathematics but also about the essential nature of what is learned in mathematics, such as:
>
> 1. People who are high in spatial ability should acquire mediating spatial concepts more readily than they do symbolic or verbal ones. This should also be true for numerical ability and verbal ability.
> 2. People who are high in verbal ability should reveal a better performance on geometric problems when they are taught verbal mediators than when they are taught spatial or symbolic ones. Similar statements could be made for the other basic abilities, spatial and numerical, using algebraic and formal logic problems as performance criteria.
> 3. When people high in spatial ability are taught the same mathematical principle in terms of spatial, verbal, and symbolic concepts, measuring retention of this principle after a period of time should reveal increasing dependence on spatial concepts. Again, similar hypotheses could be made to pertain to the other abilities.

Gagné has also proposed a research design for studying these hypothesized relationships. His statement of the problem is central to the development of the Nova Curriculum:

> Differences in fundamental abilities appear to be prominent in the learning of mathematics, as well as in the way people use mathematical concepts. Well-established factors in human abilities are spatial, numerical, and verbal. Although there are studies which have revealed moderate to high correlations between aptitude measures and grades in mathematics, no studies have been conducted in the attempt to make specific predictions concerning the facilitation of different kinds of conceptual learning by different fundamental abilities. The possession of a high degree of spatial ability should facilitate the learning of spatial concepts; and high numerical ability should facilitate the learning of symbolic concepts.

The learning of concepts of addition of directed number may be done verbally, spatially, or symbolically. Verbal rules are perhaps the best known method, occuring in most conventional textbooks. Spatial concepts have been used with considerable success, notably in the textbook of the University of Illinois Committee on School Mathematics. Symbolic concepts can readily be designed to serve the same purpose; in one form, they might resemble some of the symbolism of Boolean Algebra. Thus in this mathematical topic, the opportunity exists of relating differences in fundamental abilities to ease of learning the different types of concepts, as well as to final performance in problem solving.

Two studies have been published which give some indication of an interaction between aptitude patterns and method of teaching mathematics. Osburn and Melton (1963) administered a battery of aptitude measures to students enrolled in traditional and experimental modern ninth grade algebra classes. Three proficiency tests concerned with topics common to both types of instruction were developed and administered along with the Cooperative Algebra Test. It was found that tests of verbal reasoning and numerical ability predicted equally well for students in both types of instruction but spatial and mechanical reasoning tests were more valid for the experimental classes than for the traditional classes. A spelling test gave better predictions for the traditional group than for the experimental.

Guilford, Hoepfner, and Petersen (1965) studied the relationships between "structure of intellect factors" and achievement in general mathematics and algebra by ninth grade students. One objective of the study was to see how well the test battery could differentiate between successful students in the two courses. Another objective was to determine which factors were most relevant for predictions of success within and between courses. Twelve factors, mostly in the symbolic category, were found to be statistically significant predictors of achievement. Combinations of factor scores were found to differentiate between successful members of the two groups with approximately 90 percent accuracy. However, most factors that were relevant for one course were also relevant for the other.

A series of investigations under way at Florida State University is seeking to determine whether several forms of one content can be constructed so that achievement on each form is dependent on a different set of mental abilities. Four forms for teaching set concepts to fourth and fifth grade children have been developed. The mental abilities emphasized in each form are as follows: (1) verbal-inductive, (2) verbal-deductive, (3) figural-inductive, (4) figural-deductive. These four forms were administered to fourth grade children along with the PMA and syllogisms test of the CTMM. An achievement measure appropriate for all forms was administered also. A regression analysis of the data indicate a significant interaction between four predictors - PMA Verbal, Reasoning (induction), Perceptual speed (figural) and CTMM Syllogisms (deduction) - and form when the dependent variable was the achievement score.

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APPENDIX L

Comments by Edward L. Palmer, Florida State University

One outcome of one of our sessions there was a list of variables which appeared to be amenable to investigation in conjunction with the curriculum development and teacher training aspects of the program. In that session, it became clear that research studies could follow either or both of two distinct forms: (1) the teacher training and activity package development phases provide research leads on basic (i.e., not exclusively curriculum developmental) problems, these problems being of a sort requiring the use of subjects outside Nova School proper in order to establish experimental controls; and (2) the curriculum development aspect, as it proceeds, will come to decision points - do we proceed this way, or that? - and research may contribute to the making of the decision. In short, a research phase may be included at the very basic level, or may be restricted to practical problems associated with the curriculum development and teacher training phases, or may include both. Which to do, I see as a decision to be made there, by those who will originate and execute the whole affair.

Similarly, with evaluation, one might develop a potpourri of highly specific, homogeneous research instruments to test for objectives of specific activity packages, or he might develop a few very general, heterogeneous instruments for evaluating the objectives of "the program". I place quotes around "the program" to indicate that the objectives were not, in my opinion, readily identifiable at the conference. Certainly, we do not have standardized tests to measure them, whatever they are. This leads into the next recommendation I would make. No matter whether testing proceeds by evaluation of specific activity packages, or in a more general form, those who construct the tests will need a big headstart to get them prepared in advance of the occasions for their use. I would say that the evaluation activities could not begin any later than the teacher training and curriculum development phases. Whoever constructs the tests - and this must certainly refer to several persons - must be involved with the innovations during the entire course of the teacher training and curriculum development activities.

As a further comment in this regard, some of your evaluation problems might be solved if you propose to view the tests themselves as embodying the objectives of the whole program, and view students' performance on these tests, at some reasonable and well specified level as evidence of the attainment of these objectives. What I am suggesting here is what is referred to in testing (e.g., by Robert Ebel in EPM) as "content standardization", where the score on the test conveys information about contents (or processes) attained. One model for such a test is Guttman's scalogram model. Another, complementary, approach is to test for content mastery, in the sense that successful completion of a learning program may require mastery of all items in the program. The result of either or both of these procedures would be the possibility of saying, "X percent of our students can do this, and another X percent can do that", where "this" and "that" are highly determined.

APPENDIX M

THE ACTIVITY PACKAGE *

(A Preliminary Sketch of a Useful Idea)

By J. Robert Cleary

PREFACE

Although the Third Nova Mathematics Conference yielded several promising avenues of investigation, this participant was more personally involved, more vitally interested, and more energetic in off-hour dialogues with an idea that developed concurrently in two groups during the first day of discussion. This idea was the "activity package". It evolved and is viewed not as a critical component which holds great promise for solutions of the N-dimensional space that is mathematics curriculum research, but rather as a primitive but necessary first step if research is ever to offer partial, if not elegant, solutions to some of the problems of curriculum design in mathematics.

The following is one participant's sincere attempt to communicate the development of the idea and its description. Errors, lack of clarity, etc. are, therefore, limitations in the paper not in the discussions or in the idea itself.

The paper is organized into two main sections. The first presents considerations which led to the "activity package" and is seen as necessary back-ground information highlighting the idea. The second section directs its attention to the "activity package" itself and to its description.

CONSIDERATIONS LEADING TO THE ACTIVITY PACKAGE

Present Problems of Assessment in New Math Efforts

As participants each of us brought to the third conference varied personal experience with some phase of research directly or indirectly related to one or more of the new efforts in mathematics education. Most of us as researchers, psychologists, or whatever (not mathematics scholars) had experienced the frustrations of interdisciplinary attack on problems or the frustrations of interdisciplinary dialogue. But all of us recognized this as natural; it is natural for the vital core of concern to be mathematics for the mathematician, to be behavior for the psychologist. Further we came to Nova to support the mathematicians whom we regard as a great bunch. We believe they have already made major contributions and will make even more. We came to explore how we might be of assistance to the mathematicians in ways they would like, as well as to help others implement in practical school situations what mathematicians have already demonstrated is feasible.

Certainly none of us who has had any experience with the "new mathematics" wants to return to the "good old days" with the lock-step curriculum, the sometimes trivial content, artificial contrivances to make math palatable, teacher-centered methodology, etc. Like so many major transitions or improvements,

* The Authors feel that the following comments by Dr. Cleary merit attention and extensive treatment in the main body of the report. Unfortunately, because of other pressing demands on his time, Dr. Cleary was unable to submit this paper in time for the authors to integrate it with the rest of the report. Suffice it to say that we find little to take issue with and are most happy to have a serious void in the report filled by Appendix M.

however, we give up some things for others we value more highly. In the case of researchers and psychologists you'll forgive some of us for a casual glance backward when in blissful ignorance we had an organization in the curriculum common to most schools to study, where a sequence existed because teachers knew that young children couldn't learn very profound ideas, and when nobody much cared what mathematics really was - it was just taught. Under these conditions even we understood what was going on, and our studies, research designs, etc., were reasonably "clean", as we say, if not very profound. We had several entries for investigations in those days which we must give up. We now search for new ones for the new developments.

But now the validity of Bruner's statement that students can learn anything at any level in an intellectually honest way, if put in their language, has been amply demonstrated with young children by Suppes, Davis, and Page to mention a few. As a result we have nearly inverted some parts of the curriculum. There is no longer a sequence we can use as a "handle" or entry.

The role of content has changed, too. At least in this feasibility stage, or as one mathematician in the first conference called it, the "creative process" stage, content is more arbitrary and is seen as material to use and on which students act. In this sense it is only instrumental not an end in itself. Most of us subscribe to this view of content as means not an end, but again we have lost another "handle". Content in the old days was perhaps our biggest "handle" or entry. In those days even we disagreed with the primacy of content but we didn't complain much, because it gave us a way to study certain kinds of behavior in an organized way - remember, behavior is what we study.

Now that content is more truly means and therefore is less stable over time and across school populations, it is logical for us to ask "means for what?" This question brings us to objectives.

There has been more dialogue between mathematicians and psychologists on this aspect of math than on any other - and more misunderstanding, too. None of us close to one of the new math developments wishes to deter or violate what mathematicians are attempting to do. None of us demands that mathematicians specify everything in behavioral terms. None of us believes that paper and pencil tests can measure most of what is important in new math. None of us really believes that because new math programs lack "criterion behavior" statements that they are "mystical". In short we think we understand. We hope that mathematicians have some faith in us and that with some help from them on objectives we can begin. We may not be successful in the critical areas of the new spirit, but we need to begin where we can be successful and recognize our limitations. If we lose objectives and content, we have lost behavior entirely so far as curriculum study is concerned.

A final problem in assessment is seen by some as the most challenging. This is the new dimension, the new spirit. It is perhaps more affective than cognitive. It is this attitudinal dimension related to intellectual honesty, inductive method, and pupil attack on content which produces the excitement of the frontier for students. We can't complain about losing a handle here, because we never had one. But we need to try for one, because on balance this may well be the critical element; it already is for some programs. Several of us think our discipline is making some progress in this area which holds promise for the future.

The logistics of implementation of curricula demand data. Relevant data are gathered by attention to what students do. We brought to the conference some experience with complications arising from lack of sequence, content, objectives, and an effective dimension. By saying we have lost "handles" (my term) we mean we have lost our grasp of the conditions for adequate and relevant data collection for curriculum research.

The conceptual framework for NCMP - The Curriculum Matrix

In addition to problems which we brought with us, we also discussed more specific ones arising from our review of the material provided by Nova to facilitate our discussions.

For those of us later concerned with the activity package we found our attention was directed almost entirely to the dimension of the curriculum matrix labeled Characteristics of the Learner. This was natural since the other two dimensions are properly mathematical. It is the learner as a person learning mathematics which is of major concern to us.

We discussed each of six characteristics in detail and many more. There were differences of opinion as to what constitutes a learner characteristic. For example some of us viewed three (concrete vs. abstract materials, group size, and game setting vs. exposition) as more characteristic of method than of the learner.

In addition to a "conditions of learning" dimension the matrix could logically be extended by considering a process dimension (divorced from content), an objectives dimension, and an effective dimension.

Finally some of us suggested that attention be given to whether the basis for acceleration, tracking, or other provision for differential rates of learning, should be normative or based on proficiency, and to what extent Nova or any other school is capable of individualizing instruction.

The sense of these discussions clearly supported some broad conclusions which might be stated as follows:

1. Given the inner relationship of relevant variables, the tentative nature of some of the current math development, and the absence of means of data collection, it is quite impossible to plan research over an entire curriculum.
2. A first priority is to improve communication with respect to objectives as well as to other dimensions associated with an extended curriculum matrix. A first effort in communication will result from a meaningful description of what is operating as learners contend with materials under varying conditions or methods for given purposes or with given outcomes (performance broadly conceived).
3. Meaningful descriptions are possible and might be successful only if attempted with smaller bits of experience rather than with a curriculum as a whole.
4. Whatever the size of this manageable bit a "history" of each bit must be gathered as a result of observing how each operates under varying conditions and also how it relates to other bits.

Clearly there was a search for a module, a manageable unit, which had a coherence about it, which could be manipulated and controlled experimentally, and about which a good deal of information could be assembled under various conditions.

This thing alternately called unit, module, bit, etc. came to be known as the "activity package". The next section will attempt to describe it and how it might be used.

THE ACTIVITY PACKAGE AND ITS USE

A First Attempt at Description

What we came to call the "activity package" is essentially a body of mathematical content and experiences organized by mathematicians having a unity and coherence which make the elements of the package more like and more related with each other than with other elements outside the package. Like pipe and slippers they seem to go together. In fact they were designed to do so.

Activity packages are much more variable in length than more traditional ways of organizing learning experiences. Quite independent of each other and varying in length, they are not "modules" which are generally understood to be relatively equal in size and when joined with others form an integrated whole. Unlike modules, activity packages initially are discrete and do not necessarily join each other in pre-assigned ways to form a larger whole. Some of them may, but this would be established by investigation.

Likewise activity packages differ from the more traditional "units" generally found in elementary school organization of curricula. Units are relatively long (three to six weeks) forms of organization usually with fairly stereotyped forms of organization, with many side excursions, and with attempts to relate various subject fields. In addition to differing from units in time, activity packages are more flexible in organization. They also differ from units by virtue of their mathematical unity not their interdisciplinary character.

Actually an activity package could be as short as one episode (a mathematics game) or perhaps as long as the equivalent of a traditional unit on enumeration systems. Other examples of raw materials for activity packages might be a Madison Project "shoe-box", a week of the "equations game", use of cuisenaire rods to illustrate communitive properties, and so on.

But an activity package is seen as having more characteristics than homogeneous elements and built-in conditions of learning. It has, in addition to these, mathematical or psychological prerequisites or both, and one or more mathematical outcomes or effects directly related to the purpose for which the package was designed.

The activity package, then, consists of the input, the conditions of learning, and the output considered together as in the diagram below.

THE ACTIVITY PACKAGE

Input	Conditions of Learning	Output
Mathematical and/or psychological prerequisites	Organization of the learning experience	Mathematical outcomes (what student can now do)
Examples	Examples	Examples
can read printed directions	mathematical content	sees relationships
uses mathematical symbols	materials used	discovers patterns
performs fundamental operations of arithmetic	what students do	organizes data
follows symbolic representation during verbal exposition	what teacher does	solves equations (including affective outcomes)

The items in the diagram on the previous page are isolated illustrations of the kinds of statements which could be made about an activity package. In a description of a specific activity package the aim is to include only the critical elements of the package, and to strike the proper balance between vague generality on the one hand and unnecessary detail on the other. As you will note, the out-put dimension resembles criterion performance referred to before. This is presented not to oppress mathematicians but to encourage them to indicate what they see which satisfies them that they can go on to something else.

Attention to the critical elements of an activity package, a kind of task analysis as one participant put it, should yeild meaningful description and give a kind of stability to the package.

The effect of this approach is that the extended curriculum matrix is viewed somewhat differently than before. The relevant variables are seen as falling into two types: those within the activity package which are better known and more stable and those outside the package about which less is known. The relative stability of the package allows it to be placed in various experimental conditions so that questions may be asked of it. The answers constitute some of the other dimensions of the curriculum matrix.

Given information within an activity package including, of course, the prerequisite input, the following are illustrations of the kinds of questions which might be asked: Does this activity package work better with students who learn faster, or is it better for those who have learned more slowly in the past? Is this package too difficult for students with less verbal ability? What is the earliest level at which this package can be used successfully? Of two packages produced essentially for the same results, does the one using concrete objects work better than the one using exposition and abstraction for this group of students? Which combination of previous packages seems most efficient for success on this package just developed? Do students at earlier levels (younger) take longer to complete this package successfully than students exposed to it at later levels?

What is implied above is somewhat analogous to a possible situation in medicine in which the ingredients of a new pill are known to make positive contributions toward an eventual cure of a disease. Under experimental conditions various questions are asked about it - questions related to upper and lower limits of dosage, frequency of use, the earliest age recommended for use, effects when combined with other medicine taken by the patient, the study of side effects, etc. Thus, a medical "history" of the pill is gathered.

Histories of activity packages may be gathered likewise and perhaps catalogued, one participant suggested, with respect to their effects (outcomes) as illustrated below with the topic of <u>Proof</u>. Various activity packages are designated as AP1, AP2, etc., and the +, - or 0 indicated whether the package contributes positively, negatively, or makes no contribution to a specific outcome.

PROOF

Outcomes	AP1	AP2	AP3
(expressed in terms of what students do)			
Demands necessity of proof	+	+	
Demonstrates nature of proof	+	-	
Exhibits alternate strategies of proof	+	0	
etc. ...			

This section has attempted to describe the activity package by contrasting it with other forms of organization of curricula, by listing its additional elements, by pointing out the interrelationships of its elements, and by suggesting how it might be used. The last part of this paper discusses briefly some advantages we see in such a conceptualization.

Possible Advantages for Curriculum Study

Participants in the third conference naturally see advantages to an idea they developed. We think this approach will give us some new handles to replace those we have lost. The way may also open for retention and transfer studies under better controlled conditions and geared to mathematics curriculum rather than to nonsense syllables and other stimulus material unrelated to mathematics.

The mathematicians should see some advantages as well. For one thing the activity package approach allows the flexibility needed during curriculum development. An activity package investigation does not demand an entire curriculum to become rigid in order to study it. There are also self-adjusting features in activity packages which allow for changes in the minds of those who developed them. But here the basis for needed change would be formal information gathered by trial and should be more useful if gathered in this fashion. Finally the activity package is consistent with the manner in which many mathematicians develop the materials. They decide to treat some topic for a reason, they develop materials and techniques of presentation, they field test the ideas with some students, they revise as necessary, and when satisfied with the results, they incorporate these experiences into some more permanent form for wider use.

Perhaps the greatest advantage of all to the idea is that it faces reality. Curriculum design and implementation is a many-splendored thing. It is in great part mathematical, but it is also psychological and logistical (administrative). The activity package and its use brings these dimensions together as they really are, but in a size which can be handled. The reality of the curriculum situation recognizes different responsibilities. Matters mathematical in the activity package are, of course, the province of the mathematicians. Some others are the province of psychologists, or curriculum people.

But who knows about sequence, for example. Following are three consecutive statements from three different participants in the first conference:

> "... much of the research in this area indicates that sequencing of materials might be much more important than the way in which you present them, and I think we need to find a way to determine the best sequences for various types of students."

> "I think that today alot of the troubles we find ourselves in stem from our overconcern with sequence."

"I would say that perhaps each learner has his own sequence."

Obviously sequence is a matter for investigation. The inner relationships of a good number of activity packages might be one way to proceed. Like sequences, there are several vital questions from the point of view of design for which mathematicians, psychologists, or anyone else have no answers. The activity package approach calls for a concerted effort of several interested parties working together. This is not the first time that such a call has been made, but it seems to us that it points a way in which the task can begin. Now is the time for us to document for others what intuitively we know is working well. We think this is the time for all of us to take an active part to help those in practical school situations to make a direct attack on the logistics of implementation. Most participants in the third conference as in previous ones see Nova as the best existing field laboratory in which to begin the attack.

It is only proper that we end this rough sketch of one of the ideas with a vote of confidence for the school organization which made it possible.

APPENDIX N

ACADEMIC GAMES

By Robert W. Allen

During the past few years a new approach to instruction has been developed which makes systematic use of academic games in both the instructi .al and extra-curricular phases of the school program. Most games used in this approach fall into two general categories, programmed games and simulation (role playing) games. Each contains the features (fun and competition) which make games enjoyable, but differs from the so-called "educational" games (Scrabble, Monopoly) in that academic games are subject matter specific. In this new approach, academic games are being developed around selected units or courses of study and "packaged" as specific learning activities, whereas most games with educational implications are difficult to relate to any organized conceptual sequence. Most academic games are a sequence of sub-games, with each sub-game being of increasing difficulty and introducing new concepts and skills.

Growing interest in this approach to learning has been expressed by educators, psychologists, and sociologists around the country. Academic games projects are currently being conducted at Johns Hopkins, Yale, University of Chicago, Kansas State Teachers College, Michigan, North Carolina, Wayne State, Rutgers, Cornell, Northwestern University, The Western Behavioral Sciences Institute, and the South Florida Education Center in Fort Lauderdale, Florida. Much of this interest has been generated by recommendations made by Coleman in his book, The Adolescent Society.[9] Some of Coleman's specific recommendations will be treated later in this paper in a description of the NOVA Academic Games Project.

Experience in the use of academic games suggests that they offer a number of benefits. Among these are:

1. widespread student participation, even in large classes
2. student motivation is strong
3. material is handled in small steps, with immediate feed-back as in programmed instruction, but with the added advantage that students interact with each other rather than with machines or manuals
4. students frequently discover ideas themselves
5. students are given a concrete reinforcement (participation in the game activities) for learning a subject that may be more convincing than the injunction that "This will be useful for you when you grow up."

6. students' understanding of, retention of, and attitude toward the material appears to improve
7. mistakes can be corrected by peers
8. materials can be adapted rather easily to individual differences between students
9. the games require physical as well as mental manipulations
10. the routine work required of the teacher is decreased
11. the use of these games is relatively low in cost

Major emphasis in this paper is given to the NOVA Academic Games Project. In addition, there is some discussion of the supplementary use of two academic games - WFF'N PROOF and EQUATIONS - in classroom instruction and as tools for conducting an academic intramural program, and a brief presentation of possible contributions of academic games to the Nova Comprehensive Mathematics Project.

NOVA ACADEMIC GAMES PROJECT

The NOVA Academic Games Project is a curriculum development and research project based on the assumption that achievement can be improved by altering the structure of values and rewards evidenced in many schools. For example, the high school athlete generally is accorded recognition and prestige by his peer group whereas the scholar is often less "acceptable" in adolescent society. It is this value perspective the NOVA Academic Games Project is designed to restructure.

SCOPE

The primary objective of the project is to develop a program which will be of significant value in helping to meet the educational needs of:

1. the student classified as non-motivated, under-achiever, or less capable
2. the gifted, accelerated, or advanced student
3. the student - less capable, advanced or regular - who has formed negative attitudes about a given subject area.

The program will be initiated at the classroom level by integrating all subject areas with academic games from kindergarten to twelfth grade. It is thought that these activities should begin in the classroom so that classroom achievement is identified with general scholastic achievement. This provides a broad base of student participation necessary to insure against the isolation of academic games as an activity requiring a special talent (e.g., debate, drama, music, journalism, etc.) or as a pastime for "brains" and "eggheads" only.[9] One of the primary reasons the athletics program is so successful in schools is that almost every youngster participates in these activities at some level and therefore readily identifies with the athletic program conducted at the interscholastic level.

After familiarity with the games has been achieved in the classroom, the academic games program will then proceed through three additional stages:

1. Intramural competition
2. Interscholastic competition
3. the NOVA Academic Olympics

School and community recognition of student participation in academic games will be provided to encourage and reward achievement in the intellectual arena.

INSTRUCTIONAL METHODS

The instructional methods to be employed for classroom use, teacher training and wide-scale dissemination are:

1. the development of a series of television tapes which will be used for classroom instruction and also a separate series to be used for teacher training purposes
2. the development of a series of slides accompanied by audio explanations on records for classroom instruction and teacher training
3. the conducting of in-service programs for teachers by the Academic Games Administrator in Broward County and in selected school systems throughout the country
4. the local telecasting of interscholastic competition in academic games
5. an invitational academic games tournament in which teams from various school systems in the state and nation compete at NOVA.

MATERIALS

1. Simulation (role assuming) games
   a) The Great Game of Legislature
   b) International Relations
   c) The Career Game
2. Programmed Games
   a) EQUATIONS, The Game of Creative Mathematics
   b) SET THEORY I.
   c) WFF'N PROOF, The Game of Modern Logic

NOTE: Other games still in the developmental stage, such as The Presidential Role Game, Euro-Card, and Propaganda will be added as they are made available.

NEED

Three studies are especially pertinent to the need for an academic games program. The first is Coleman's The Adolescent Society [9]. Along with many other provocative ideas, Coleman suggests that competitive games can be used, not only as a new mode for learning, but also as a means for positive reconstruction of the present system of adolescent values and peer rewards within the schools. Coleman points out further:

> "There is a failure to recognize that the fundamental competition among children, adults, or anyone, is a competition for respect and recognition from others around them. In different systems, different achievements will bring this respect and recognition. The removal of scholastic achievement as a basis of comparison does not lessen the amount of competition among adolescents; it only shifts the arena from academic matters to non-academic ones..."

A second study by Tannenhause [20] considers the structure of values and rewards evidenced in schools today. He asked students in a large high school in a middle class neighborhood of New York City to rank eight imaginary characters. These range from the brilliant non-studious athlete to the brilliant studious non-athlete. In terms of peer acceptability, the top four choices were athletes. Within both athlete and non-athlete groups, non-studious types were deemed more acceptable than studious types.

A third study was undertaken in 1964 by the Burbank Unified School District, Burbank, California [7] to determine the effects on students of specific games used in the classroom. Two games were used in this research, a game of modern logic (two value, propositional logic) called WFF'N PROOF and a game of basic mathematics called EQUATIONS. After three weeks of intensive exposure to the WFF'N PROOF materials an experimental group of 43 junior and senior high school students achieved an average increase in non-language I.Q. score of 20.9 points - significantly greater than the 6.6 point increase achieved by the control group. After four months of instruction using the EQUATIONS Game, a basic math group consisting of 84 ninth grade students had an average increase of 1.3 years in arithmetic reasoning - again significantly greater than the .6 year increase achieved by the control group.

EVALUATION

Initial research in the use of academic games will attempt to determine the ability level required by the games in their most rudimentary and their most complex forms, as well as the extent of motivational and learning involvment occurring among the students of high, average and low scholastic ability.

A second phase of research is planned to determine the contributions of academic games to the total educational program. In this second phase of the research program, the following hypotheses will be tested:

1. students who participate in academic games built around the key concepts of the course curriculum will have greater long term retention of these concepts than students who do not participate in the games.
2. students in experimental classes will evidence an improvement in attitudes toward the course following the introduction of the games.
3. students who excel in the academic games, especially those who are selected to participate in intramural and interscholastic competition, will increase in peer acceptance and show an accompanying increase in self-acceptance.

CURRENT USE OF ACADEMIC GAMES IN THE NOVA MATH PROGRAM

Two games - WFF'N PROOF (a game involving propositional calculus) and EQUATIONS (a game involving elementary mathematical concepts) - are currently being used as part of the mathematics program at NOVA. Every student, grades 1 - 12, has played these games at some level as part of his classroom instruction. An intramural program has been initiated at grades 7 - 12, and applications for the program far exceed the number of openings. Fifteen leagues, consisting of 378 students, are involved in this year's program. The winners from each league will compete in an all-school tournament at the end of April. Those emerging victorious will represent NOVA in the NOVA ACADEMIC OLYMPICS to be held May 9 - 14, 1966.

Excerpts from the Academic Games Newsletter which follow will illustrate the nature of intramural competition in Academic Games.

ACADEMIC GAMES NEWSLETTER January 7, 1966

The NOVA Academic Games Intramural Program got off to a fast start this week. Forty-two teams representing five classes have been formed, and 126 students are involved in the competition. These teams will compete until February 4th when five new leagues will be formed. This second group will compete until March 18th. A third group will then compete until April 22. At this time all of the winners representing each league will compete in an all-school tournament, and those emerging victorious will represent NOVA in the NOVA ACADEMIC OLYMPICS to be held May 9 - 14, 1966.

* * * * * * * * * *

Michigan got off to a fast start in the BIG TEN CONFERENCE by winning all of their games this week. Dick Burke was chosen PLAYER OF THE WEEK by leading his team in games won and points scored; he also led the Professors Division in point spread. Other outstanding players included Richard Prete of Michigan State, Judy Houghton of Indiana and Guy Pietrobono of Michigan State.

In the KARMOS CONFERENCE there was also a tie for the team leader. The Karmos Koolies and the Empty Set forged ahead with identical 5 - 1 records. Because the Koolies earned a point spread of 13 points to the Empties 11, the Koolies were awarded first place at the end of the first week's activity. Mark Rosenstein of the Koolies was named PLAYER OF THE WEEK when he scored 7 points and had a point spread over his opponents of 8 points. Pete Suni of the Oxymorons; Leslie Hill, Empty Set; and Pat Novak, Koolies, were others who achieved honors.

* * * * * * * * * *

Anyone not completing his games during the past week due to absenteeism should do so as quickly as possible. This will enhance your team's opportunities for advanced standing and your opportunities for success in the individual competition. Be sure to indicate on the score sheet which game you are making up, otherwise it will entail a great deal of research for the Academic Games Department to determine into which slots the statistics should fit.

Don't be discouraged if your team has gotten off to a slow start. Remember each team plays two matches per week, and each match involves three games. It is conceivable that a last place team could overtake a first place team in one match. For example, in the Ivy League, Cornell plays Pennsylvania this Tuesday. Pennsylvania is in first place with four wins and no losses. Cornell is in last place with one win and three losses. If Cornell wins all three games in Tuesday's match, each team will have an identical record of four wins and three losses. Cornell's team leader, Dan Oberlin, should work very closely with his teammates prior to this match to increase the possibility of this outcome.

CONGRATULATIONS TO LAST WEEK'S LEADERS !!! Practice and YOU can be among this week's leaders !!!

(Statistics Chart for second week of play in one of the five leagues)

SOUTHWEST LEAGUE — Week ending 1-14-66

TEAM STANDINGS

Name	W	L	T	Pct	Points	Points Against	Spread
Texas Tech	6	1	4	.857	16	11	5
Arkansas	6	2	1	.750	12	-7	19
Houston	4	4	2	.500	11	14	-3
S. M. U.	5	6	1	.455	4	11	-7
Rice	4	5	0	.444	0	2	-2
Baylor	2	5	2	.286	9	14	-5
Texas A & M	2	6	0	.250	7	14	-7

INDIVIDUAL STATISTICS

Point Spread Leaders (by divisions)

Professors	Associate Professors	Assistant Professors
15 Wheeler-Arkansas	7 Foster - Arkansas	3 Berger - Texas Tech
5 Powell - Baylor	4 Katz - Rice	1 Slusher - Baylor
	4 Williams - Houston	

WON-LOST Percentage (by league)

Name	Team	W	L	T	Pct
Jim Wheeler	Arkansas	3	0	0	1.000
Jim Murphy	Independent	3	0	0	1.000
Gary Foster	Arkansas	2	0	1	1.000
George Berger	Texas Tech	2	0	1	1.000
Tina Osterman	Texas Tech	2	0	2	1.000
Janette Williams	Houston	1	0	2	1.000
Hal Katz	Rice	2	1	0	.667

LEADING SCORERS (by league)

Jim Wheeler	Arkansas	9
Jim Murphy	Independent	8
Tina Osterman	Texas Tech	8
Karen Powell	Baylor	6
Janette Williams	Houston	6
Linda Thompson	Houston	5
George Berger	Texas Tech	4
Debbie Slusher	Baylor	4
Kathy Ferris	S.M.U.	4
Roger Varian	Texas Tech	4

PLAYER OF THE WEEK - - - - - JIM WHEELER — W 2 L 0; Points 7; Spread 11

It is felt, in terms of methodology, that some part of the curriculum can be presented better through a series of academic games which can be embodied in activity packages. These should be planned and developed as an integral part of the total curriculum rather than imposed as a fait accompli. Such games-centered activity packages probably should be made available at all levels and constitute some part of each student's mathematics program. The extent to which the academic games are used will, of course, depend on each student's individual needs and inclinations.

Maximum opportunities for discovery obviously should be provided for in NCMP, yet it is difficult to provide for discovery through most modes of instruction. One of the basic limitations of most textbooks, for example, is that they are too rigidly bound to dissemination of information. The text usually explains a concept and then sets forth a series of problems which enables the student to rehearse that concept. By contrast, experience gained in the use of academic games in Burbank, California, mentioned above indicates that the games approach is highly conducive to discovery in classroom learning. In this instance the teachers used the games to encourage discovery by allowing the students to create their own problems involving the concepts. Thus, the role of the teacher became that of (1) designing experiences that encourage the student to explore for himself and (2) creating an environment which will motivate a student to test and apply his discoveries in many new situations.

In brief, there are certain advantages of the academic games approach which are of value to NCMP:

1. games provide a greater opportunity to individualize instruction on a group basis
2. students can discover ideas for themselves under the direction and scrutiny of the teacher
3. games provide immediate intrinsic reinforcement as well as long-term social reinforcement for learning mathematical concepts and ideas
4. more flexibility is added to the curriculum by an additional mode of instruction
5. the attitude of the learner toward the subject matter is greatly improved.

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